CS 301: Recursion

The Art of Self Reference

Tyler Caraza-Harter

Goal: use self-reference is a meaningful way

Hofstadter's Law: *"It always takes longer than you expect, even when you take into account* ***Hofstadter's Law****."*

(From Gödel, Escher, Bach)

good advice for CS 301 assignments!

https://en.wikipedia.org/wiki/Circular_definition

Goal: use self-reference is a meaningful way

***Hofstadter's Law**: "It always takes longer than you expect, even when you take into account **Hofstadter's Law**."*

(From Gödel, Escher, Bach)

***mountain**: "a landmass that projects conspicuously above its surroundings and is higher than a **hill**"*

***hill**: "a usually rounded natural elevation of land lower than a **mountain**"*

(Example of **unhelpful** self reference from Merriam-Webster dictionary)

https://en.wikipedia.org/wiki/Circular_definition

Overview: Learning Objectives

Recursive information

- What is a **recursive definition/structure**?
- What is a **base case**?

Recursive code

- What is **recursive code**?
- Why write recursive code?
- Where do computers keep local variables for recursive calls?
- What happens to programs with **infinite recursion**?

Read: Think Python 5
(“Recursion” through “Infinite Recursion”)

Read: Think Python 6
(“More Recursion” through end)

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What is Recursion?

Recursive definitions contain the term in the body

- Dictionaries, mathematical definitions, etc

A number x is a positive even number if:

- x is 2
  OR
- x equals another positive even number plus two

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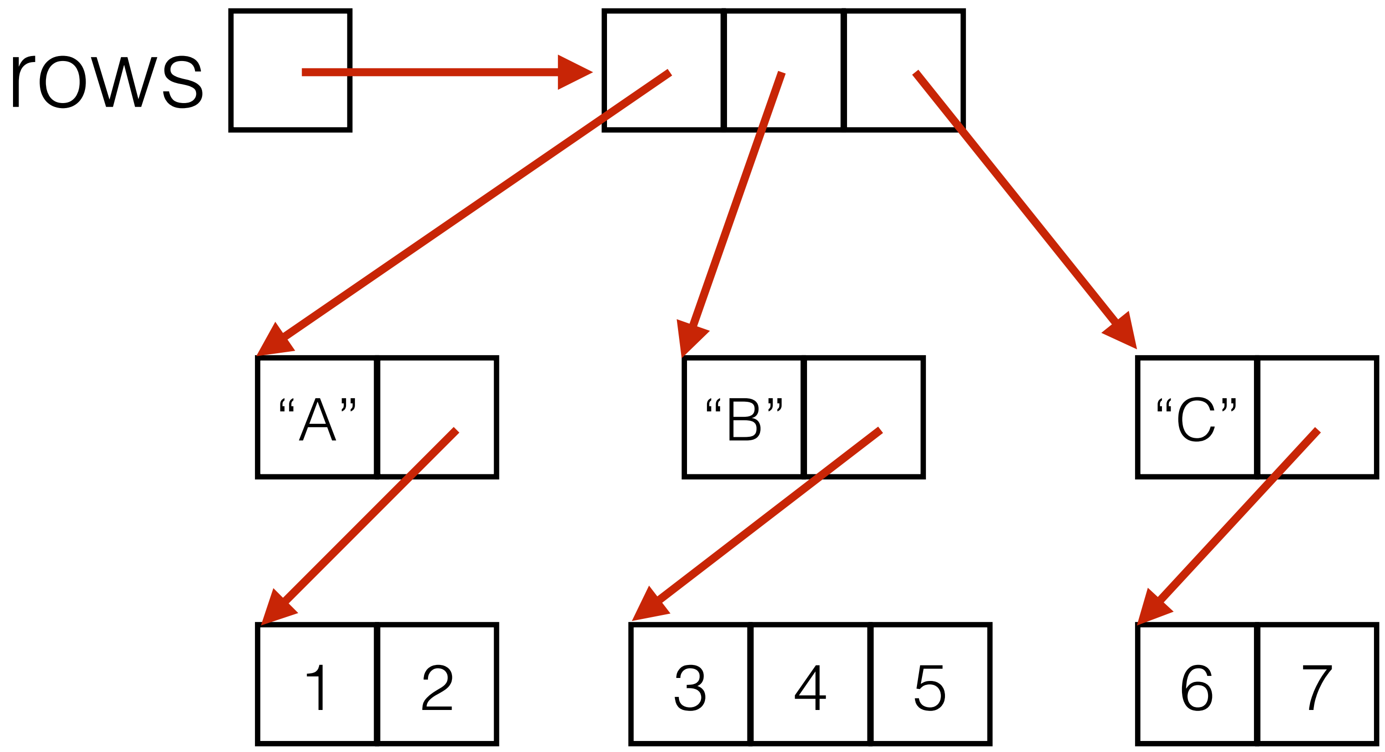
Recursive definitions contain the term in the body

- Dictionaries, mathematical definitions, etc

Recursive structures may refer to structures of the same type

- data structures or real-world structures

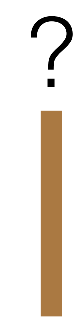
```
rows = [
  ["A",[1,2]],
  ["B",[3,4,5]],
  ["C",[6,7]]
]
```



Example: Trees (Finite Recursion)

Term: branch

Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf



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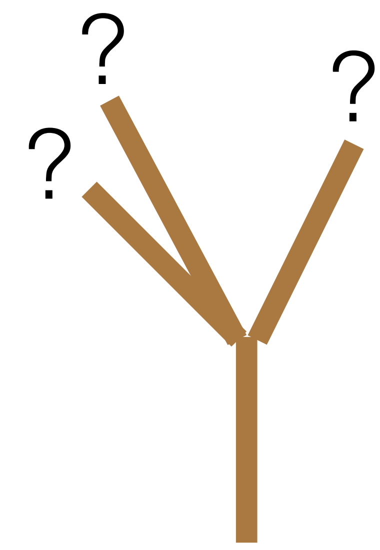

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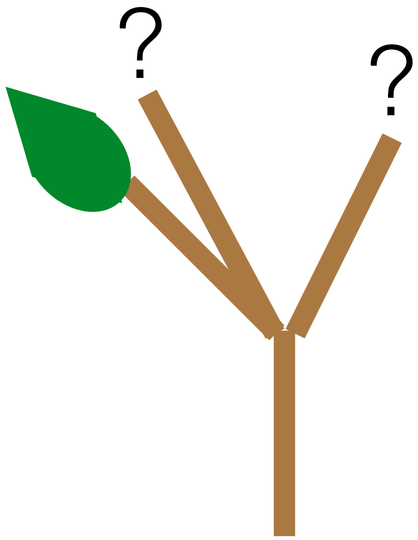

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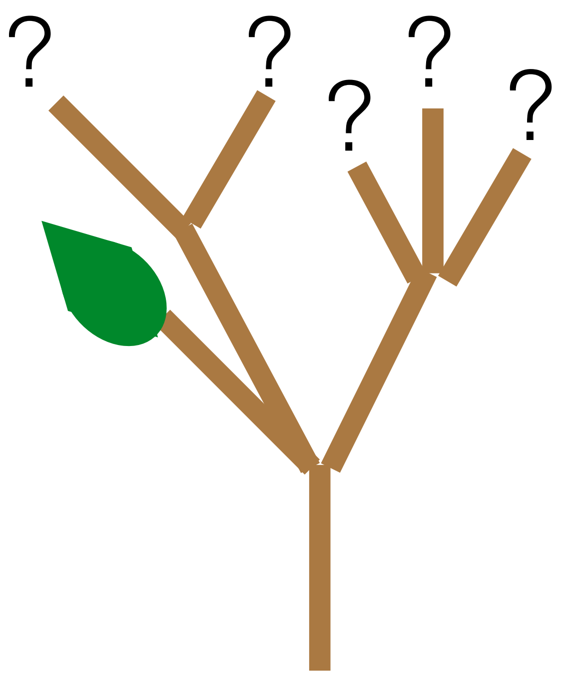

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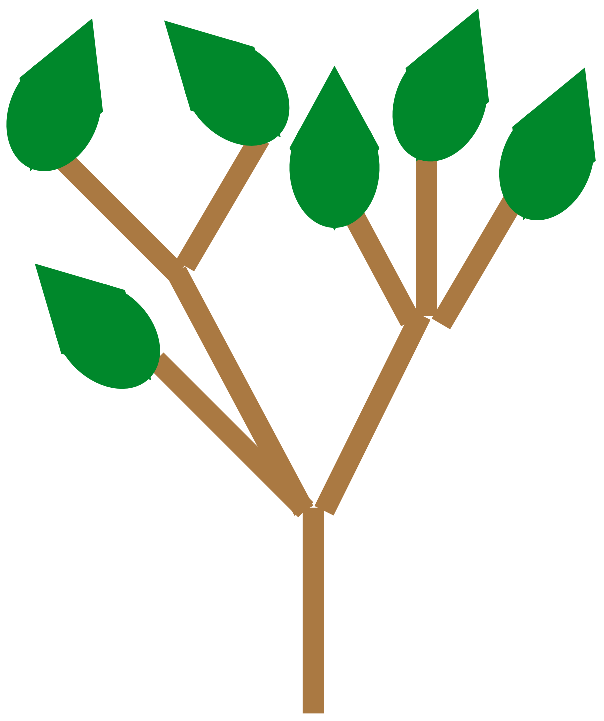

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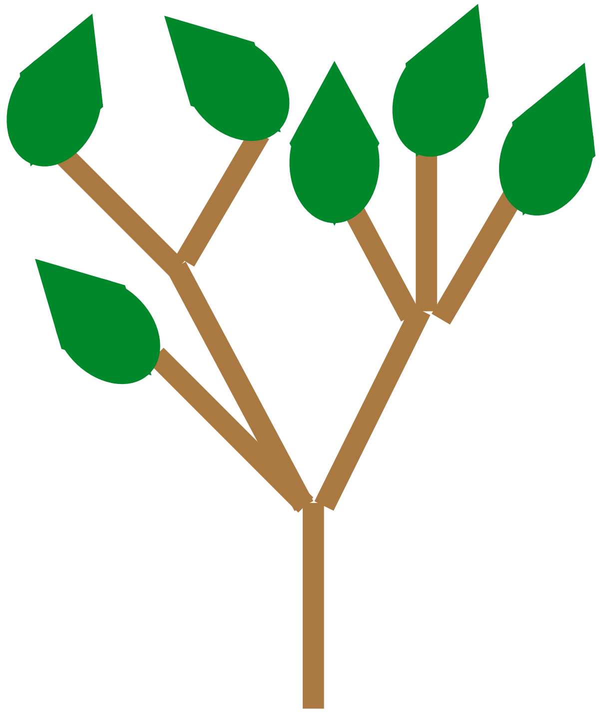
trees are arbitrarily large:
recursive case allows
indefinite growth



Example: Trees (Finite Recursion)

Term: branch

Def: wooden stick, with an end
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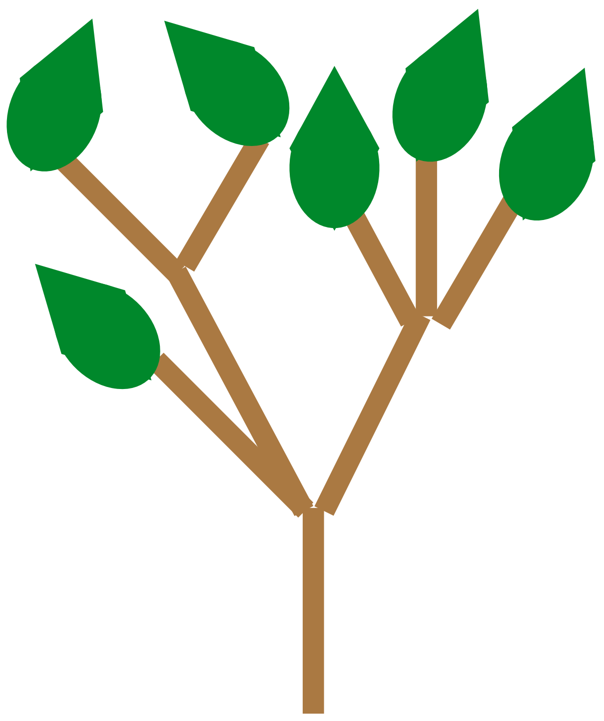
trees are finite:
eventual **base case**
allows completion

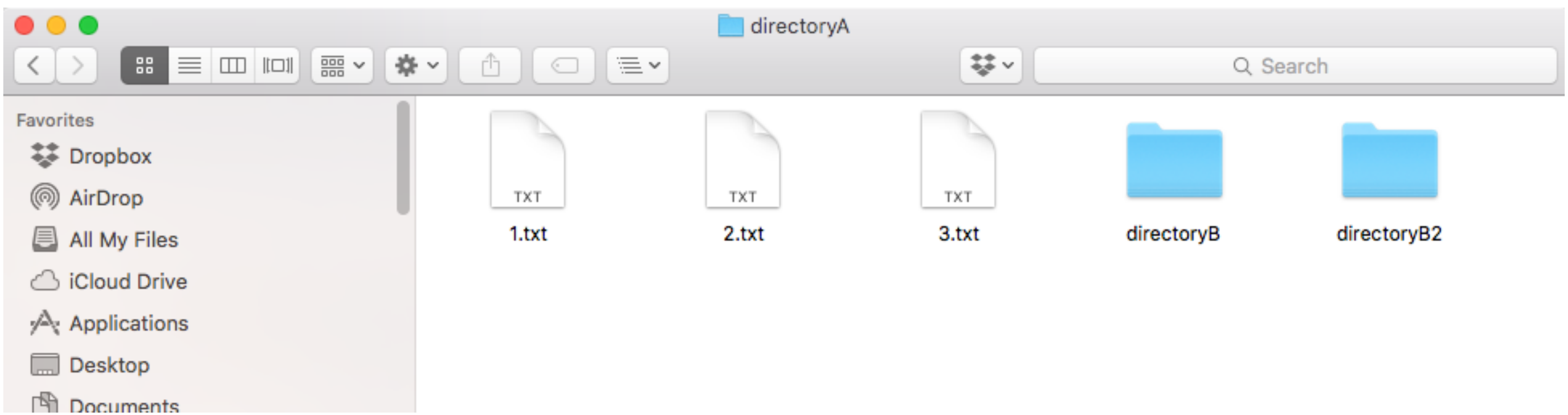
trees are arbitrarily large:
recursive case allows
indefinite growth



Example: Directories (aka folders)

Term: directory

Def: a collection of files and directories



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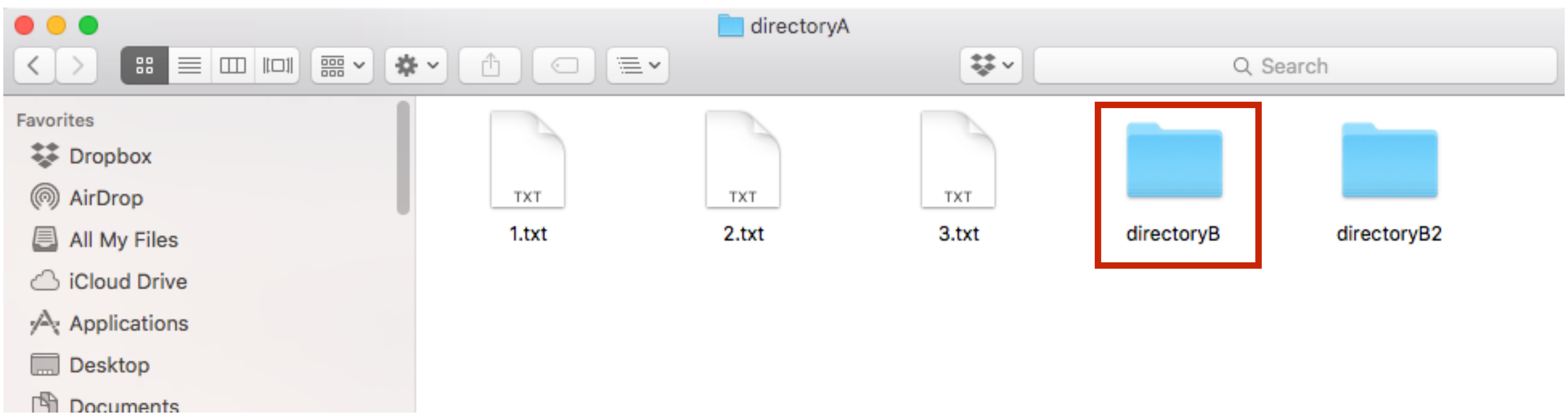

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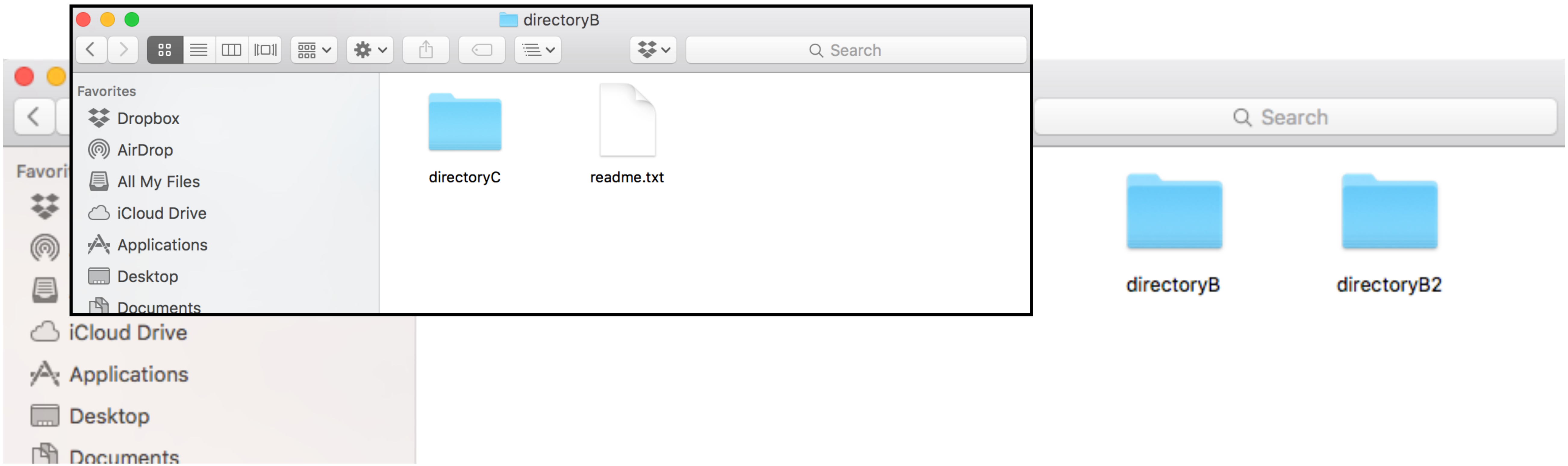

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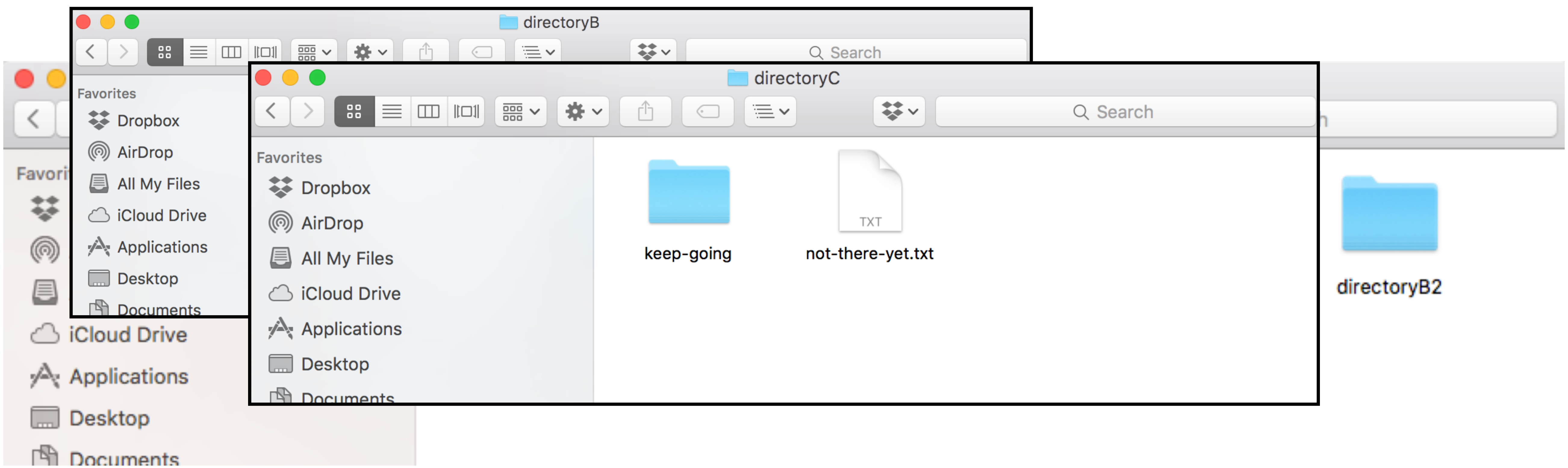

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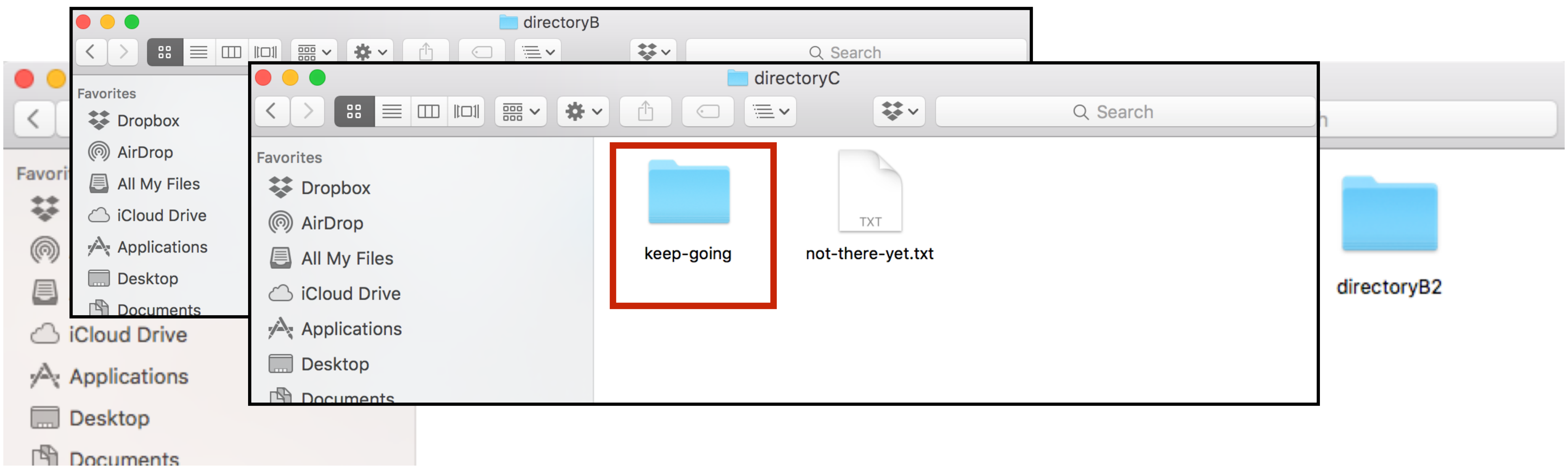

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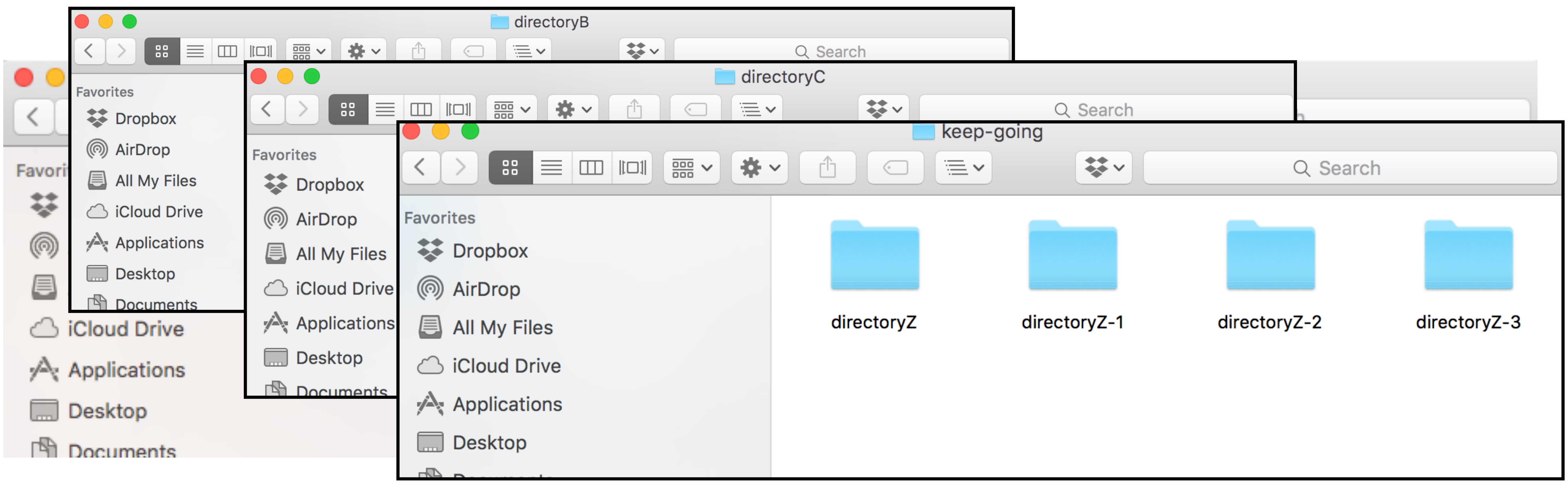

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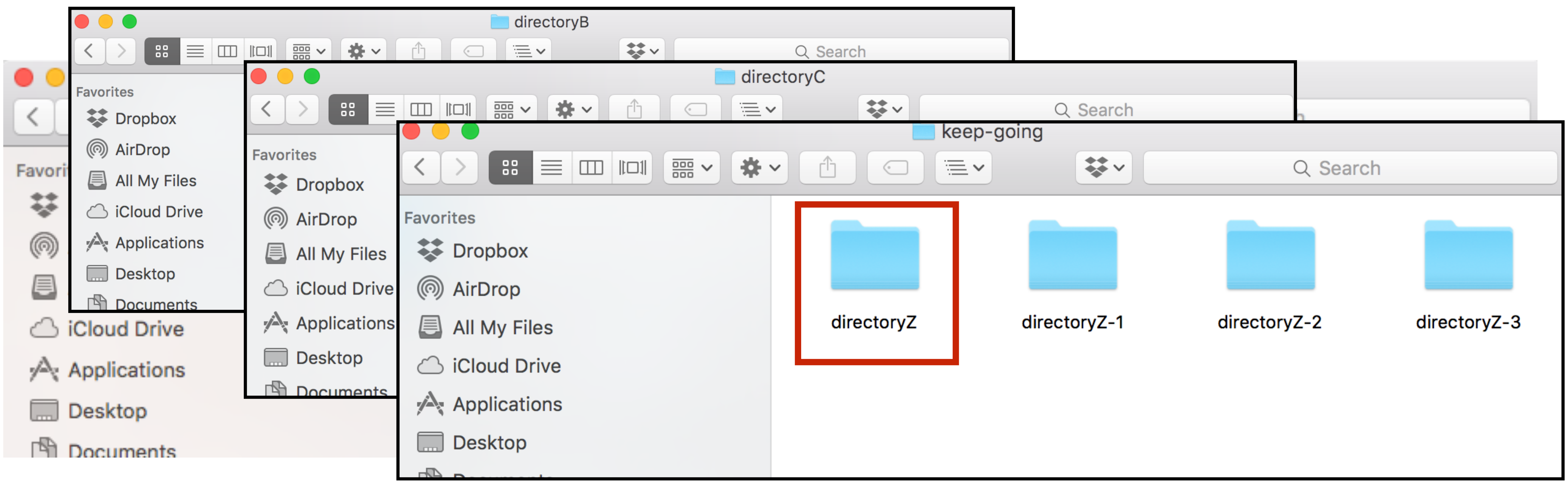

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Term: directory

recursive because def contains term

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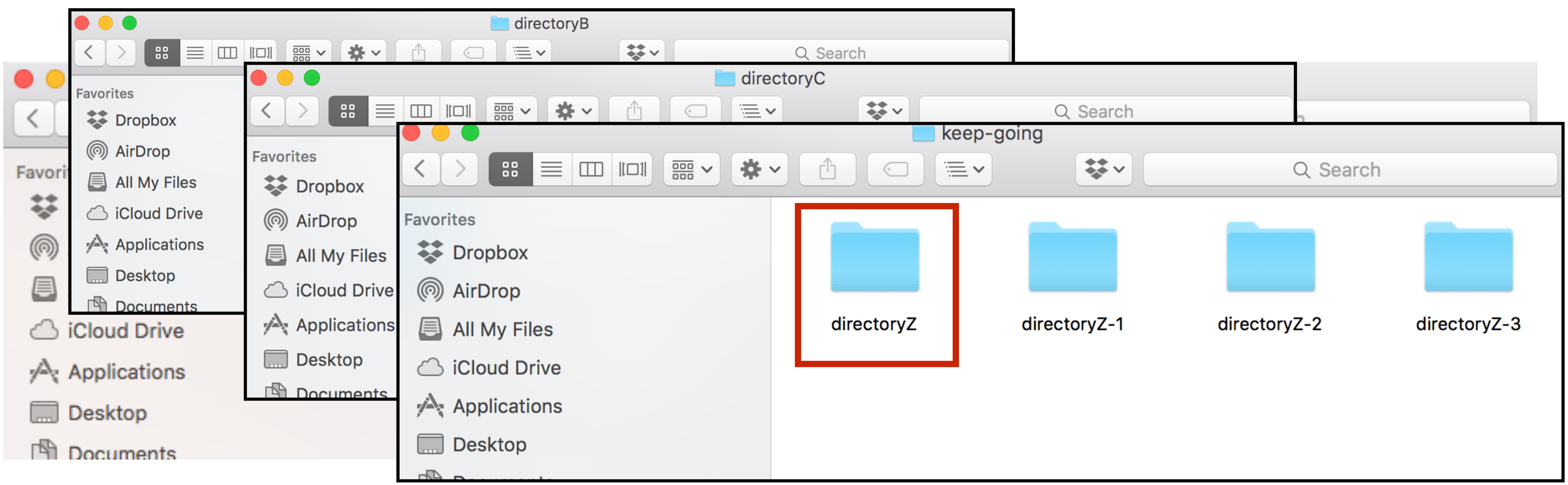

Example: JSON Format

Example JSON Dictionary:

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{
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keys values

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note: complete JSON is slightly more flexible

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recursive self reference isn't always direct!

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Example: JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96,
 “exams”: {
   “midterm”: 94,
   “final”: 98
 }
}
```

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 “name”: “alice”,
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 “exams”: {
   “midterm”: {“points”:94,
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             “total”: 100}
 }
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Recursive Code

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- A function that calls itself (possible indirectly)

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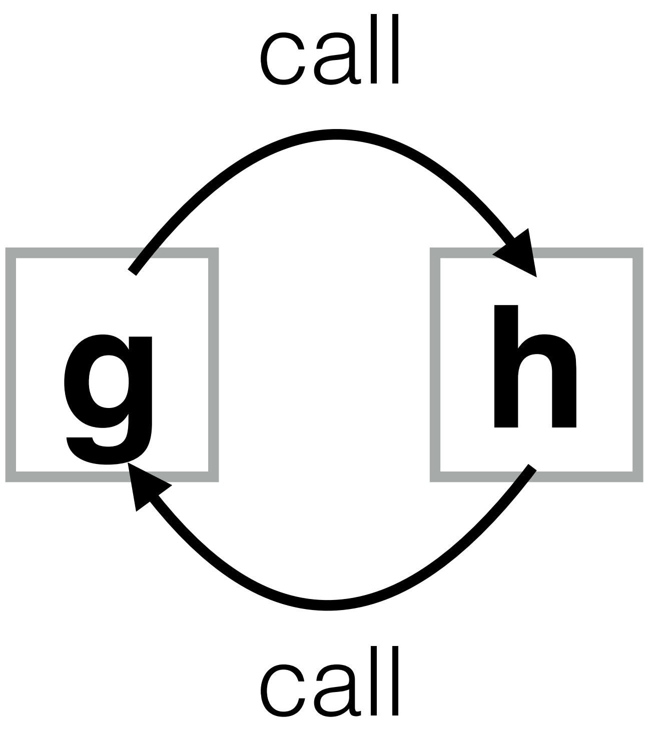
```python
def f():
    # other code
    f()
    # other code
```



Recursive Code

What is it?

- A function that calls itself (possible indirectly)

```
def f():
    # other code
    f()
    # other code
```

```
def g():
    # other code
    h()
    # other code

def h():
    # other code
    g()
    # other code
```

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What is it?

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Motivation: don’t know how big data is before execution

- Need either **iteration** or **recursion**
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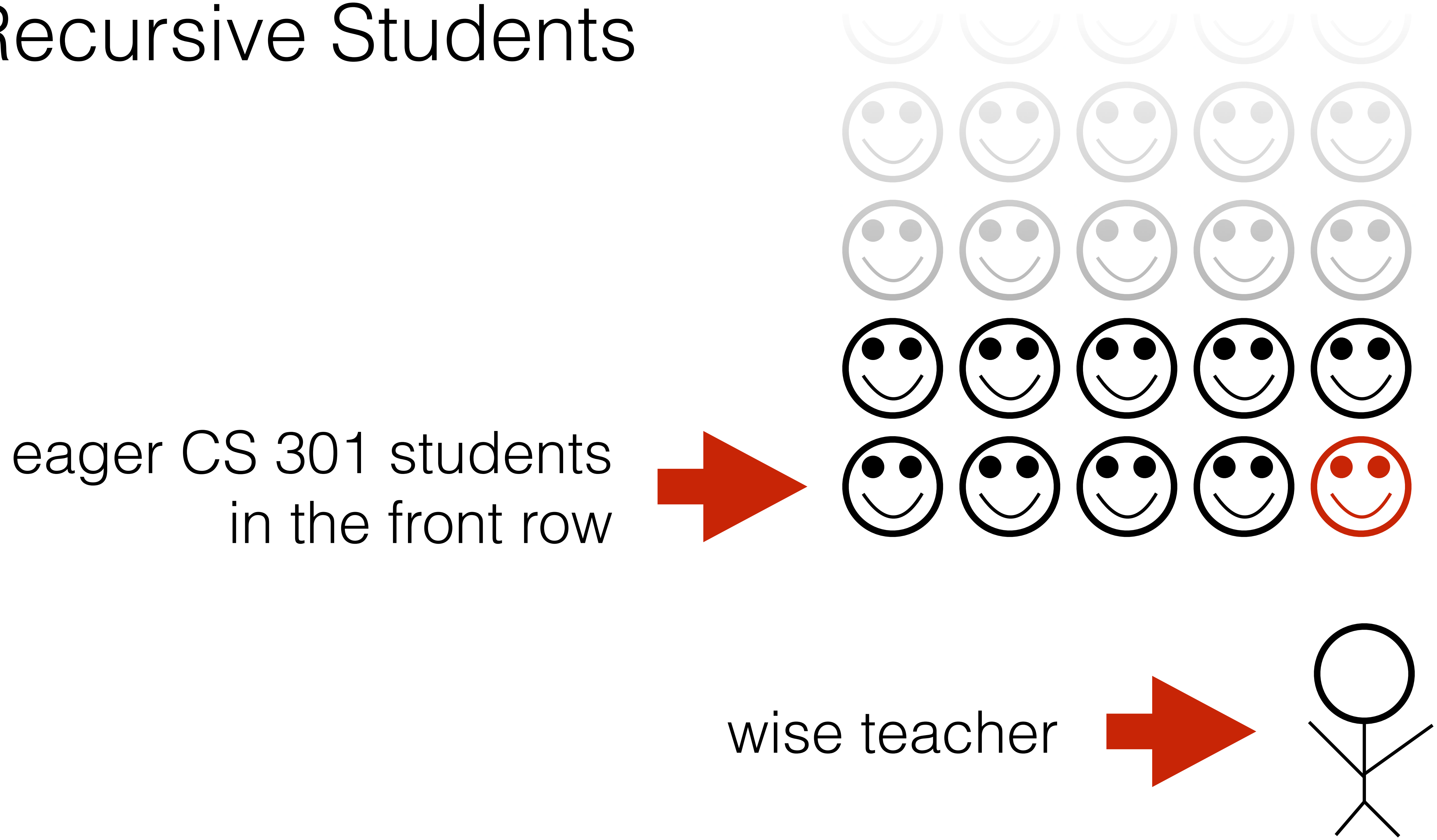
Why recurse? (instead of always iterating)

- recursive code corresponds to recursive data
- reduce a big problem into a smaller problem

Recursive Students

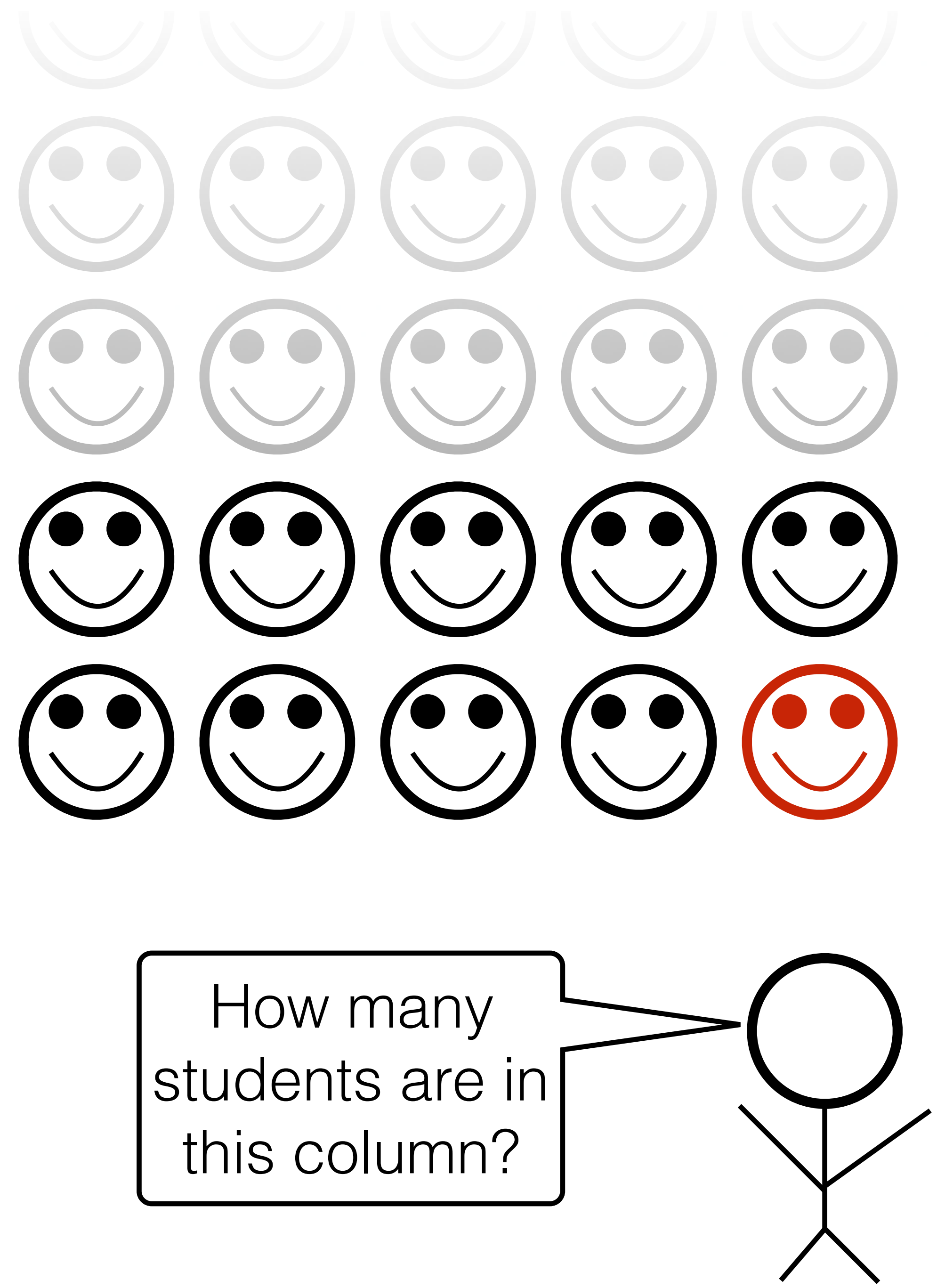


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Imagine:

A teacher wants to know how many students are in a column.
How can the students answer?



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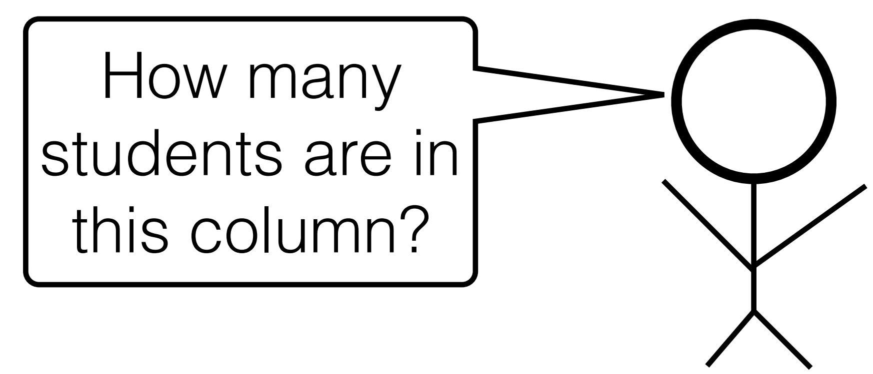
Recursive Students

Imagine:

A teacher wants to know how many students are in a column. **How can the students answer?**

Constraints:

- It is dark, you **can't** see the back
- You **can't** get up to count
- You **may** talk to adjacent students



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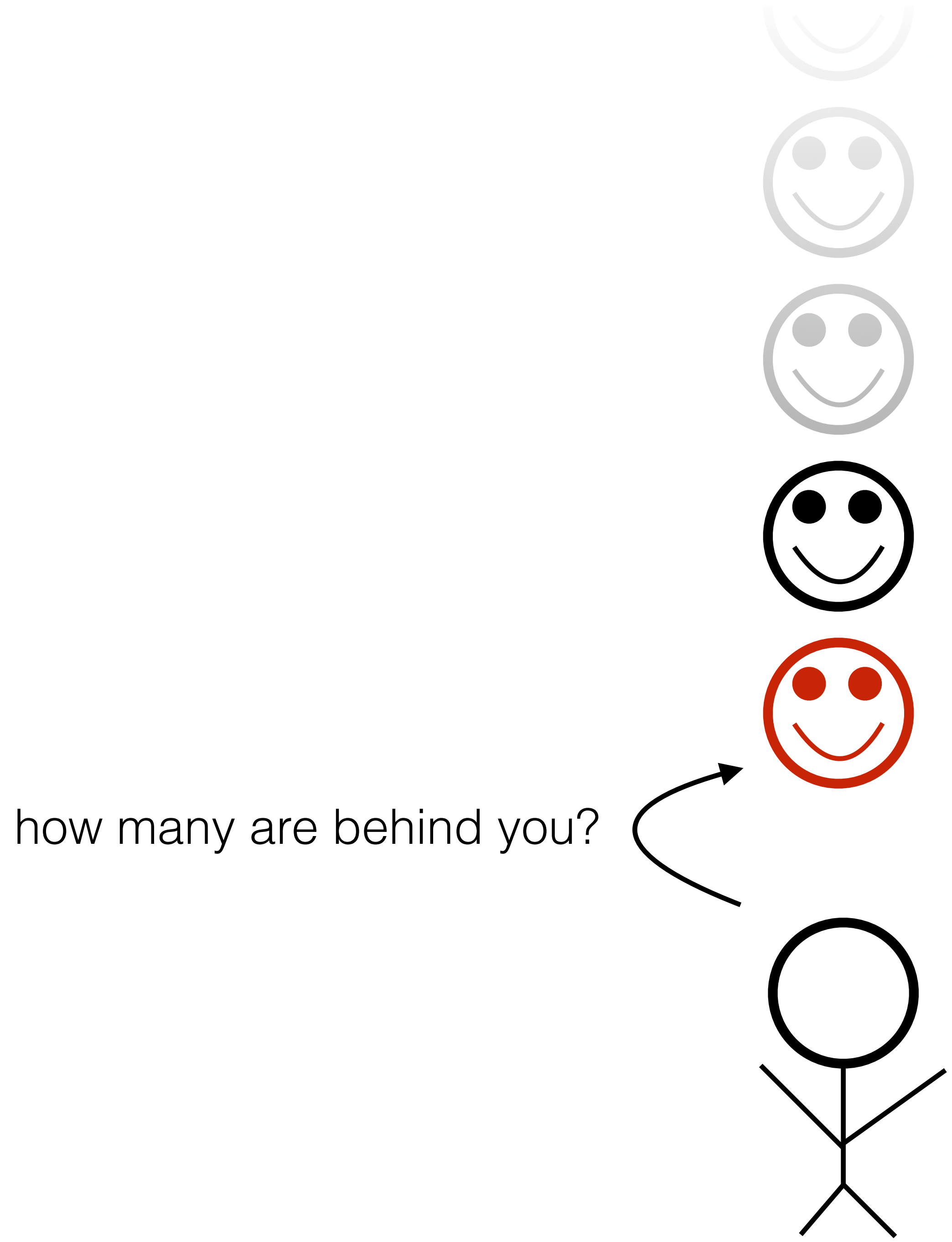
Recursive Students

Strategy: reframe question as *"how many students are behind you?"*



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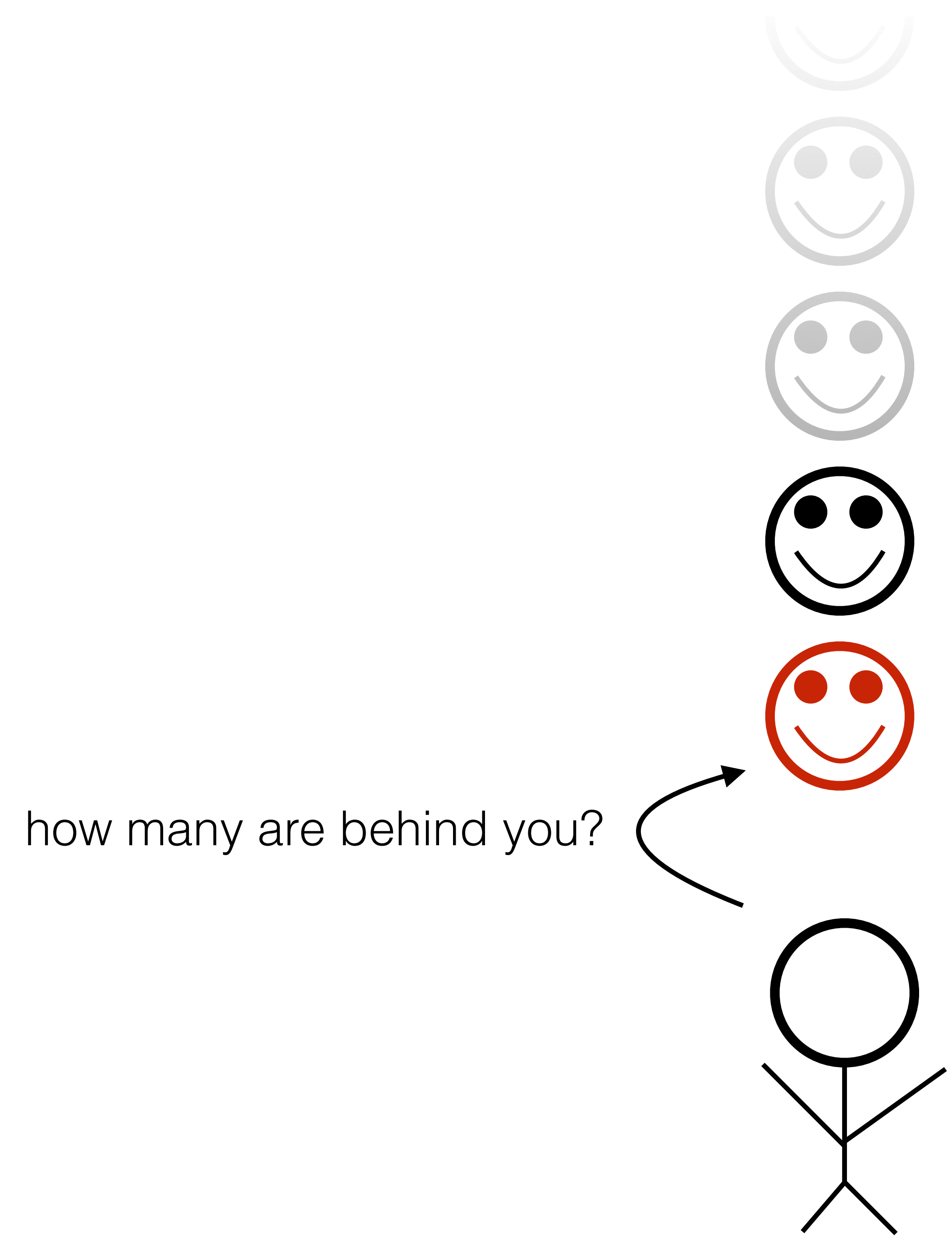
Reframing is the hardest part



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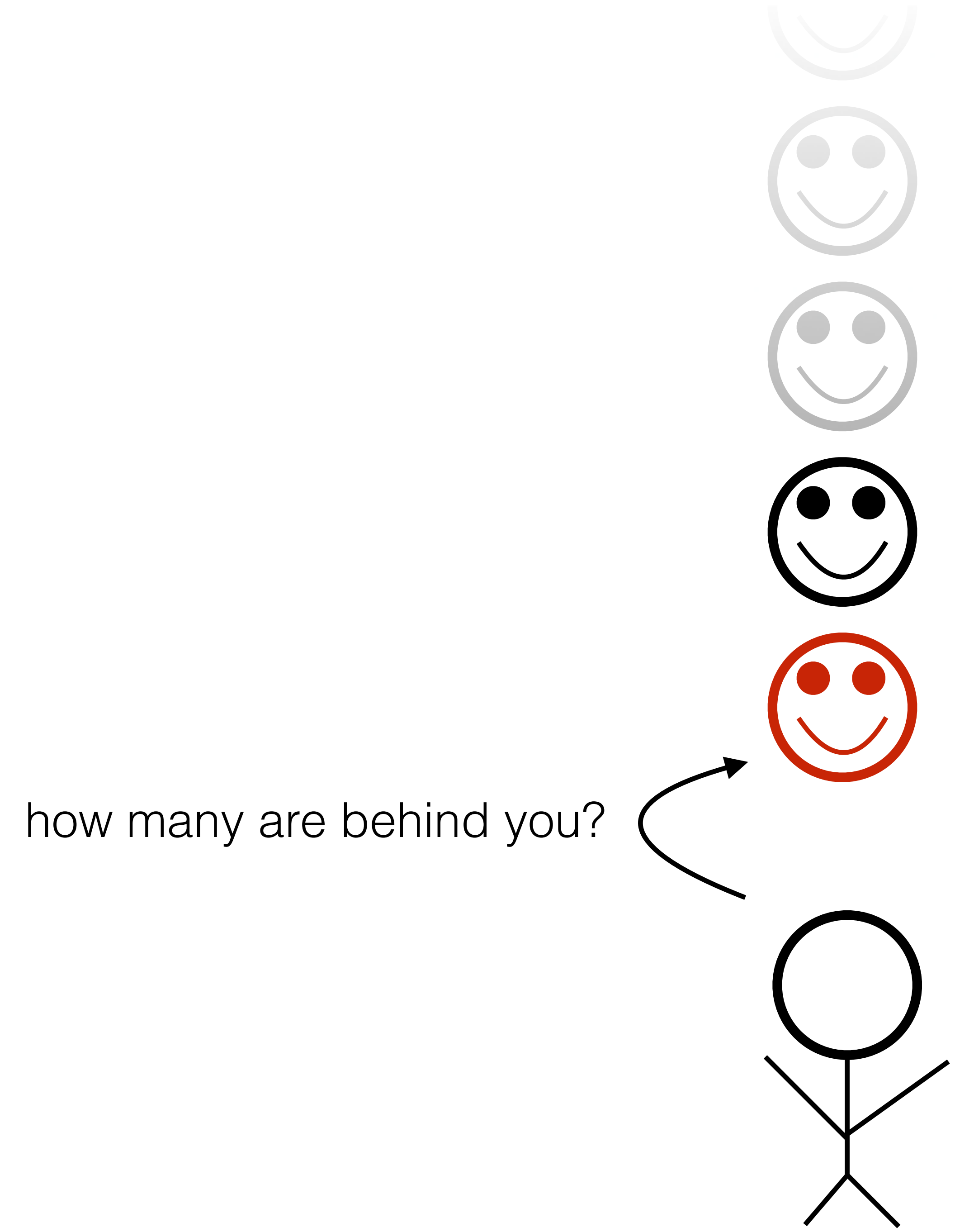
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1



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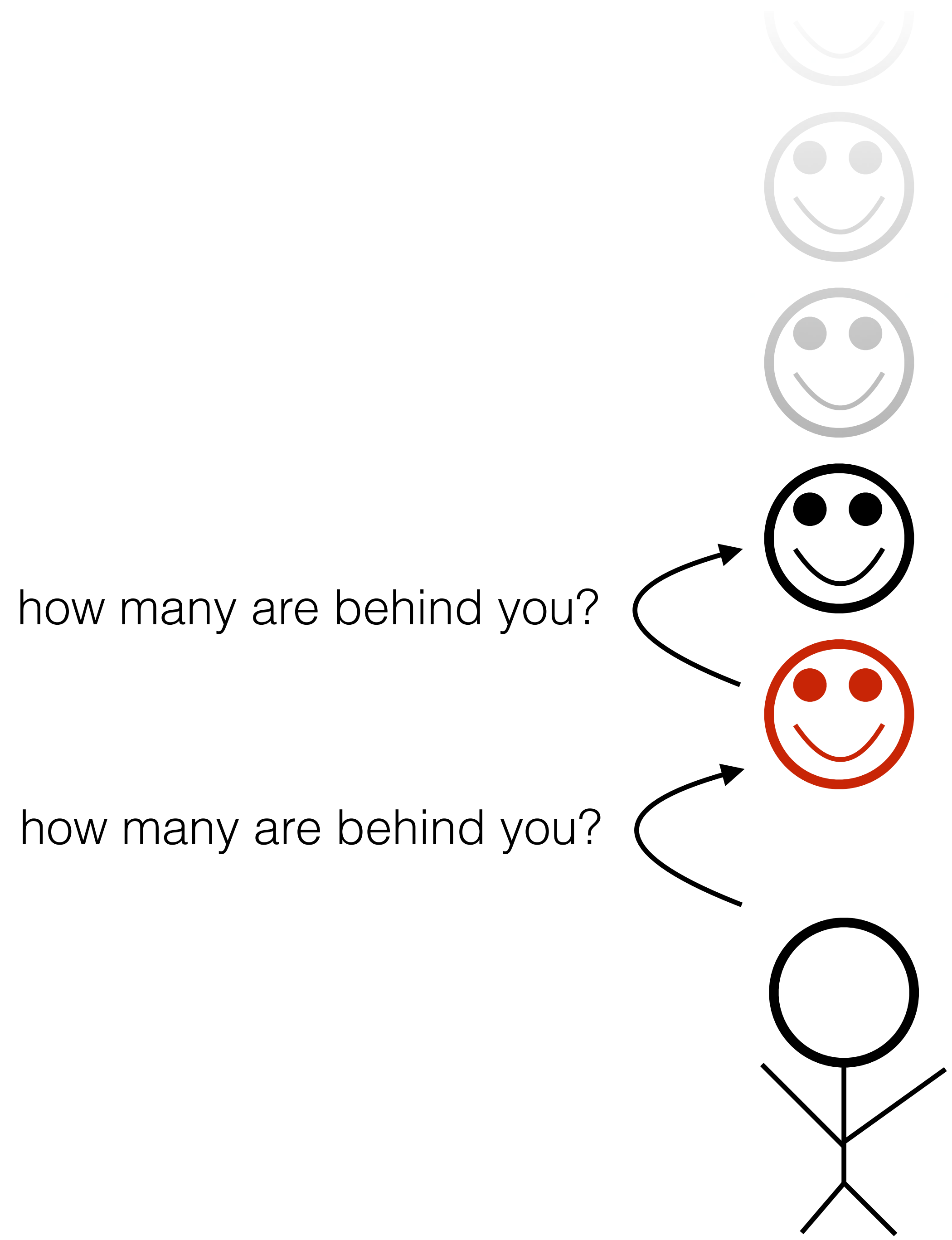

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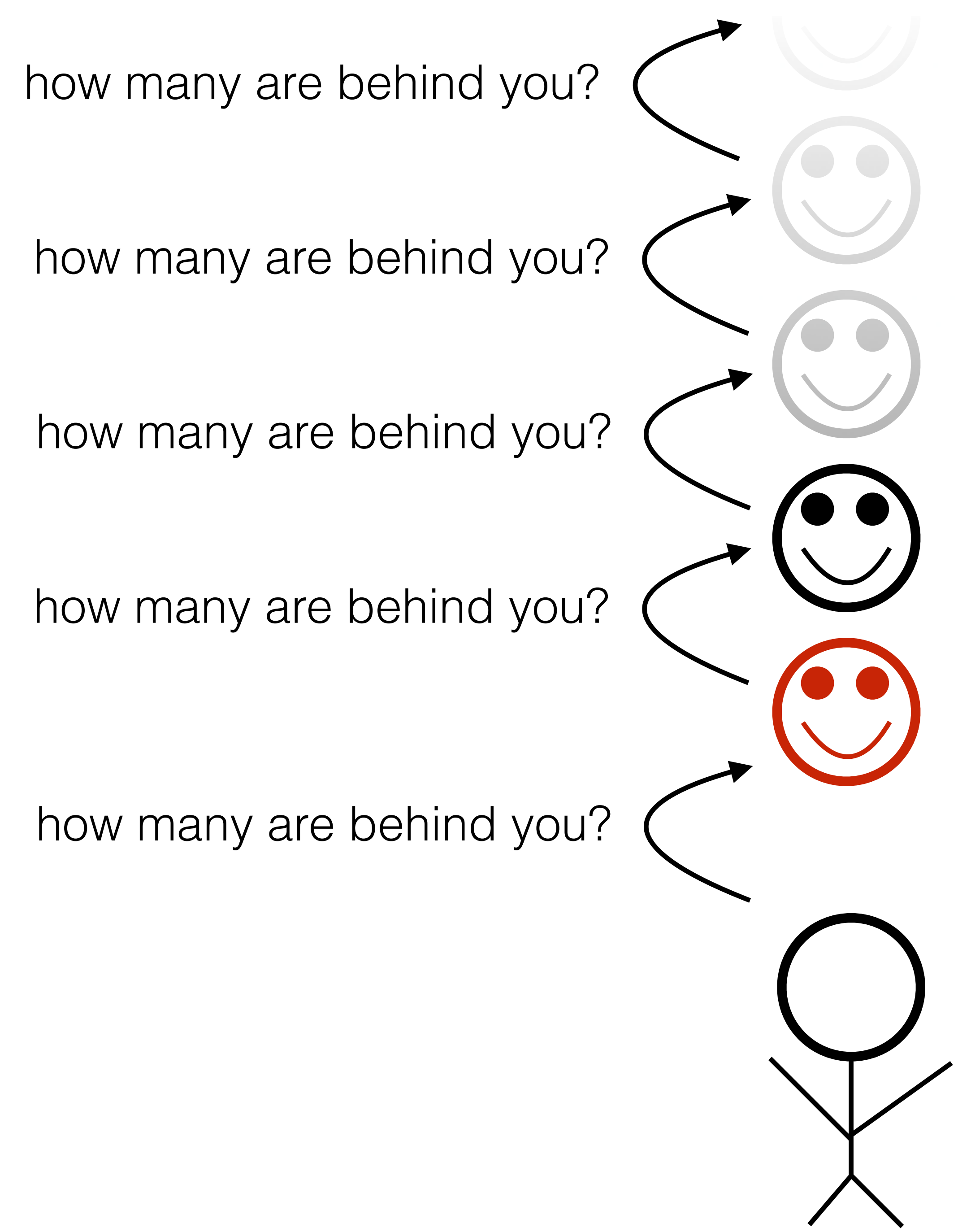

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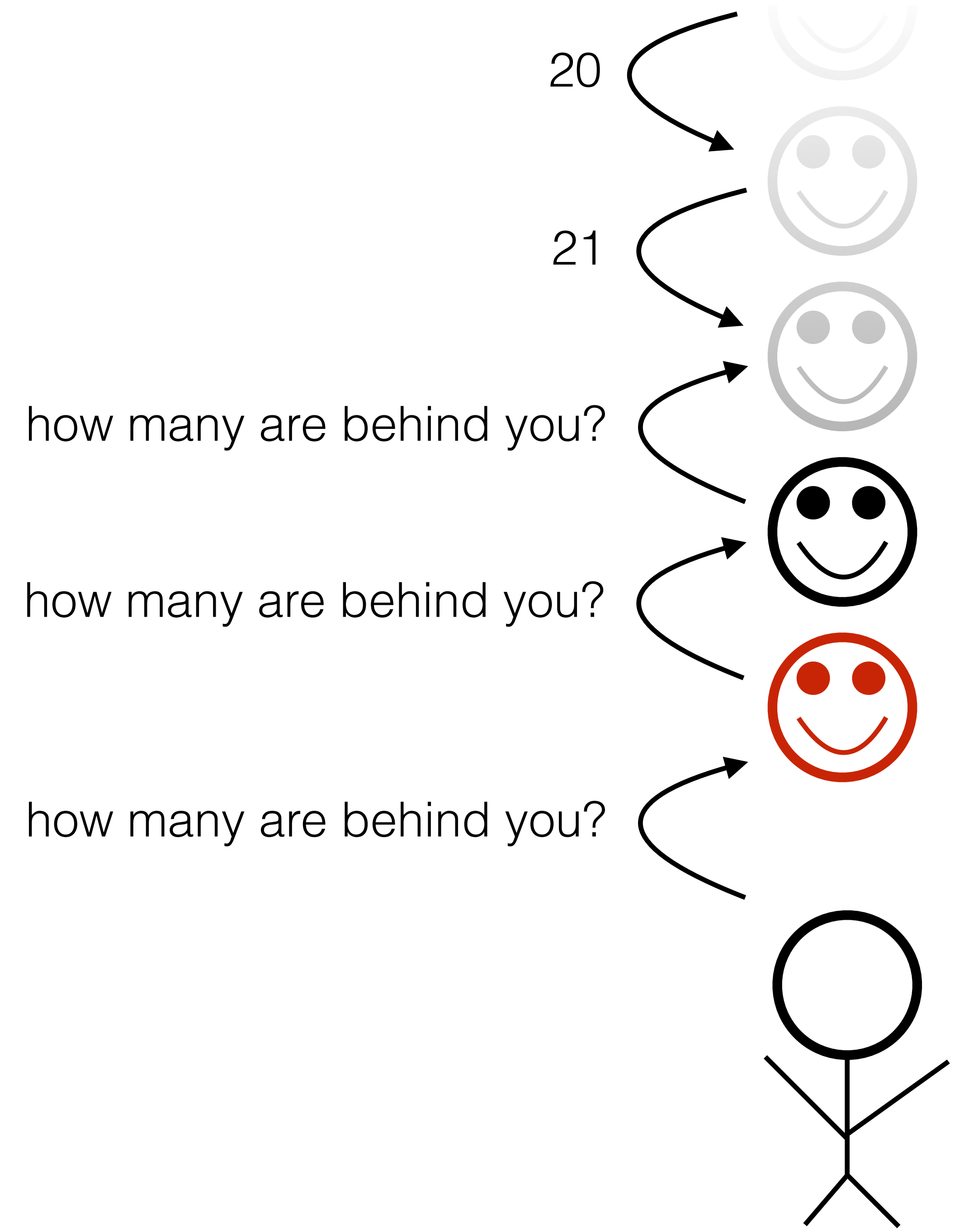

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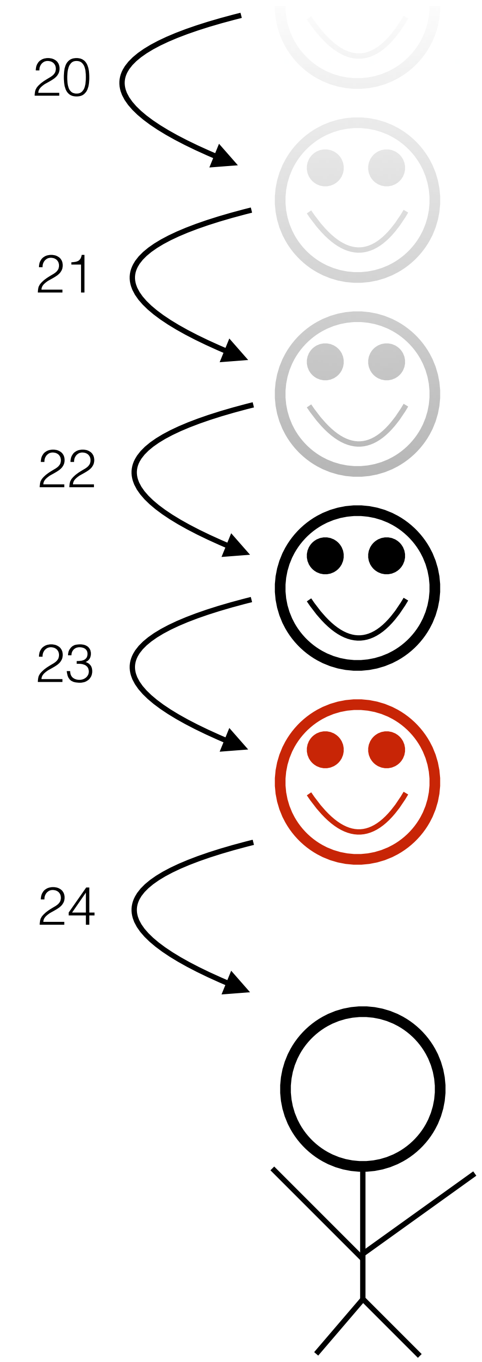

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Observations:

- Each student runs the same "code"
- Each student has their own "state"

Example: Factorials

$N! = 1 \times 2 \times 3 \times \ldots \times (N-2) \times (N-1) \times N$

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

3. Recursive Definition:

4. Python Code:

```
def fact(n):
    pass # TODO
```

Goal: work from examples to get to recursive code

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look for patterns that allow rewrites with self reference

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don't need a pattern at the start

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convert self-referring examples to a recursive definition

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`N!` is `????` for N>1

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2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

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1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
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```

3. Recursive Definition:

`1!` is `1`

`N!` is `(N-1)! * N` for N>1

4. Python Code:

```
def fact(n):
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```

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```

Let's "run" it!

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

fact(n=4)

Tracing Factorial

```
def fact(n):
    if n == 1:
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    p = fact(n-1)
    return n * p
```

fact(n=4)

if n == 1:

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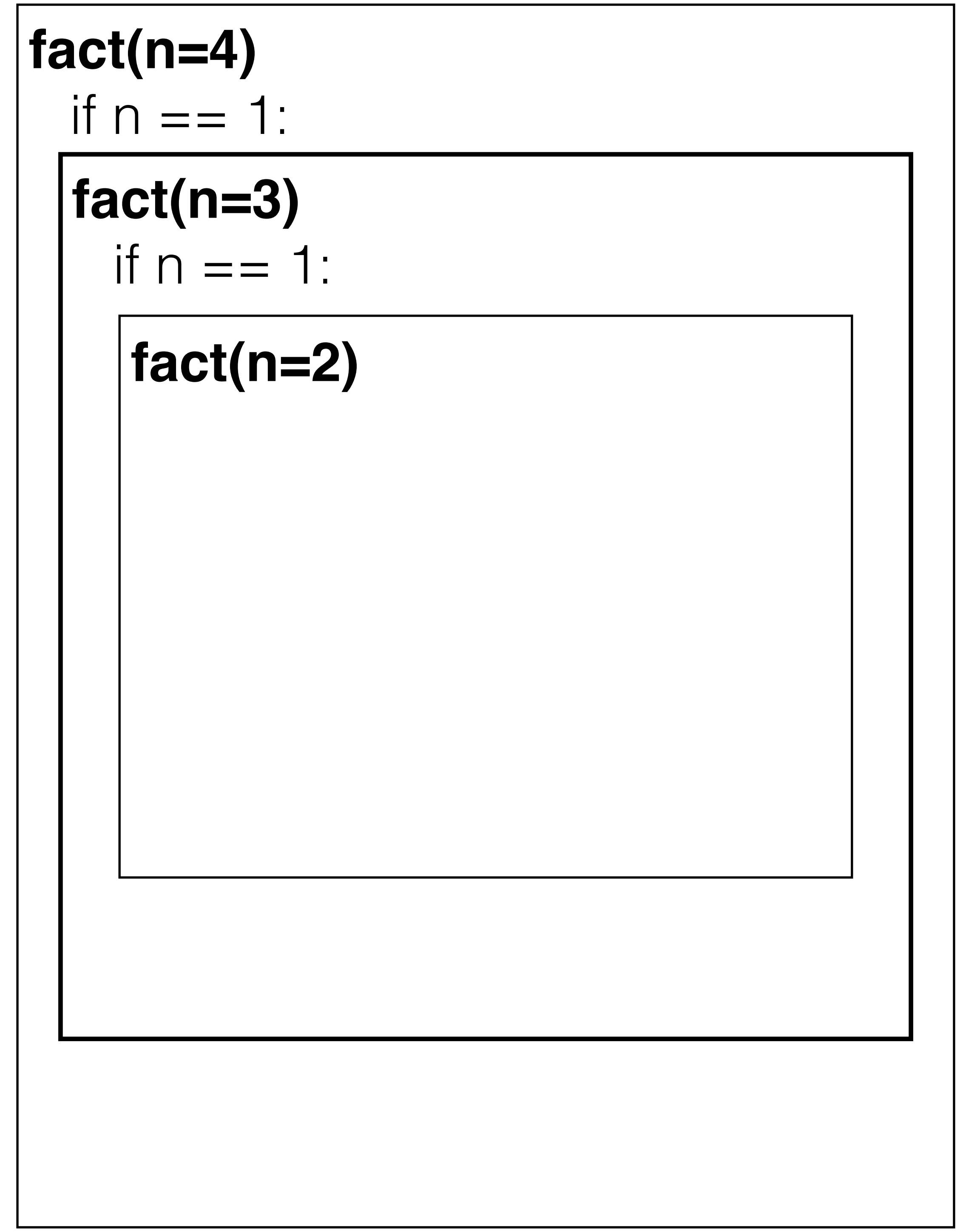

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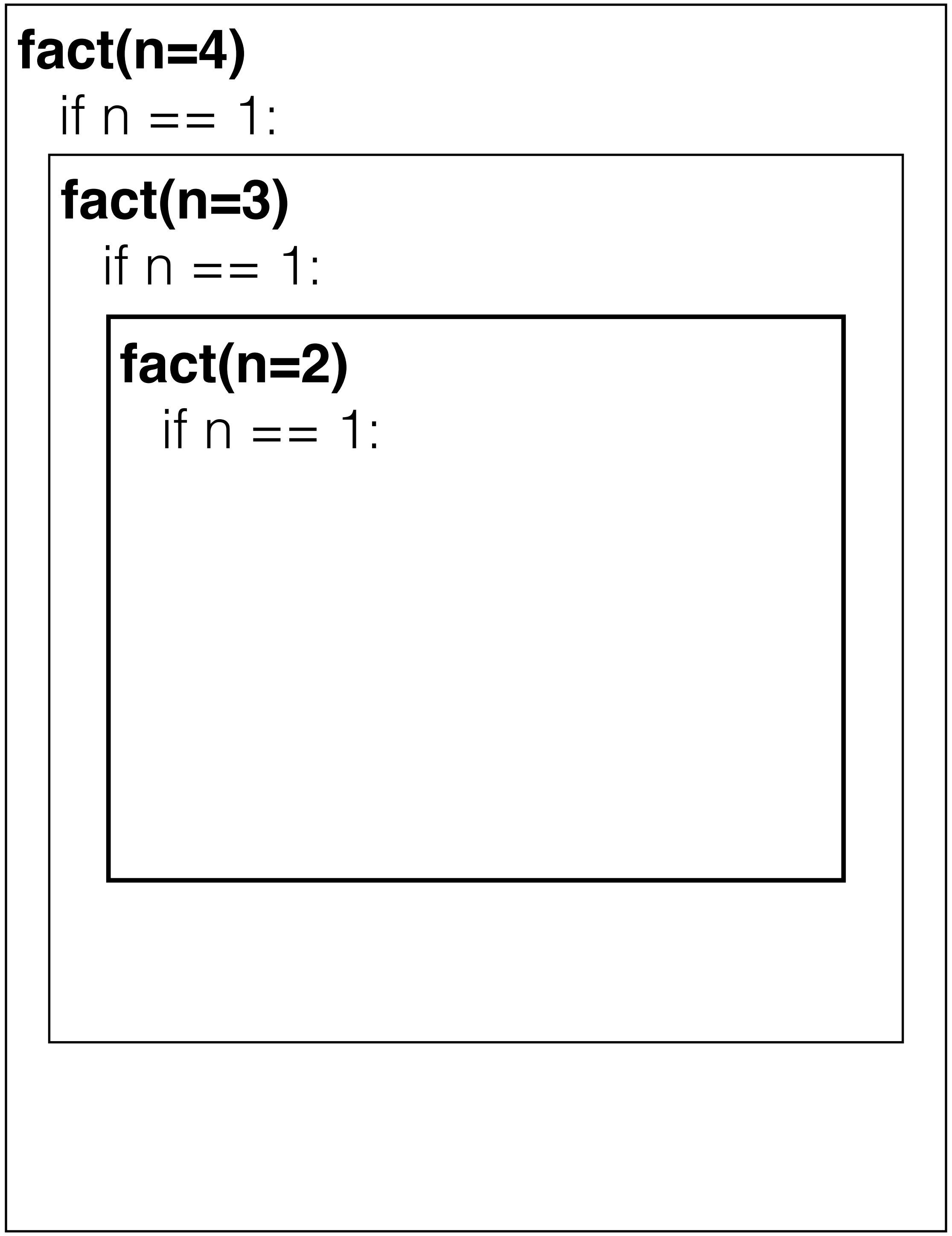

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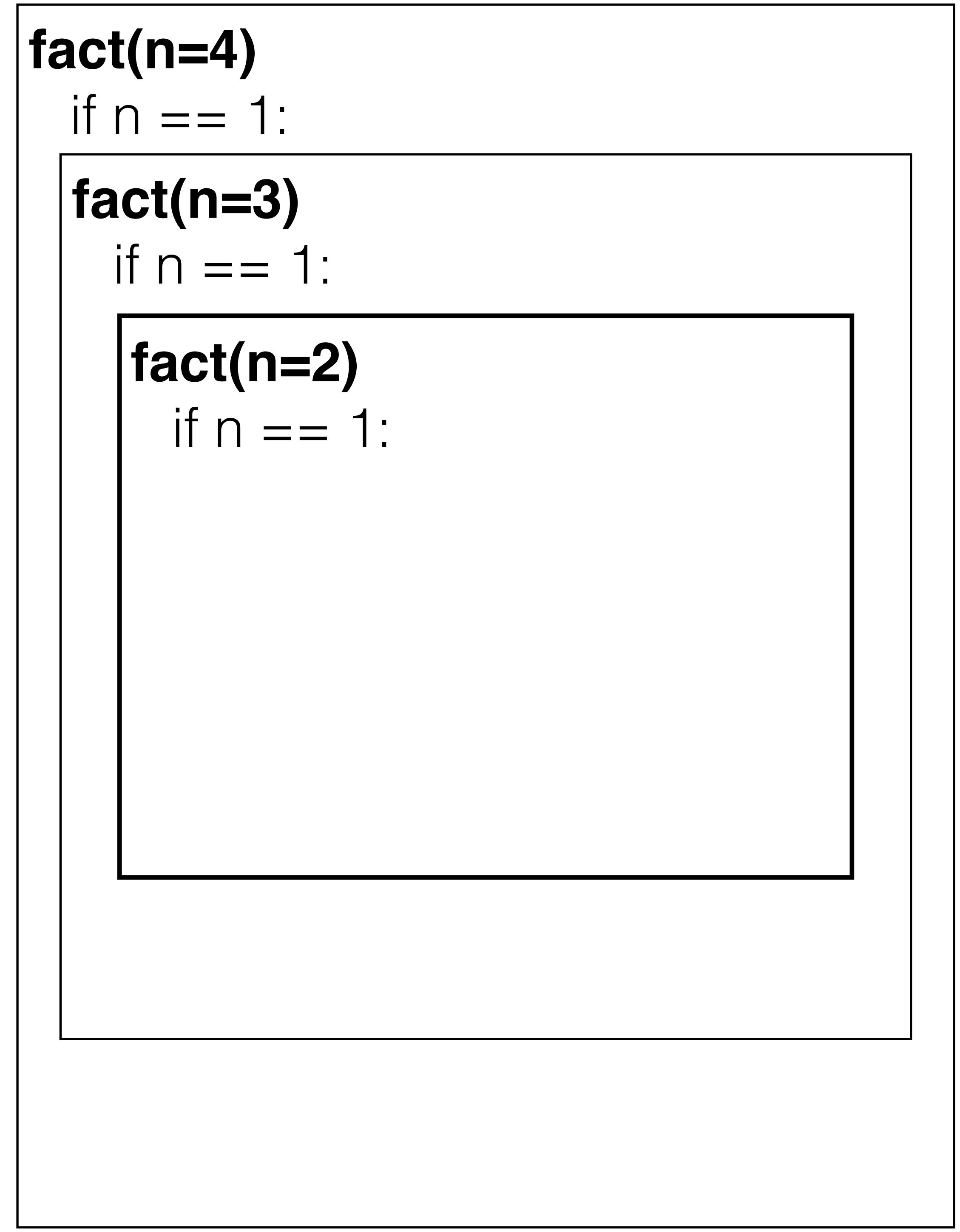

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fact(n=4)

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fact(n=3)

if n == 1:

fact(n=2)

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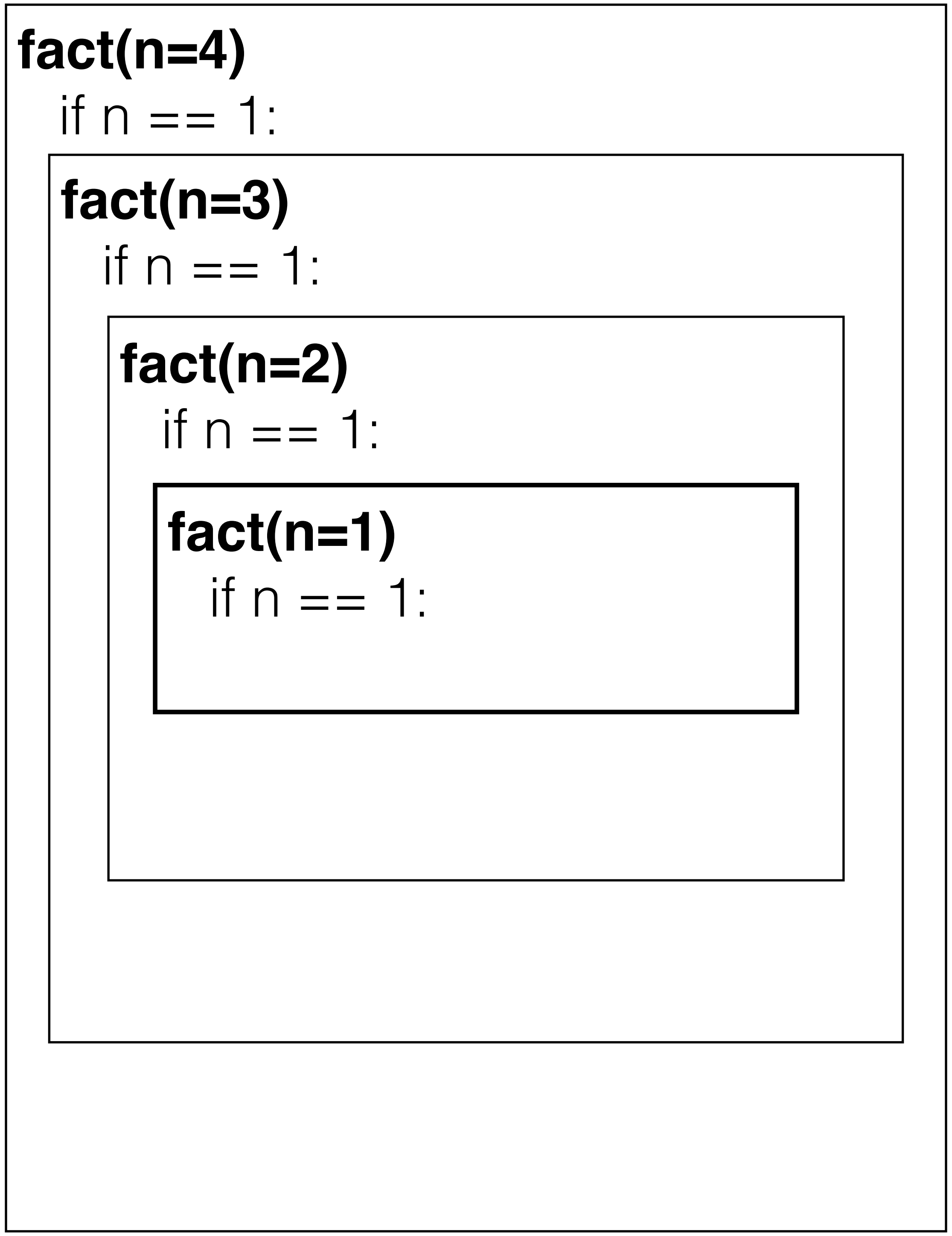

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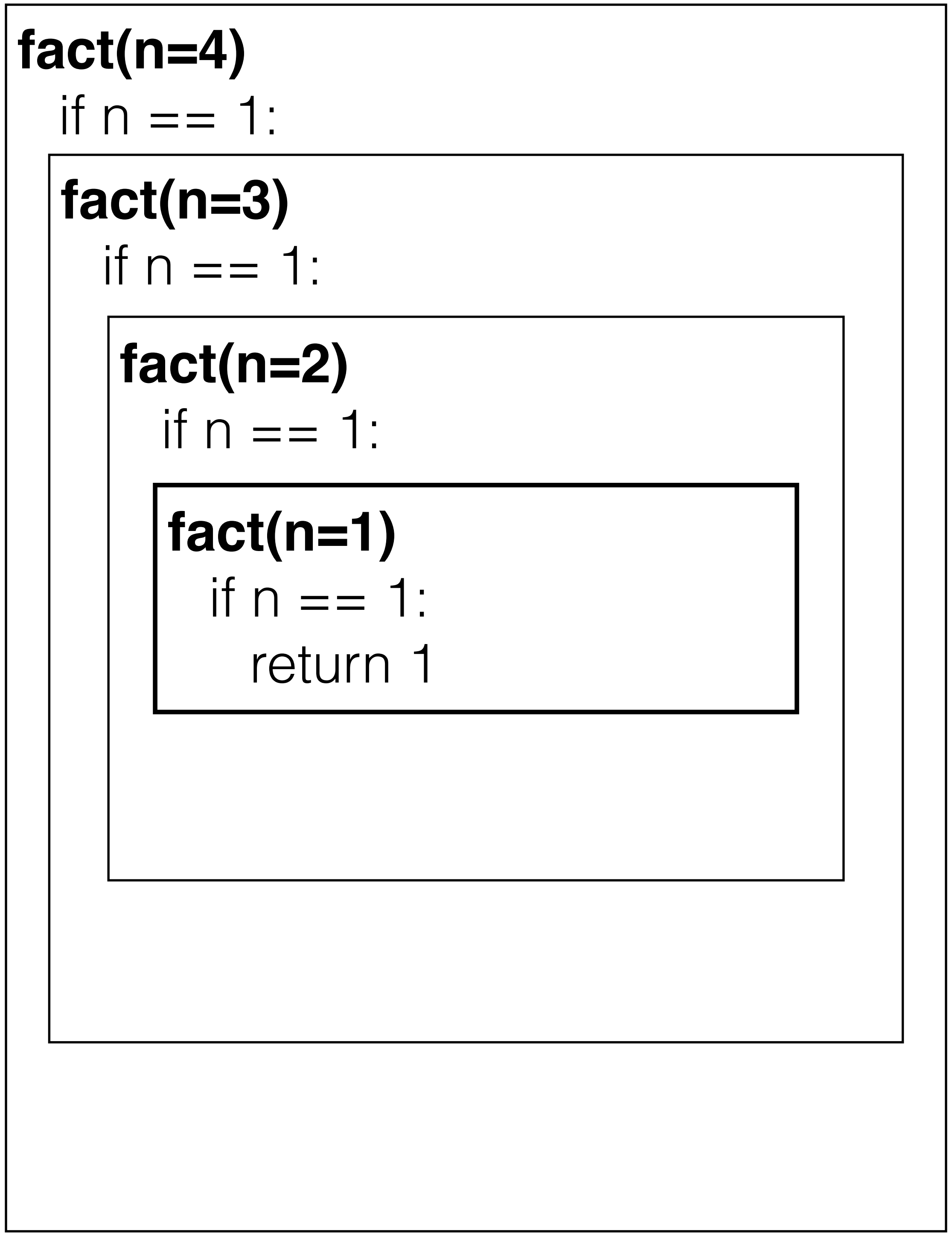

Tracing Factorial

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```

fact(n=4)
if n == 1:

fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)
if n == 1:
return 1

p = 1

Tracing Factorial

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```

fact(n=4)
if n == 1:
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fact(n=2)
if n == 1:
fact(n=1)
if n == 1:
return 1
p = 1
return 2

Tracing Factorial

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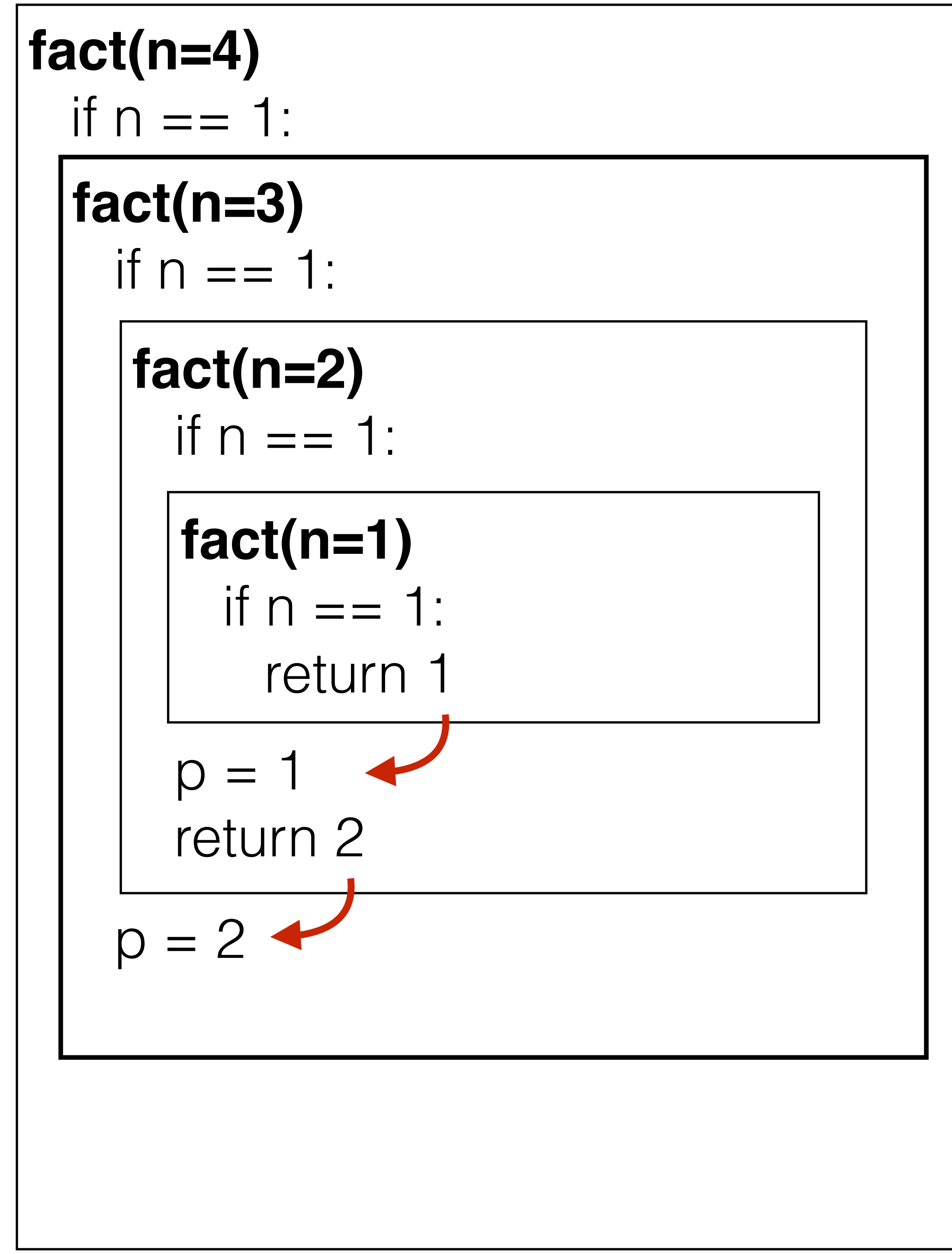
    return n * p
```



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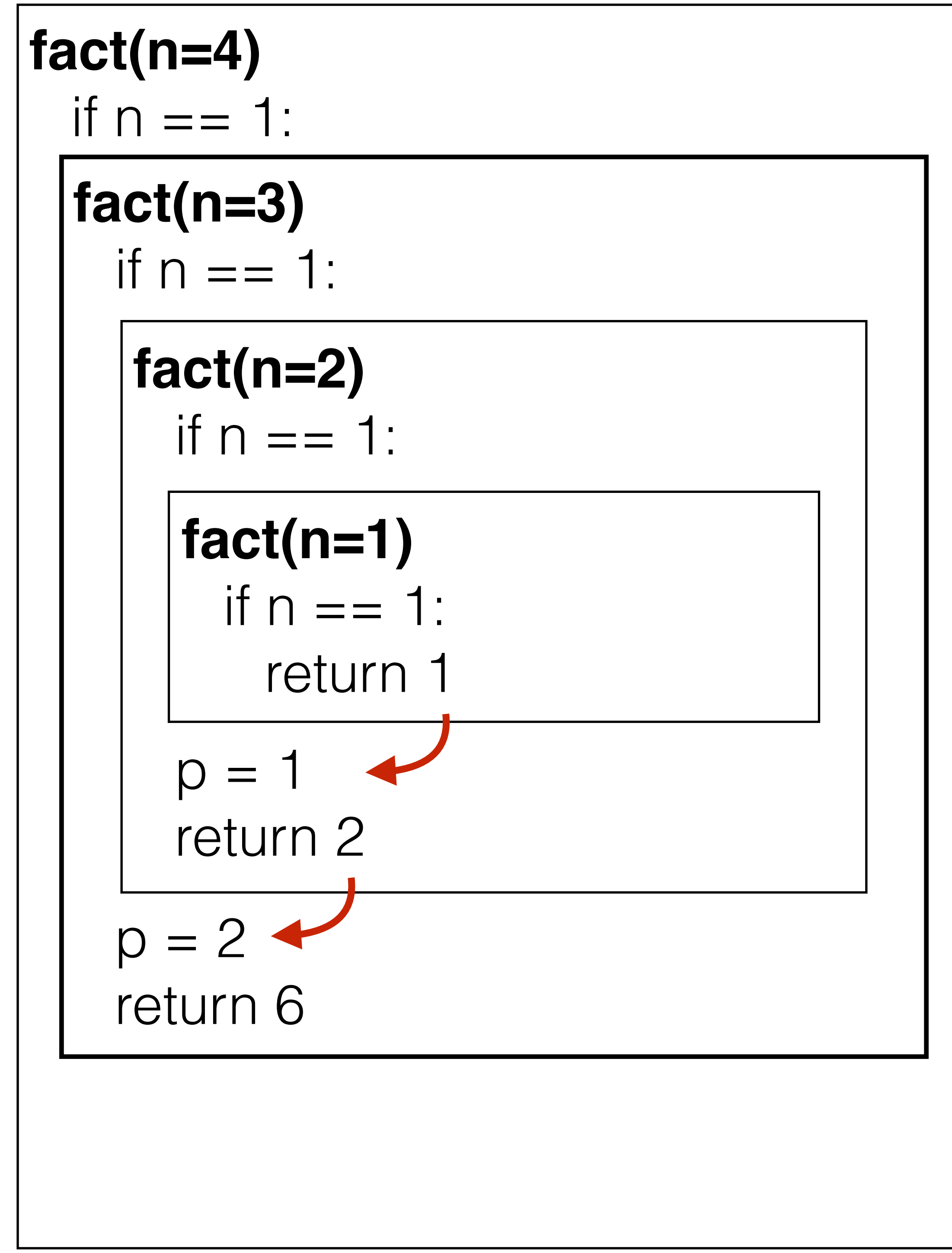

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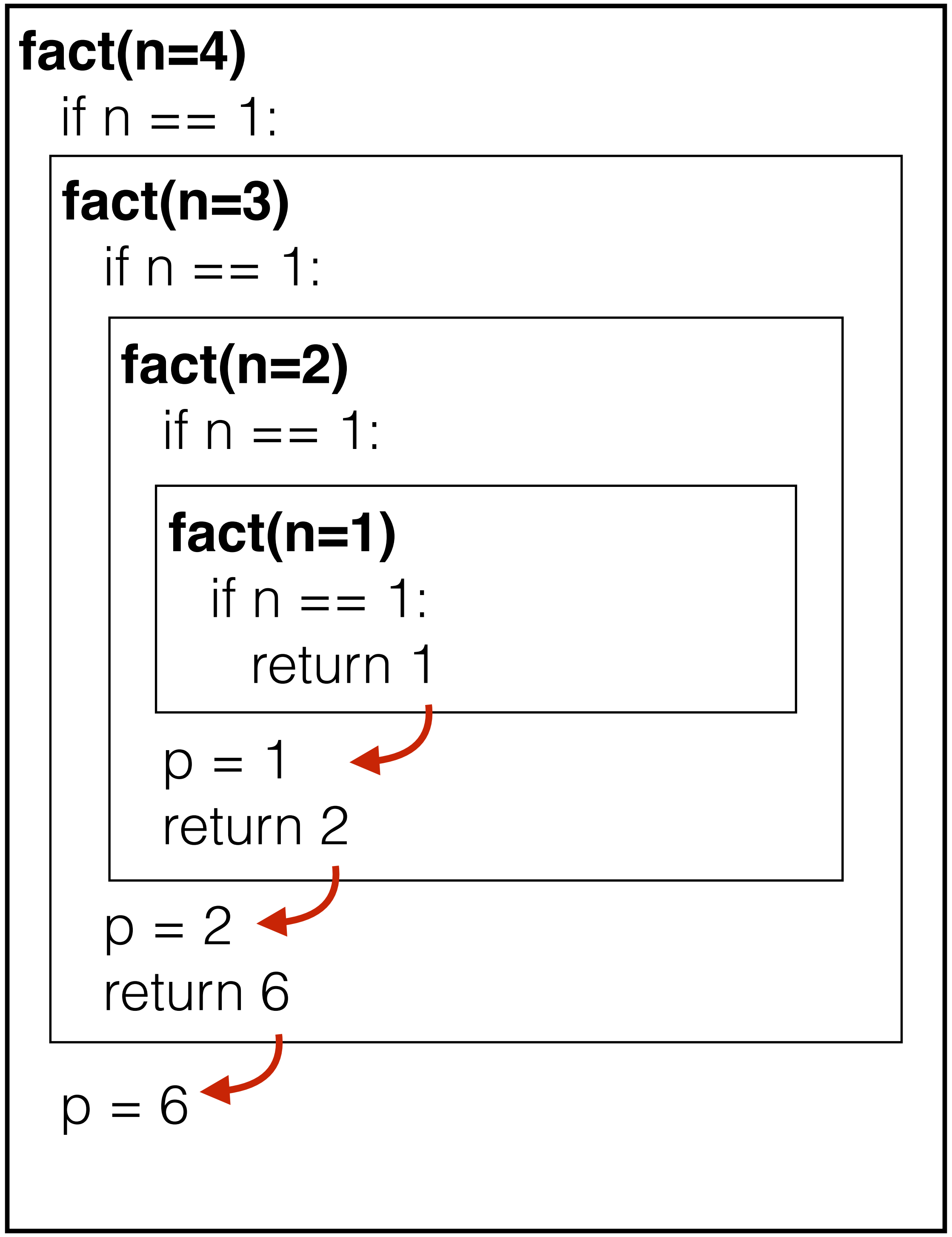

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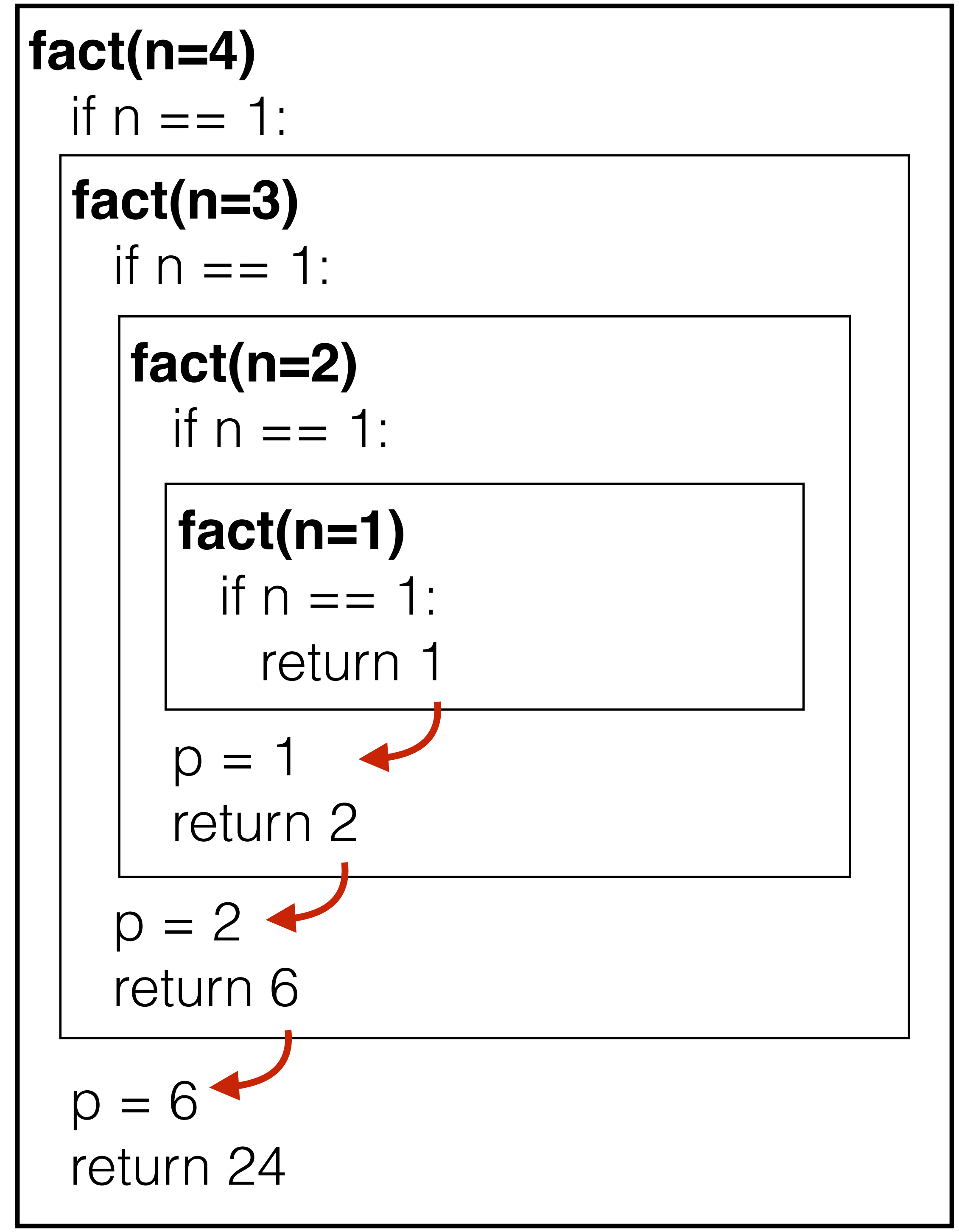

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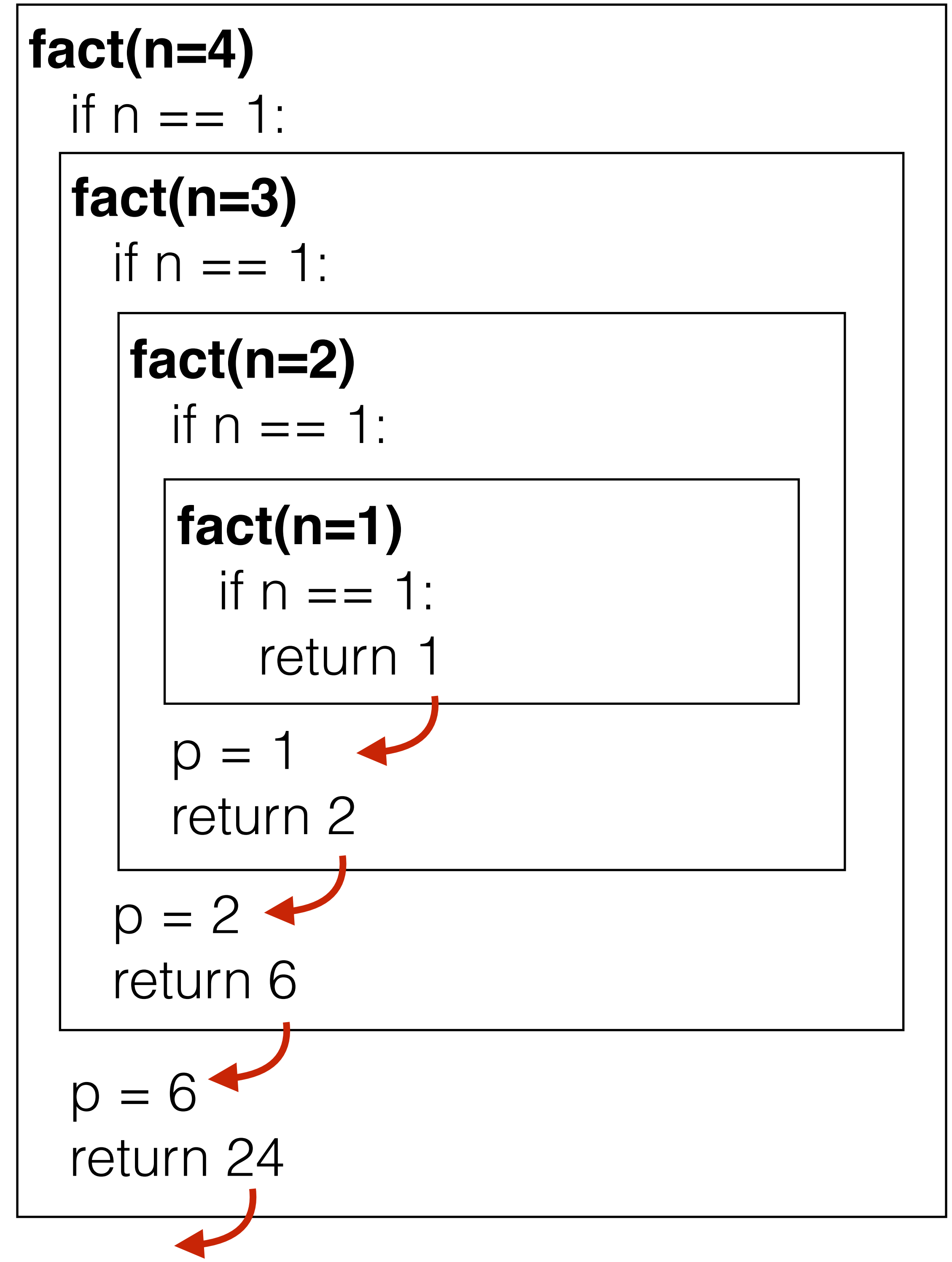

Tracing Factorial

```
def fact(n):
    if n == 1:
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    p = fact(n-1)
    return n * p
```

How does Python keep
all the P variables separate?

fact(n=4)
if n == 1:

fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)
if n == 1:
return 1

p = 1
return 2

p = 2
return 6

p = 6
return 24

Deep Dive: Invocation State

In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

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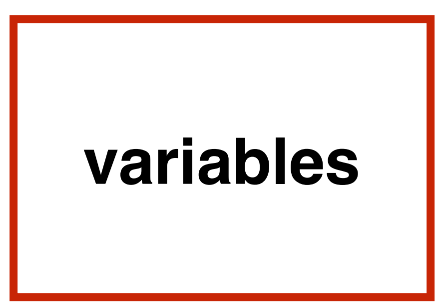
Variables for an invocation exist in a ***frame***

frame:



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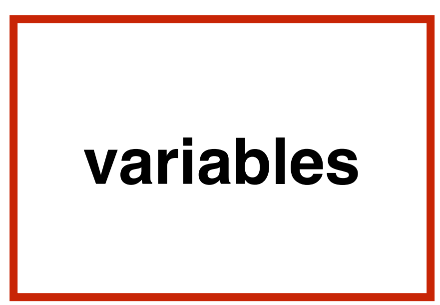
Variables for an invocation exist in a ***frame***

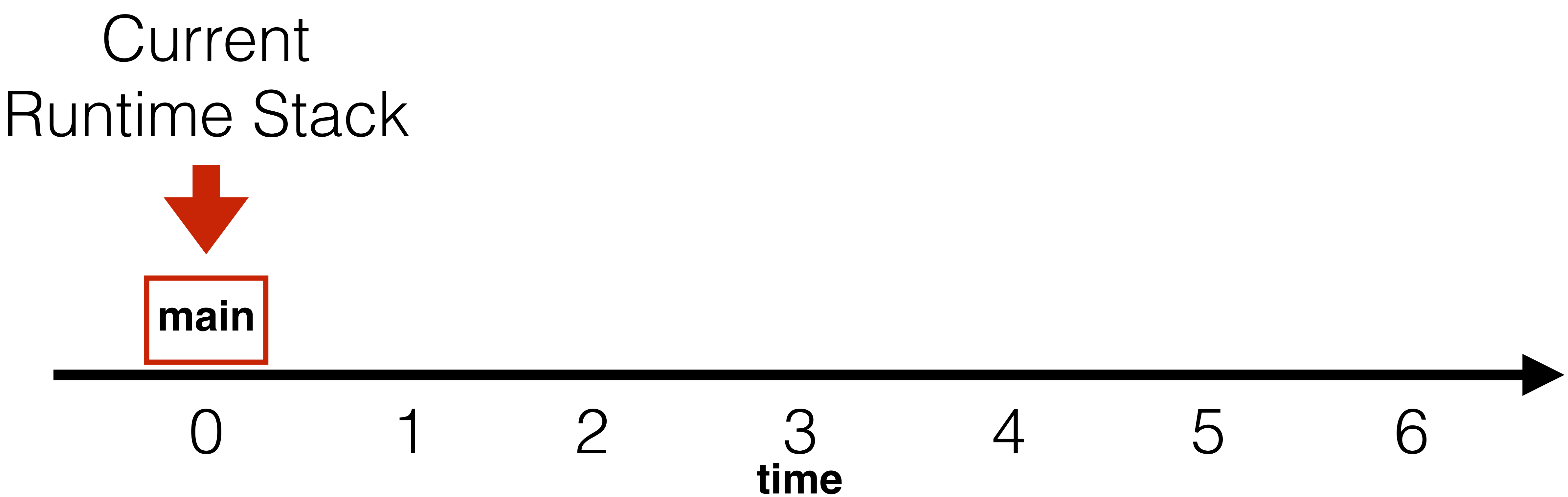
- the frames are stored in something called the **runtime stack**
- one invocation is active at a time: its frame is on the top of stack
- if a function calls itself, there will be multiple frames at the same time for the multiple invocations of the same function

frame:



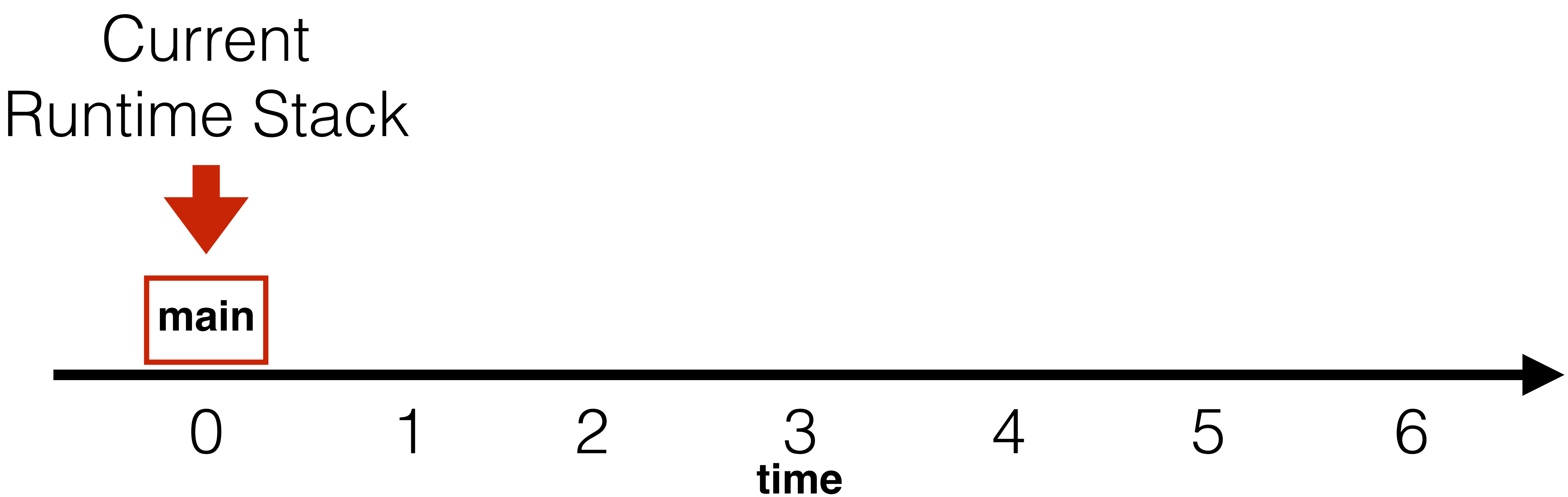
stack:

Deep Dive:
Runtime Stack



Deep Dive: Runtime Stack

`main()` calls `fact(3)`



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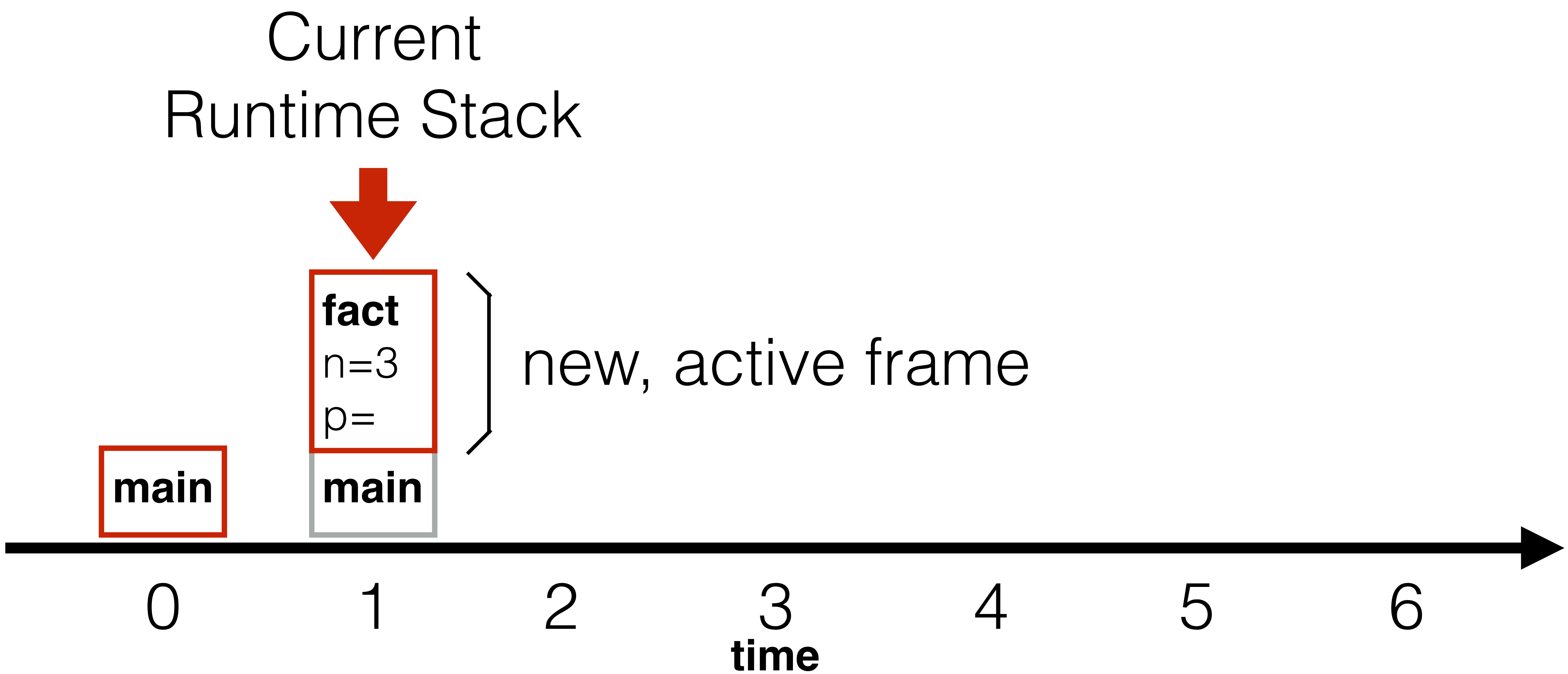

Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
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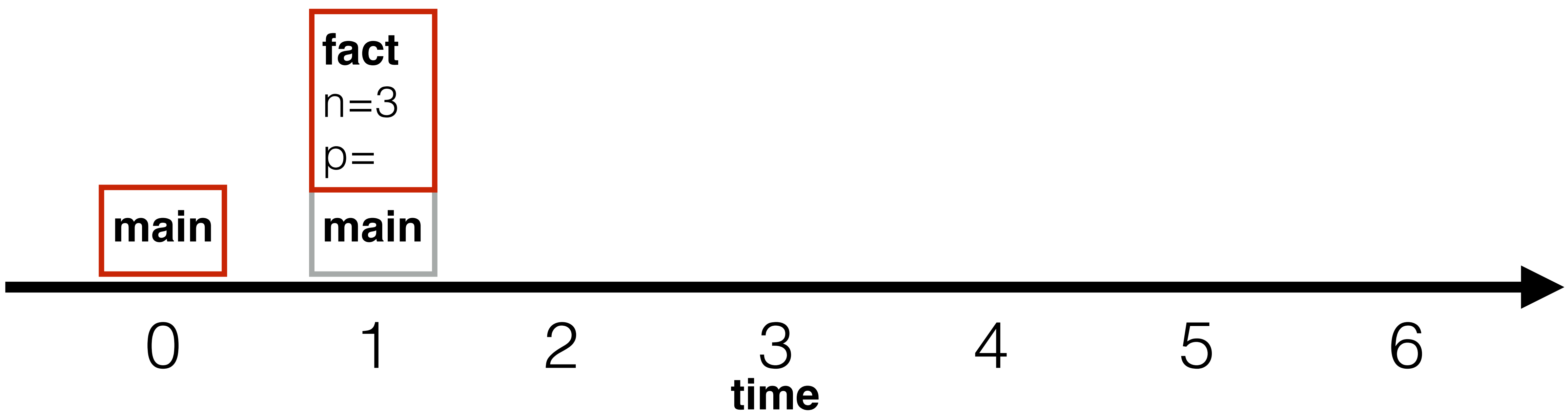

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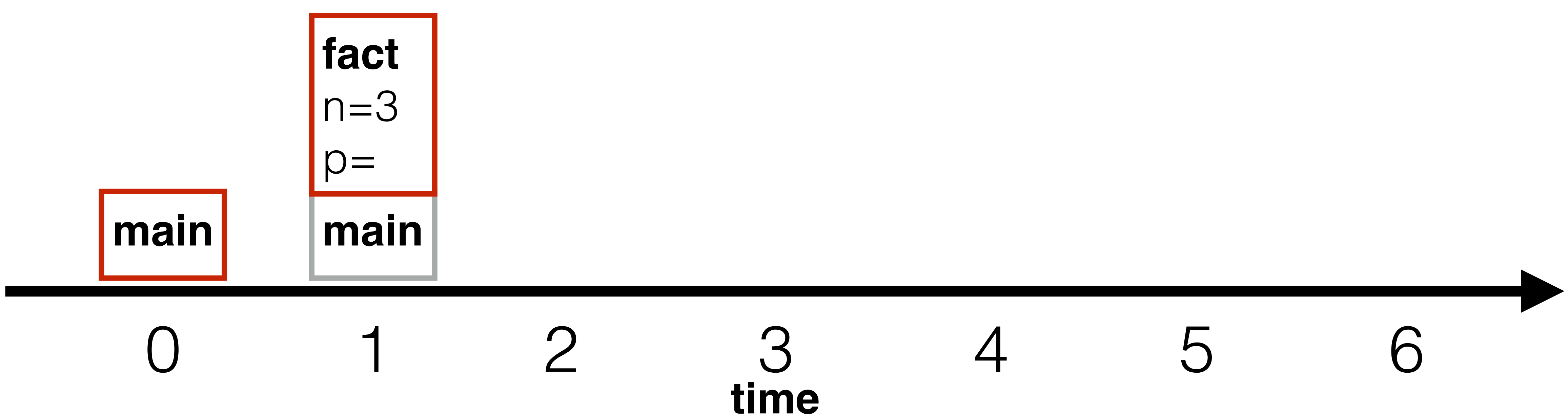

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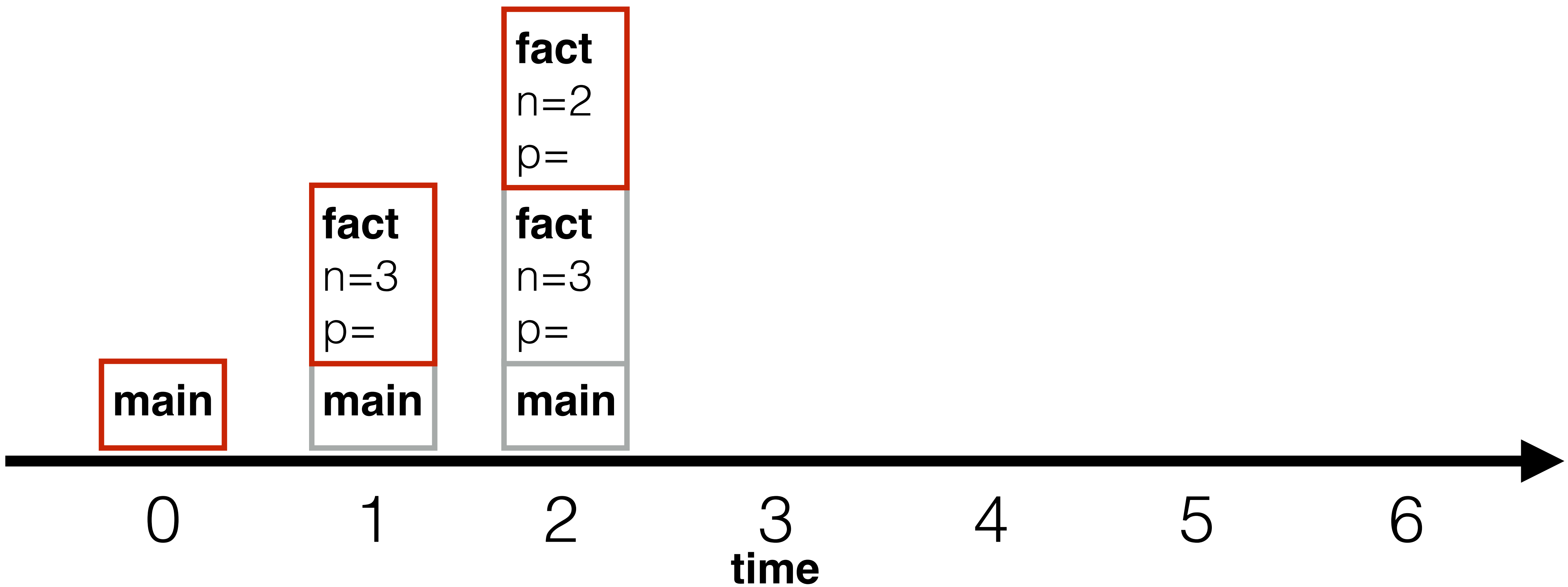

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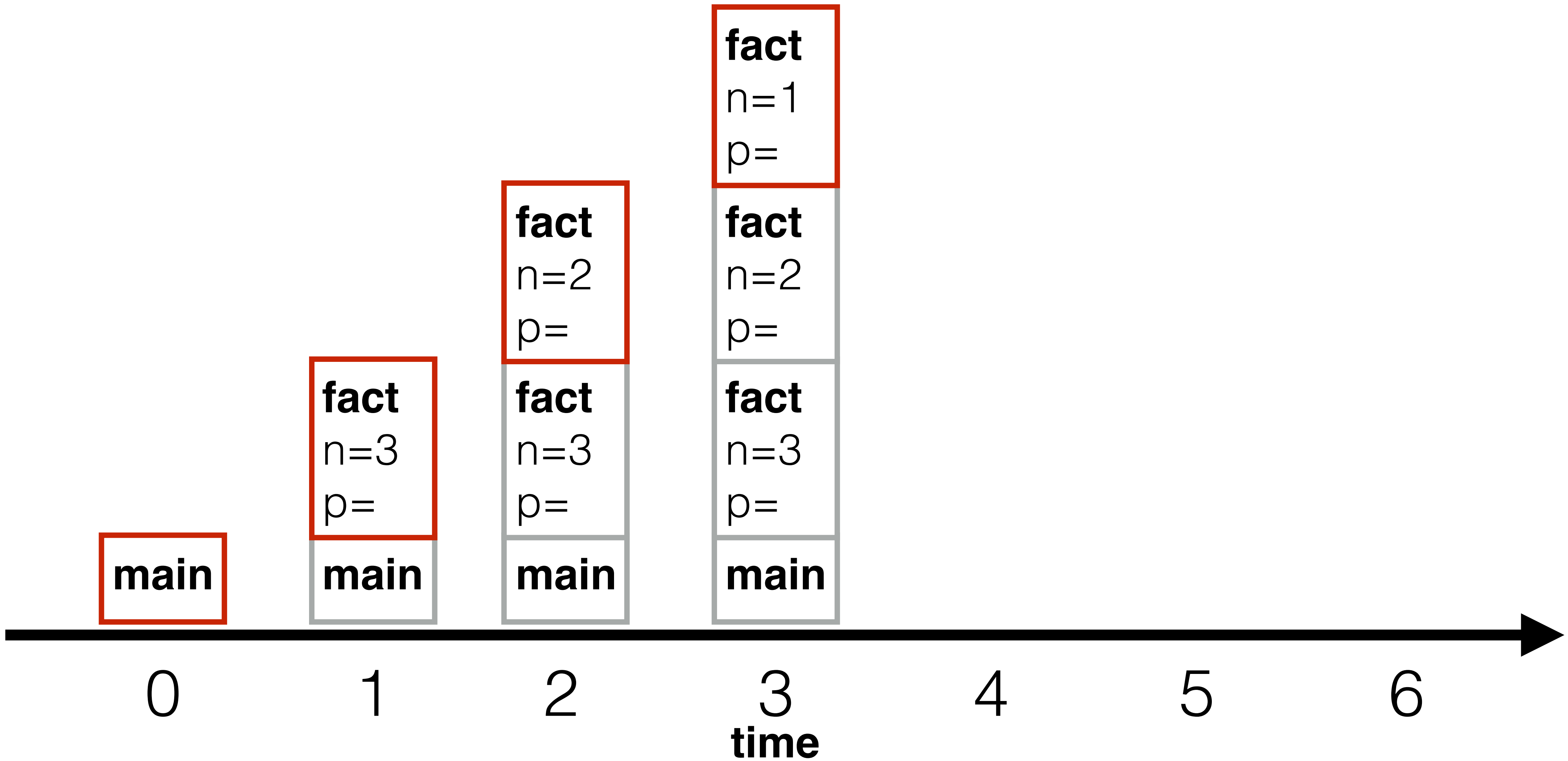

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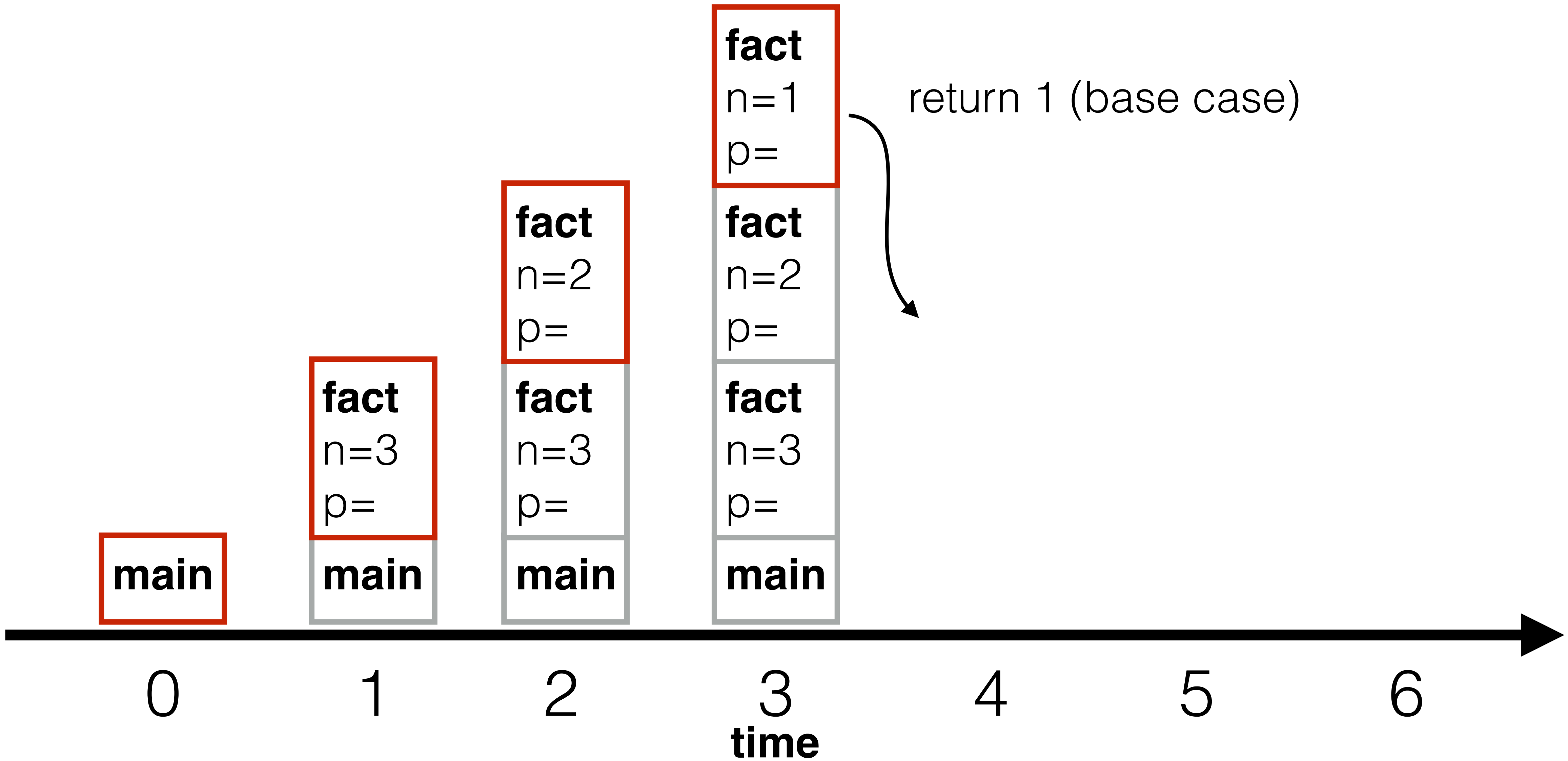

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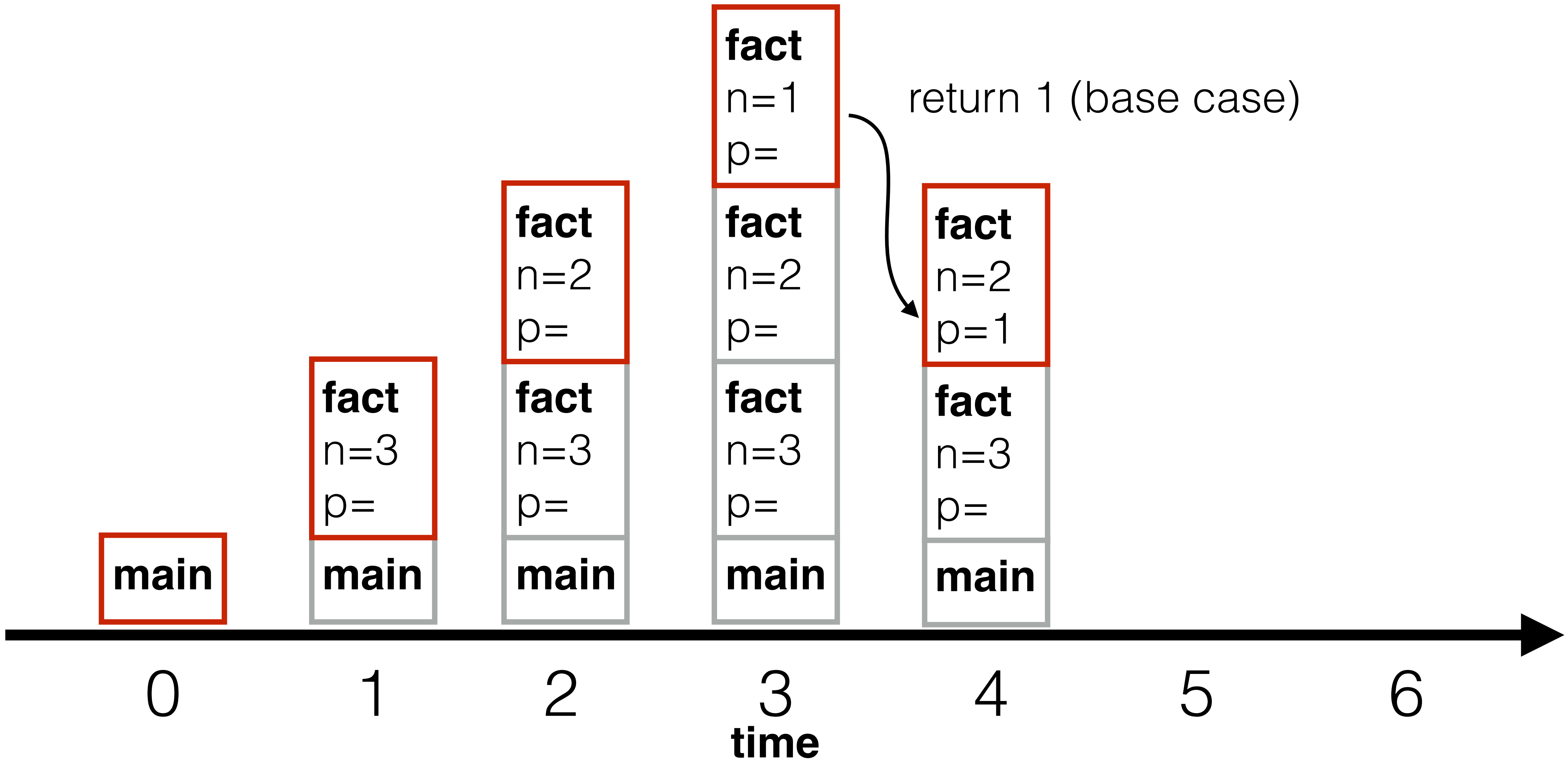

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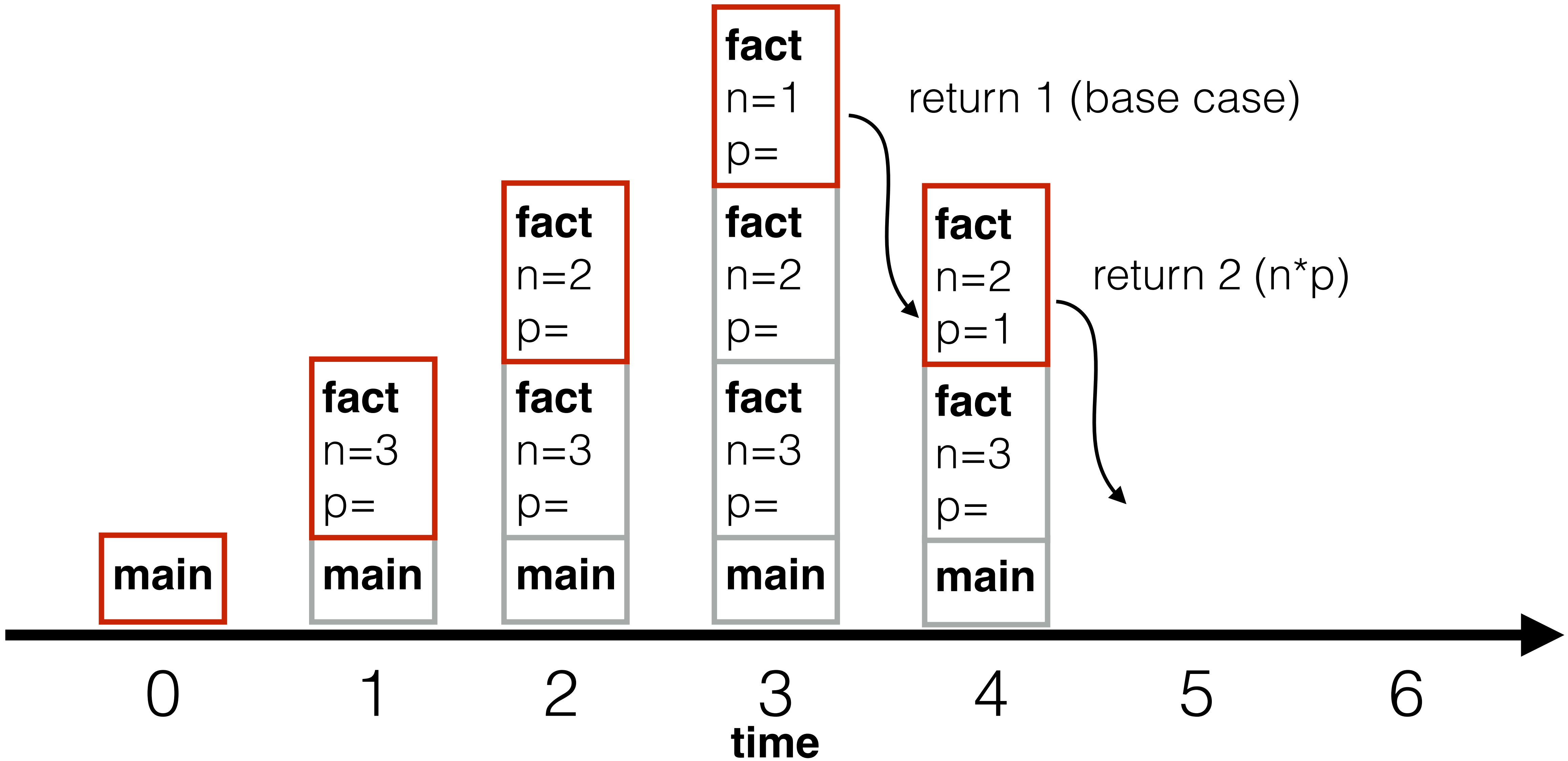

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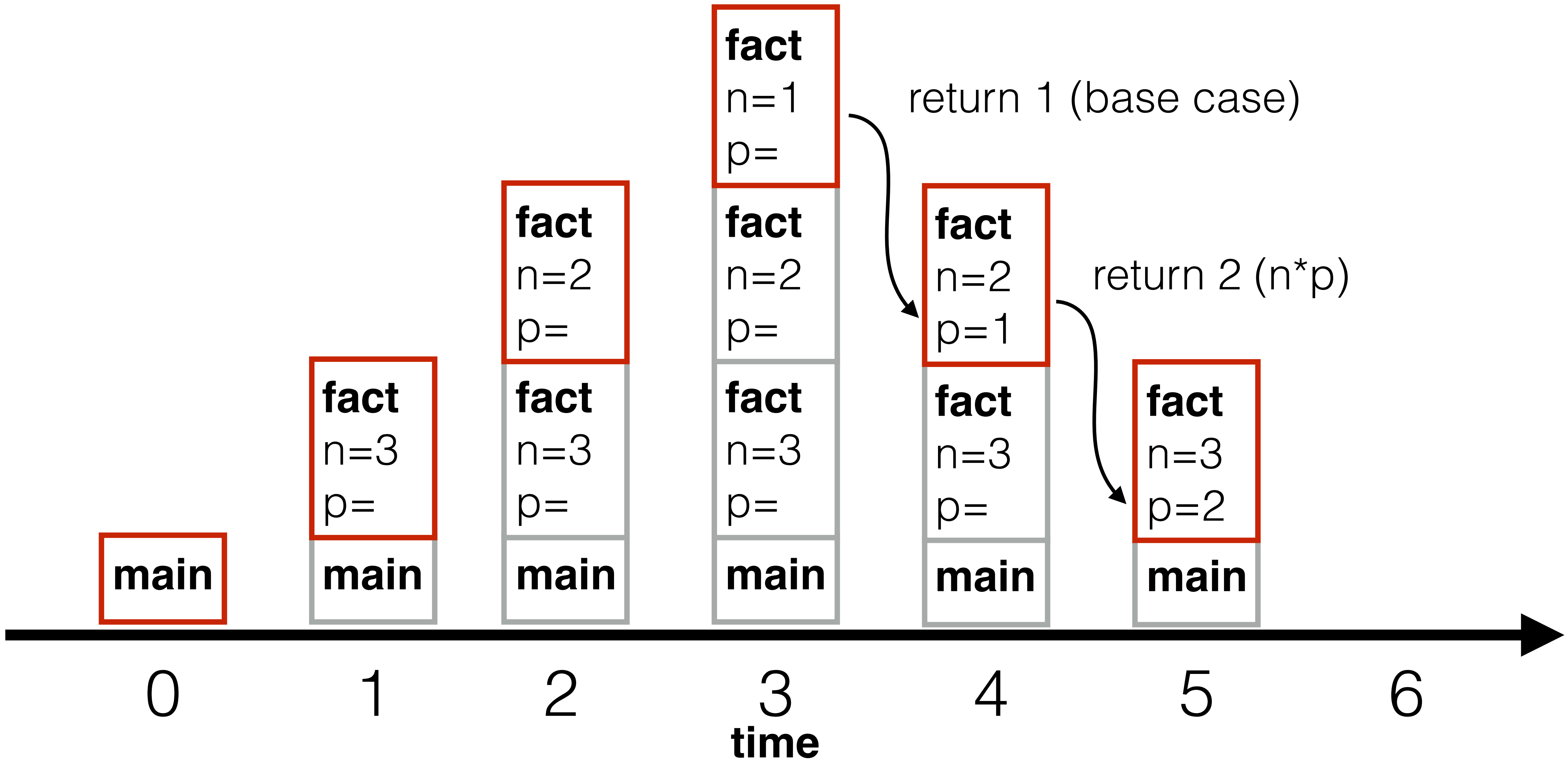

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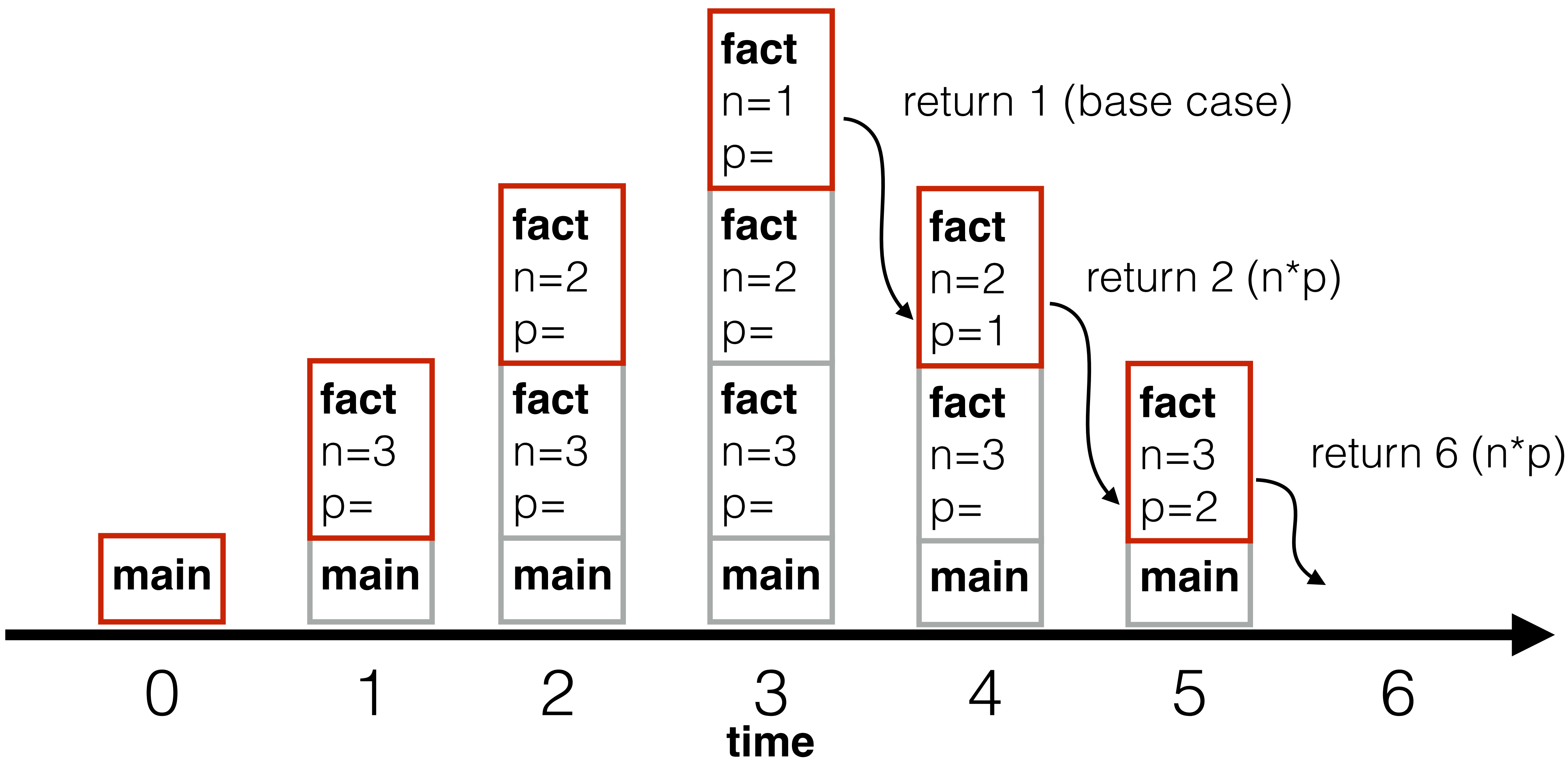

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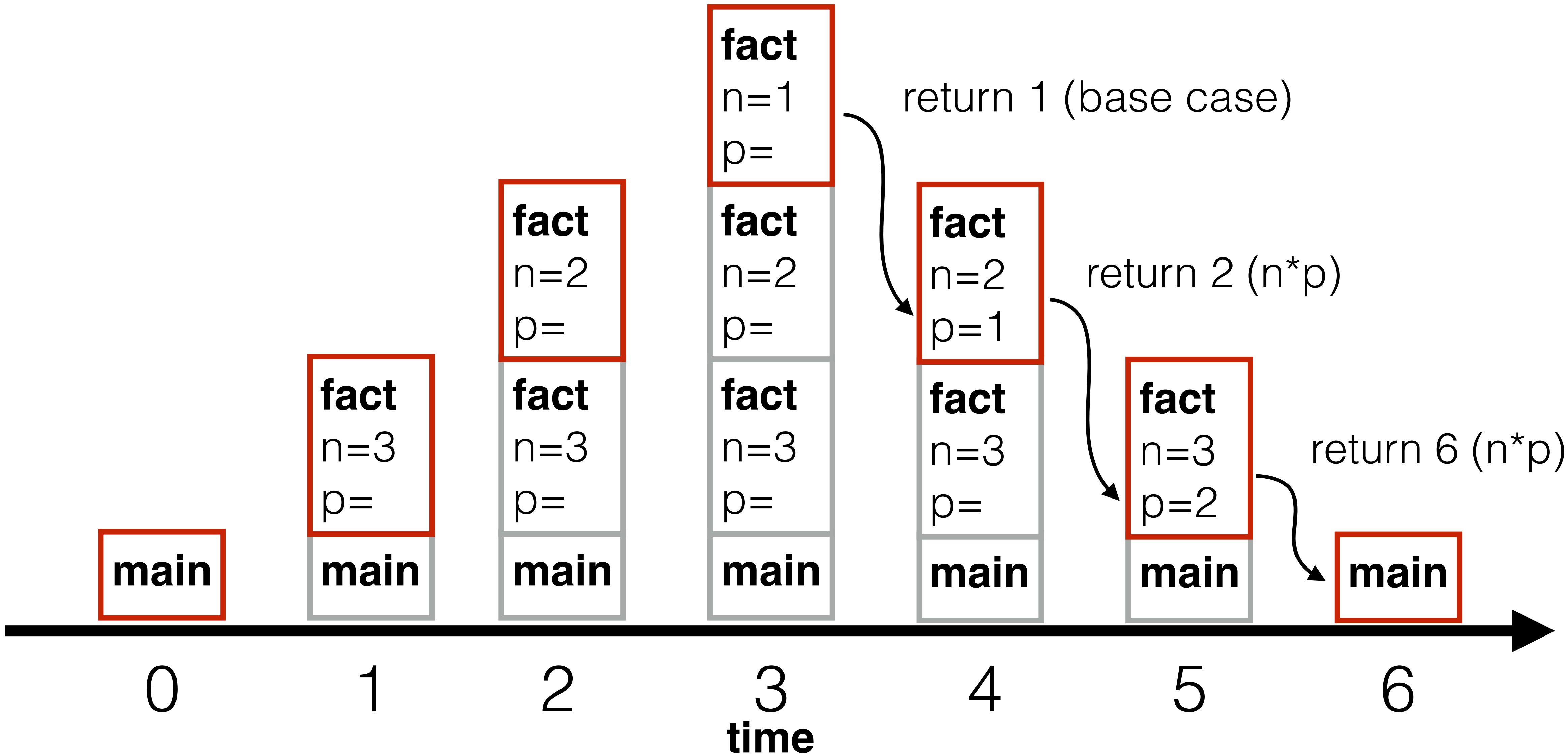

“Infinite” Recursion Bugs

What happens if:

- we forgot the “n == 1” check?
- factorial is called with a negative number?

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`def fact(n):`
~~`    if n == 1:`~~
~~`        return 1`~~
never terminates `    p = fact(n-1)`
`    return n * p`

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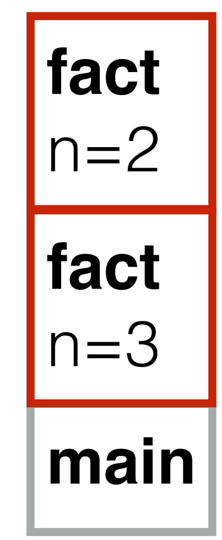

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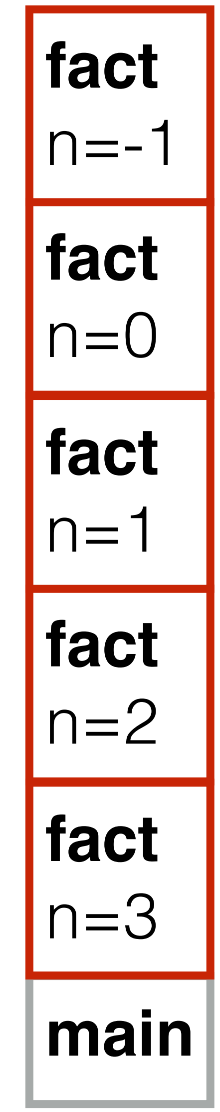

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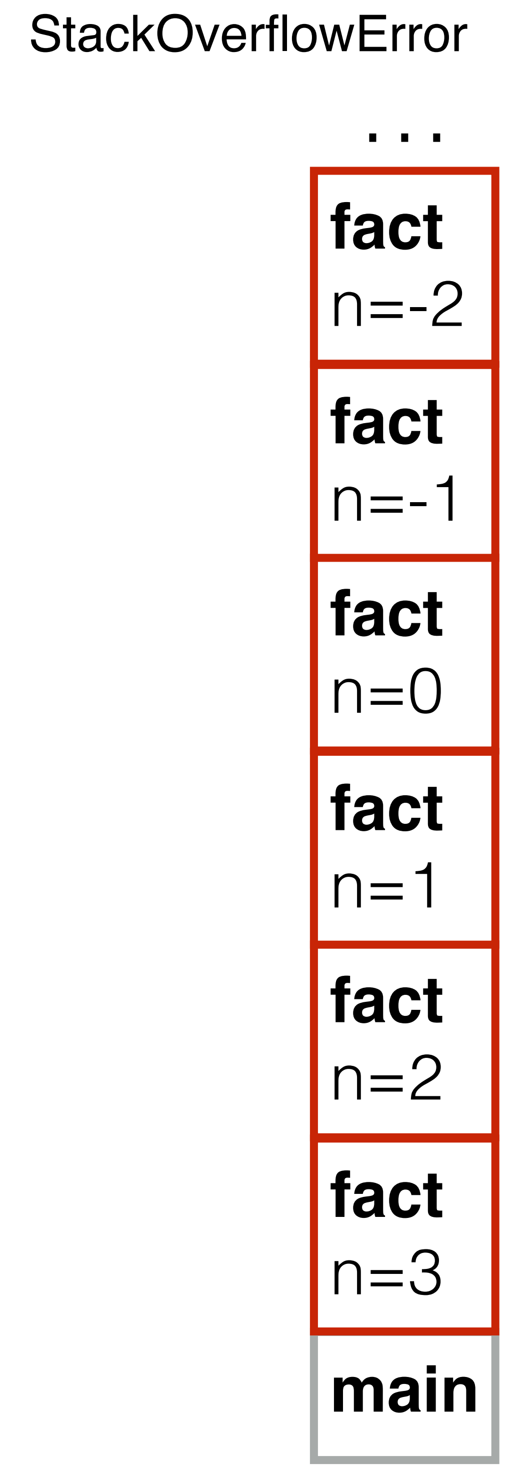


Coding Demos

Demo 1: Pretty Print

Goal: format nested lists of bullet points

Input:

- The recursive lists

Output:

- Appropriately-tabbed items

Example:

```
>>> pretty_print([“A”, [“1”, “2”, “3”,],
                  “B”, [“4”, [“i”, “ii”]]])
*A
  *1
  *2
  *3
*B
  *4
    *i
    *ii
```

Demo 2: Recursive List Search

Goal: does a given number exist in a recursive structure?

Input:

- A number
- A list of numbers and lists (which contain other numbers and lists)

Output:

- True if there's a list containing the number, else False

Example:

```
>>> contains(3, [1,2,[4,[[3],[8,9]],5,6]])
True
>>> contains(12, [1,2,[4,[[3],[8,9]],5,6]])
False
```

Conclusion: Review Learning Objectives

Learning Objectives: Recursive Information

What is a recursive definition/structure?

- Definition contains term
- Structure refers to others of same type
- Example: class **X** is defined to have members of type **X**

What is a base case?

- Recursion terminates without further self reference

Learning Objectives: Recursive Code

What is recursive code?

- Function that sometimes itself (maybe indirectly)

Why write recursive code?

- Real-world data/structures are recursive; intuitive for code to reflect data

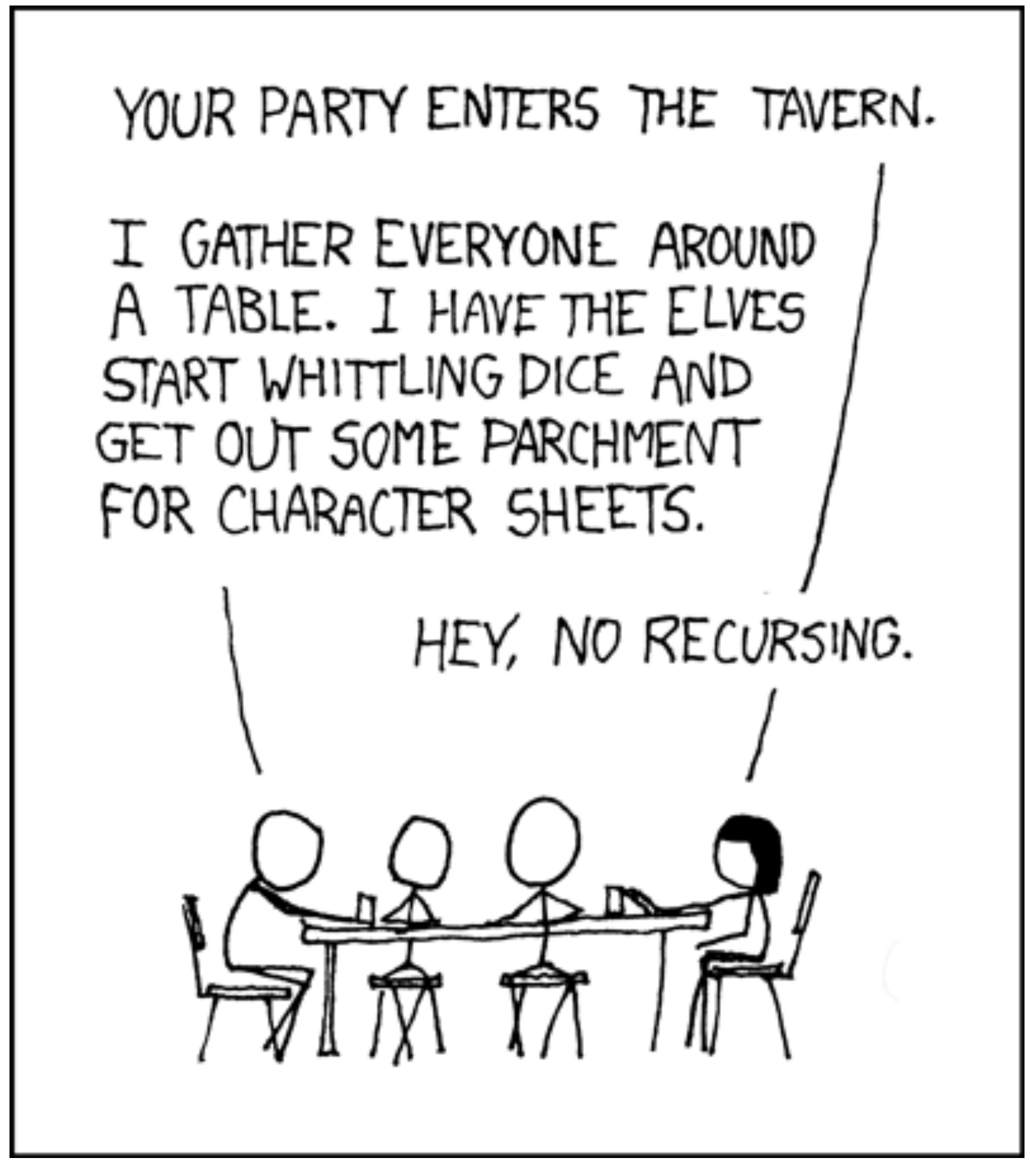
Where do computers keep local variables for recursive calls?

- In a section of memory called a “frame”
- Only one function is **active** at a time, so keep frames in a stack

What happens to programs with infinite recursion?

- Calls keep pushing more frames
- Exhaust memory, throw **StackOverflowError**

Questions?



https://xkcd.com/244/