[301] Regression

(and numpy)

Tyler Caraza-Harter

Learning Objectives Today

History of regression

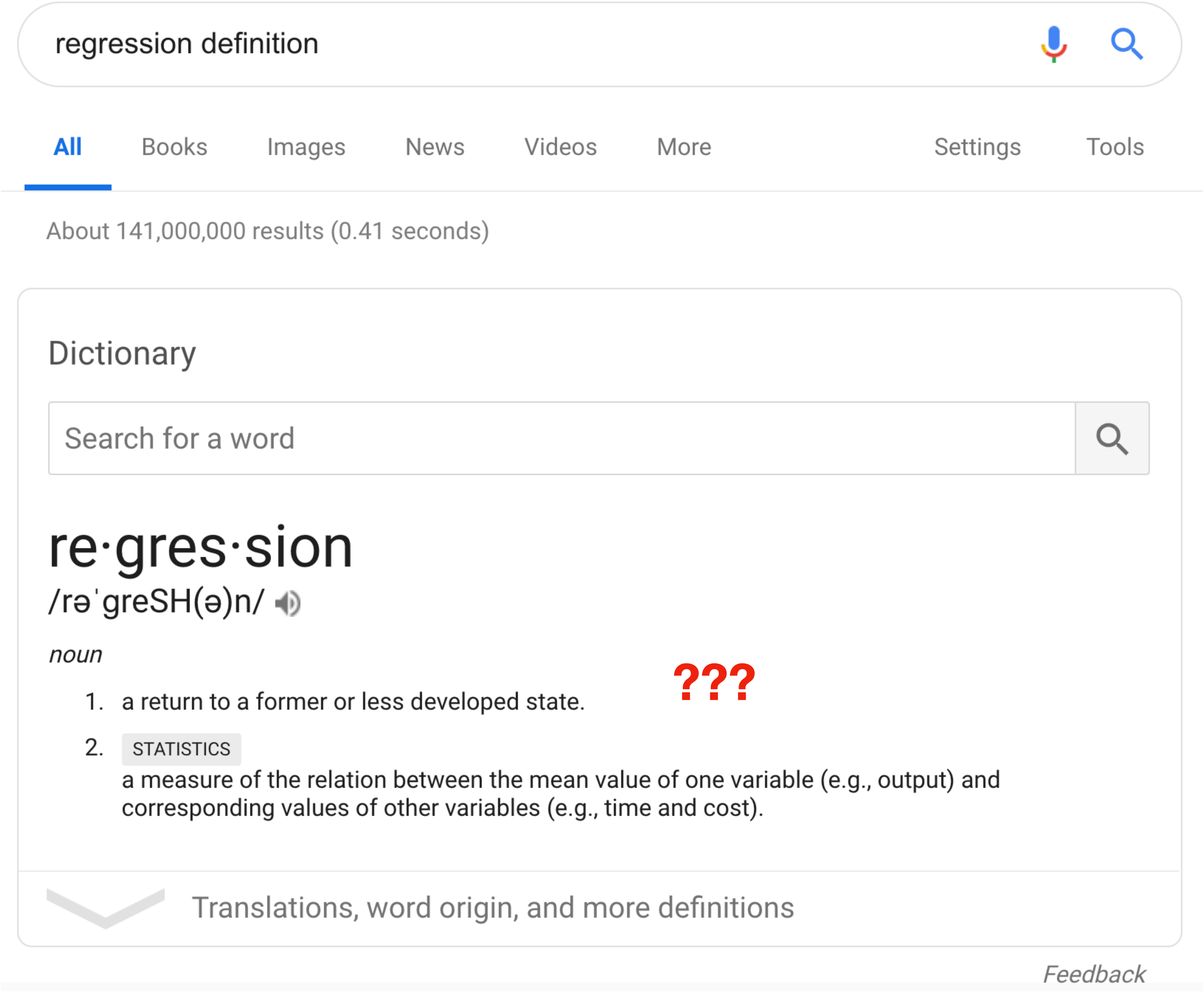
Drawing a fit line

Finding the slope/intercept w/ least squares method

Numpy introduction

Using numpy.linalg.lstsq

Definition



About 141,000,000 results (0.41 seconds)

Dictionary

re·gres·sion

/rəˈgreSH(ə)n/

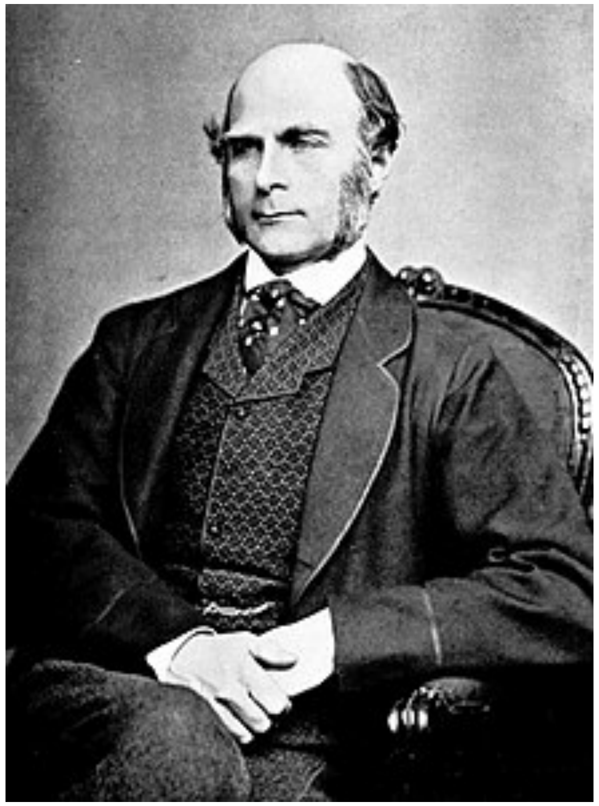
noun

1. a return to a former or less developed state.
2. STATISTICS
   a measure of the relation between the mean value of one variable (e.g., output) and corresponding values of other variables (e.g., time and cost).

???

History of Regression

Francis Galton

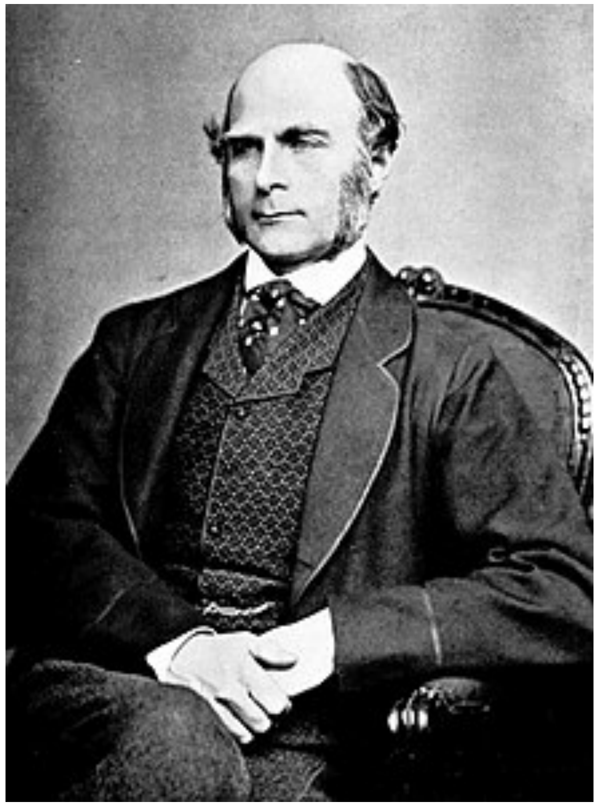


https://en.wikipedia.org/wiki/Francis_Galton

Question: what is the relationship between a parent’s and child’s height (both as adults)

History of Regression

Francis Galton

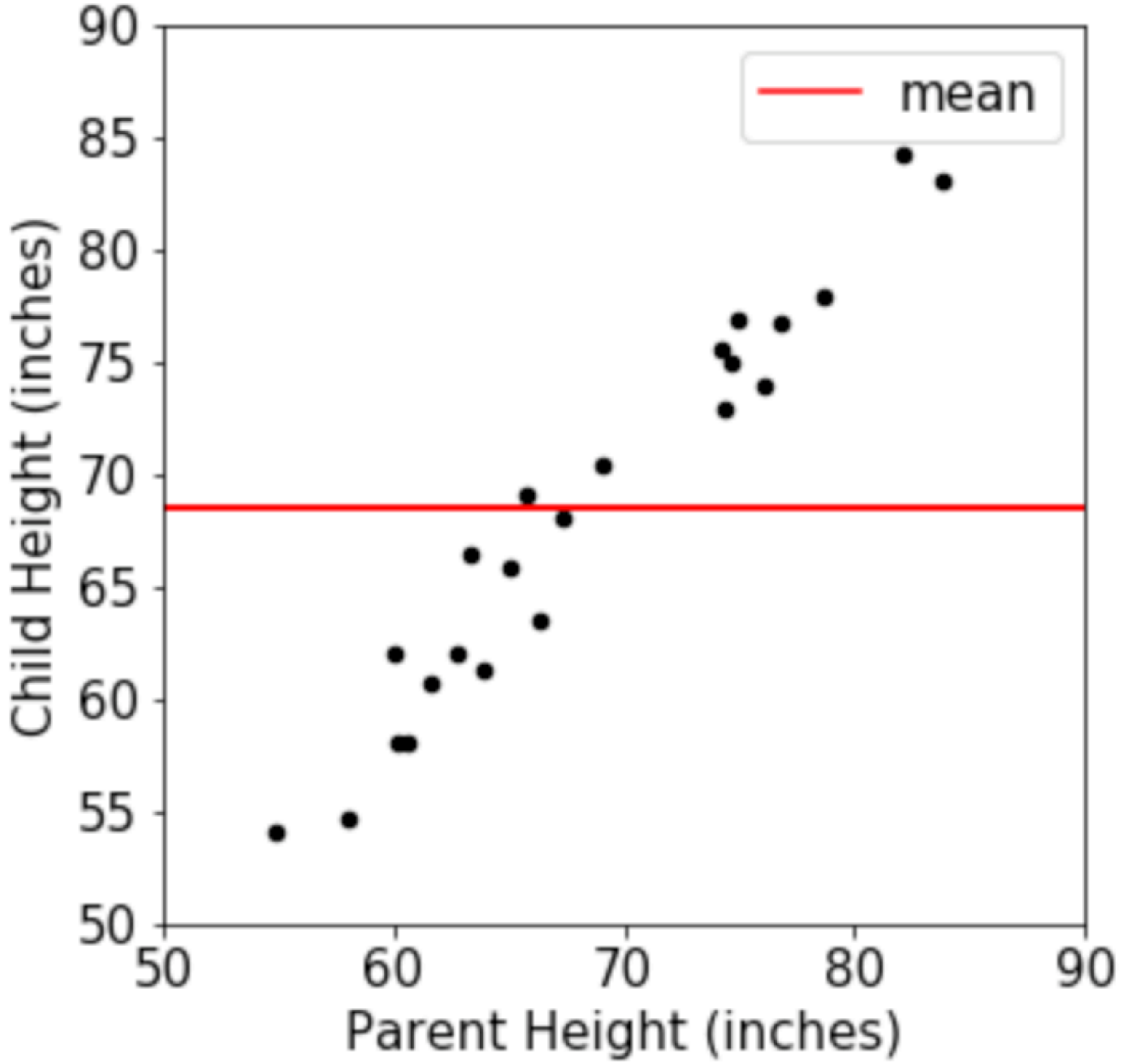


https://en.wikipedia.org/wiki/Francis_Galton

Question: what is the relationship between a parent’s and child’s height (both as adults)

What kind of plot should we make?

Result you might expect

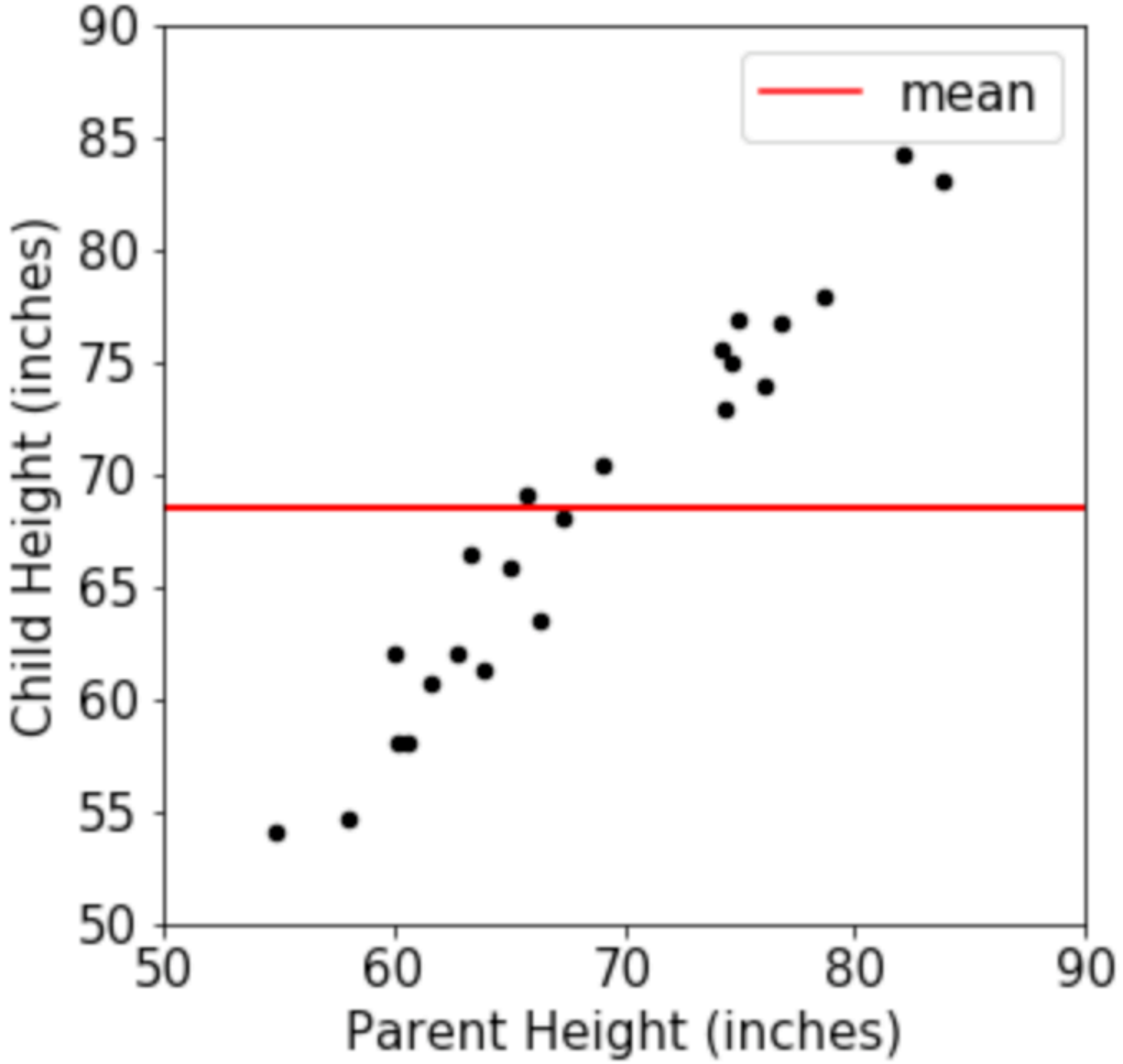


Observation:

- child height equals parent height (plus some noise)

Note: all these height plots contain fake data

Result you might expect



Observation:

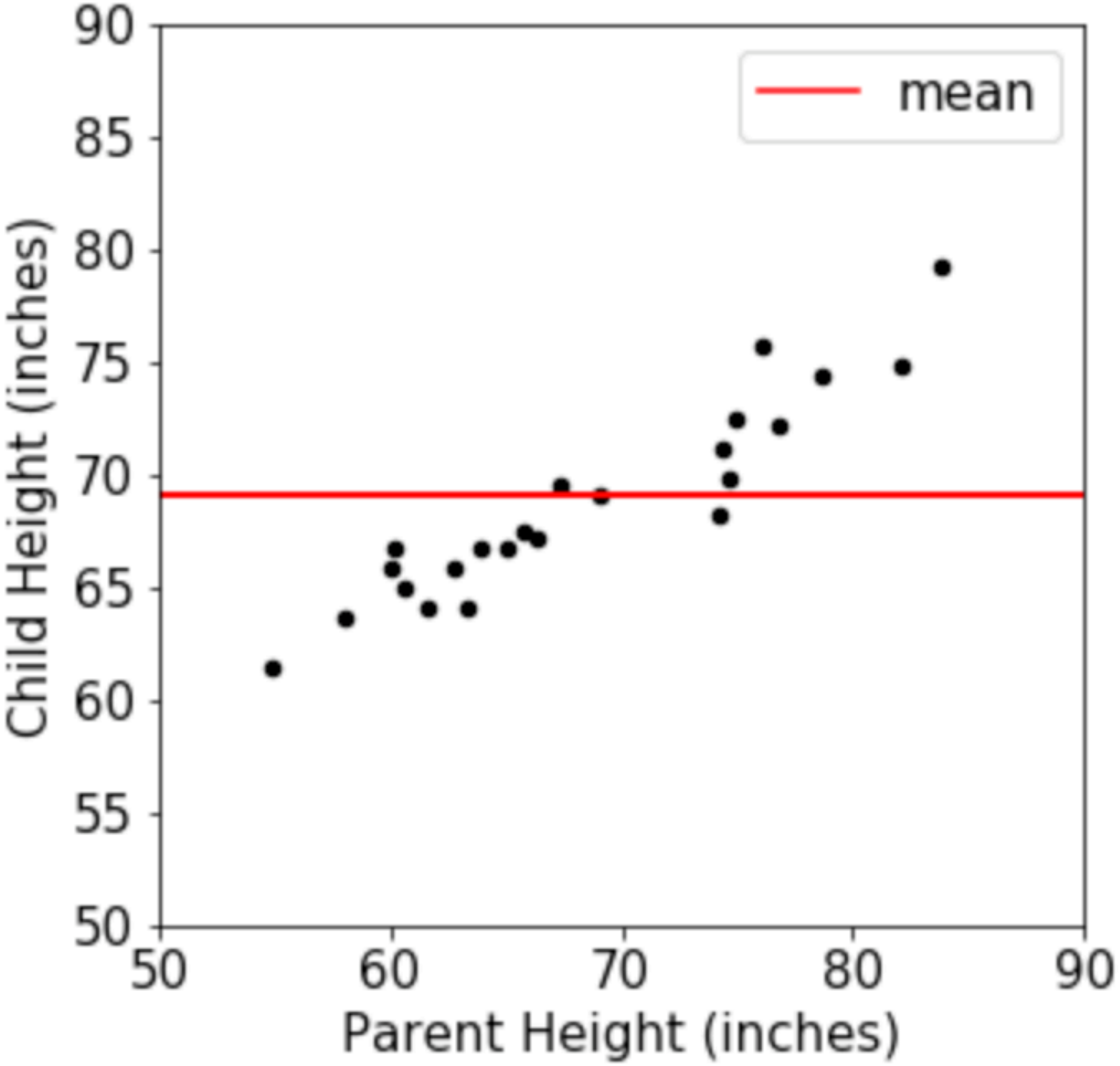
- child height equals parent height (plus some noise)

What about other factors?

- height of other parent
- nutrition
- etc

Note: all these height plots contain fake data

More realistic results

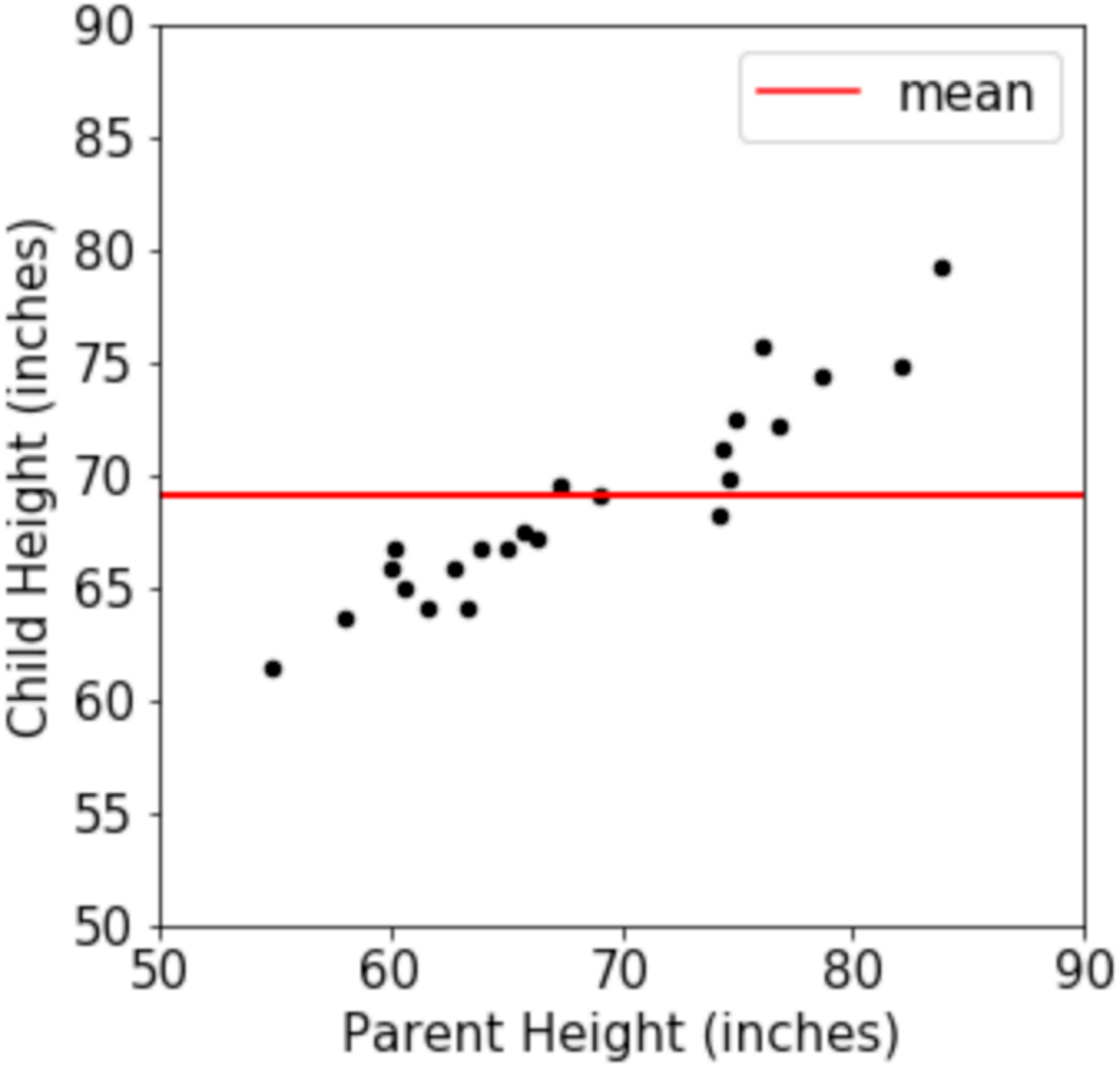


Observation:

- heights are correlated
- tall parents tend to have shorter children
- short parents tend to have taller children

Note: all these height plots contain fake data

More realistic results



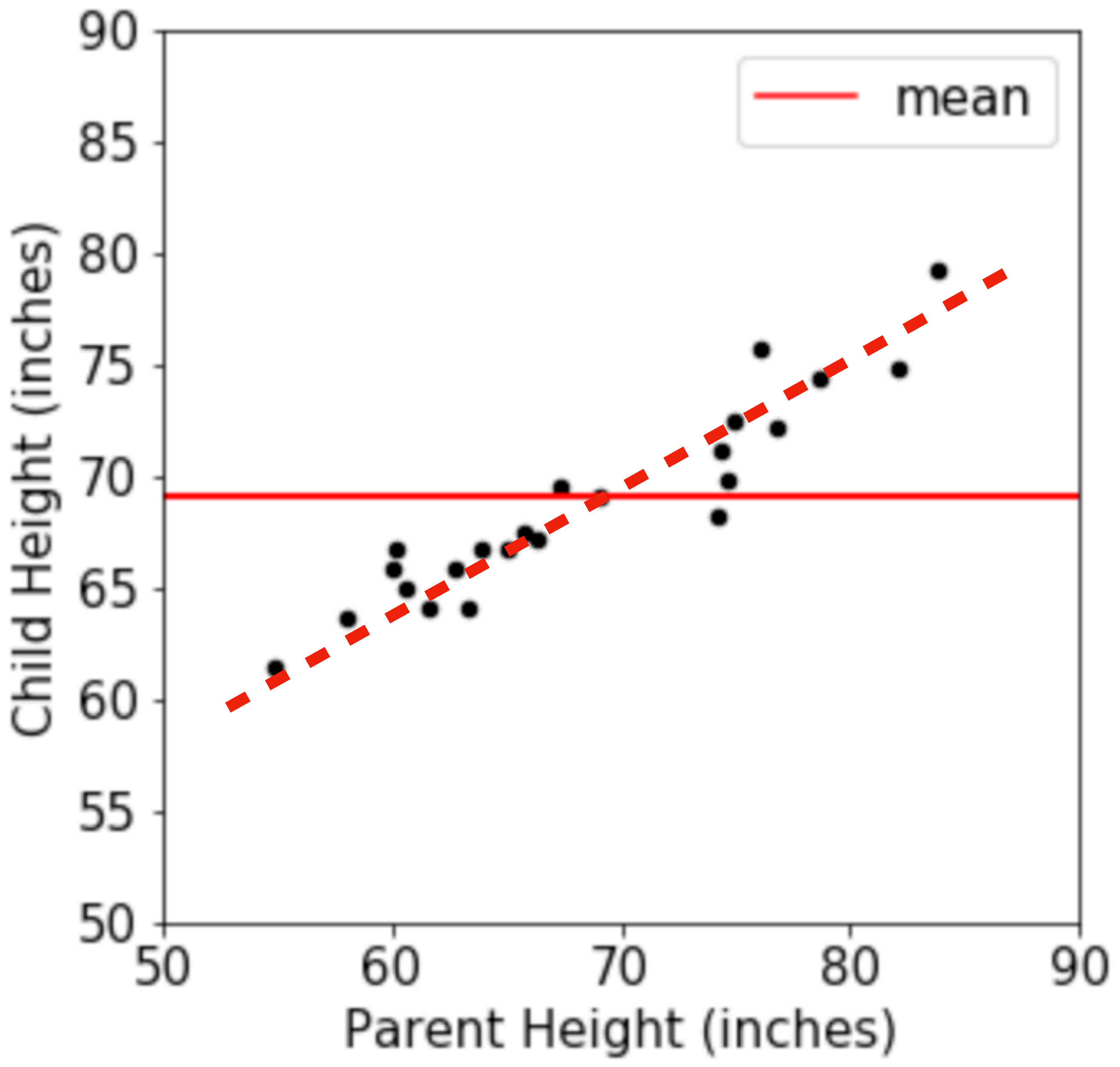
Observation:

- heights are correlated
- tall parents tend to have shorter children
- short parents tend to have taller children

Galton referred to this phenomenon as “regression to the mean”.

Note: all these height plots contain fake data

More realistic results



Observation:

- heights are correlated
- tall parents tend to have shorter children
- short parents tend to have taller children

Galton referred to this phenomenon as “regression to the mean”.

Nowadays, “regression” can refer to any fitting of a line to the points.

Note: all these height plots contain fake data

Learning Objectives Today

History of regression

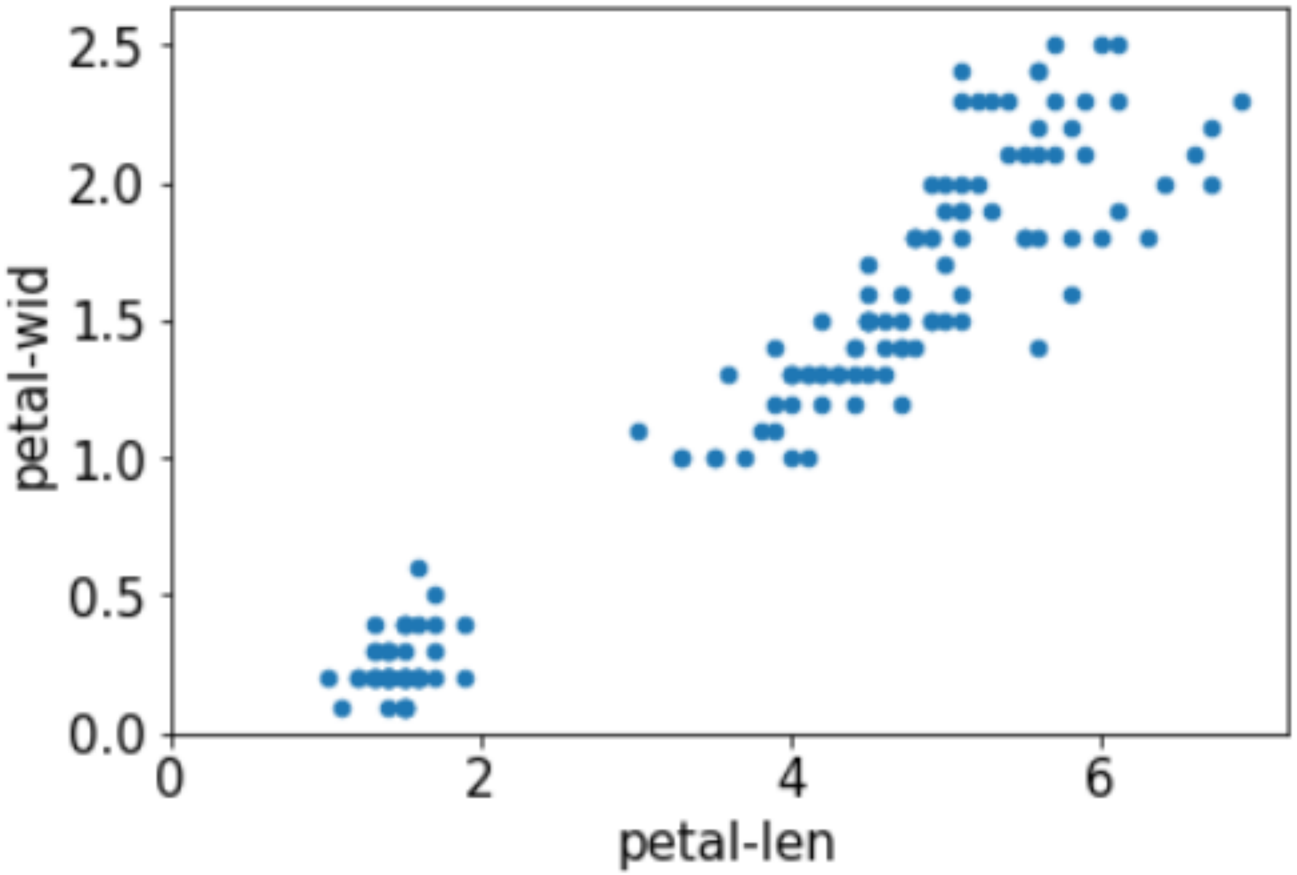
Drawing a fit line

Finding the slope/intercept w/ least squares method

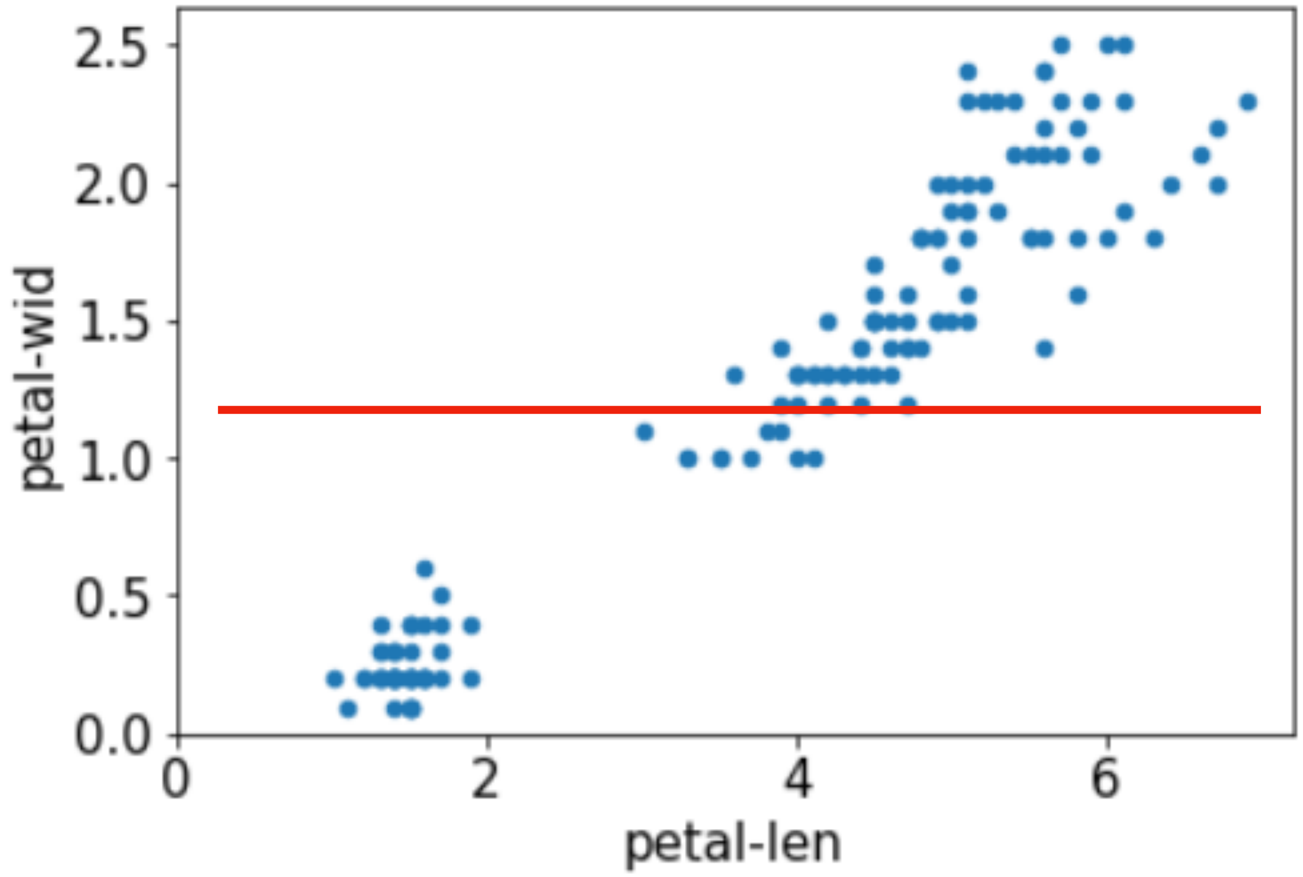
Numpy introduction

Using numpy.linalg.lstsq

Demo 1: annotate Iris data

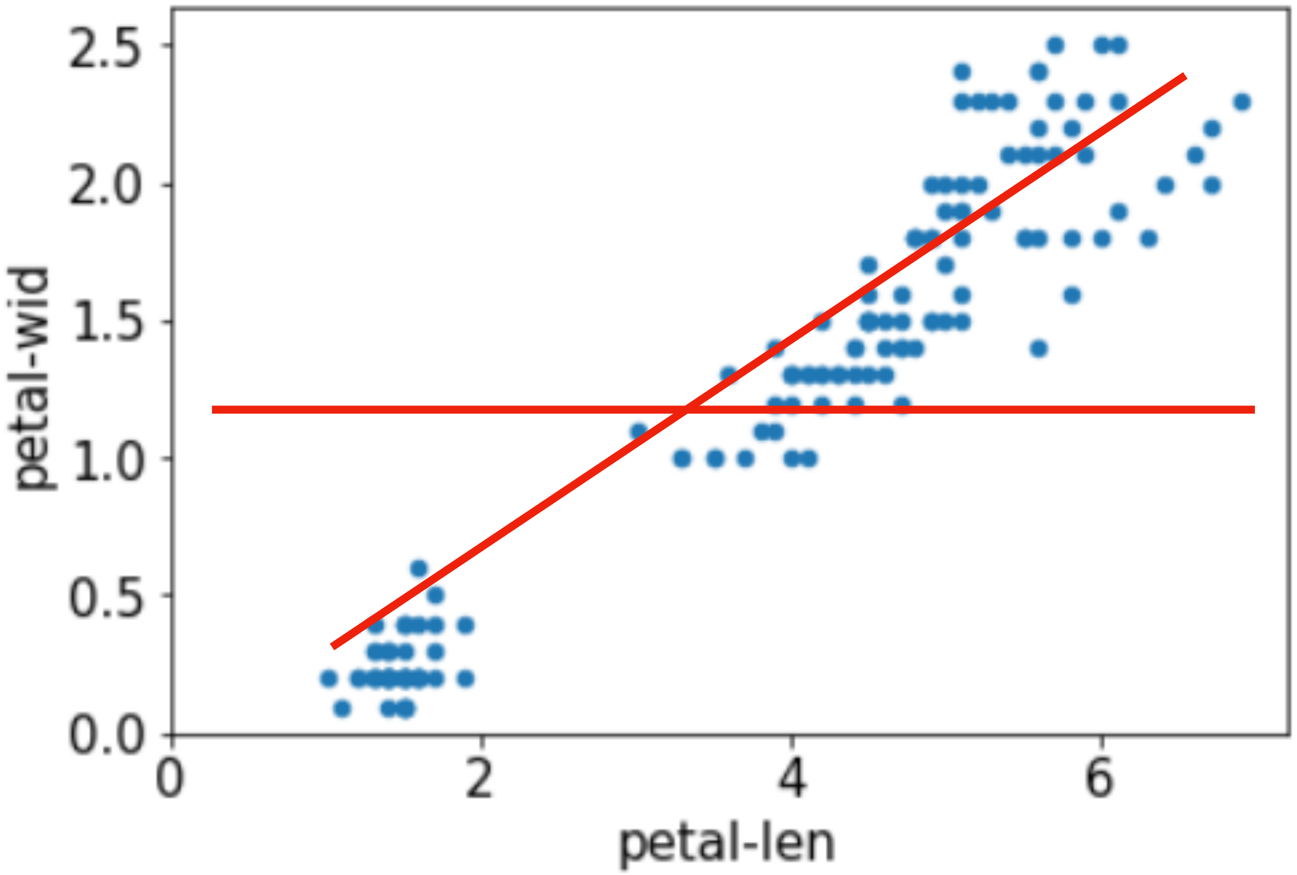


Demo 1: annotate Iris data



Annotation 1: mean line

Demo 1: annotate Iris data



Annotation 1: mean line

Annotation 2: fit line

- assume slope=1/3
- y-intercept=0

not necessarily the best line

Learning Objectives Today

History of regression

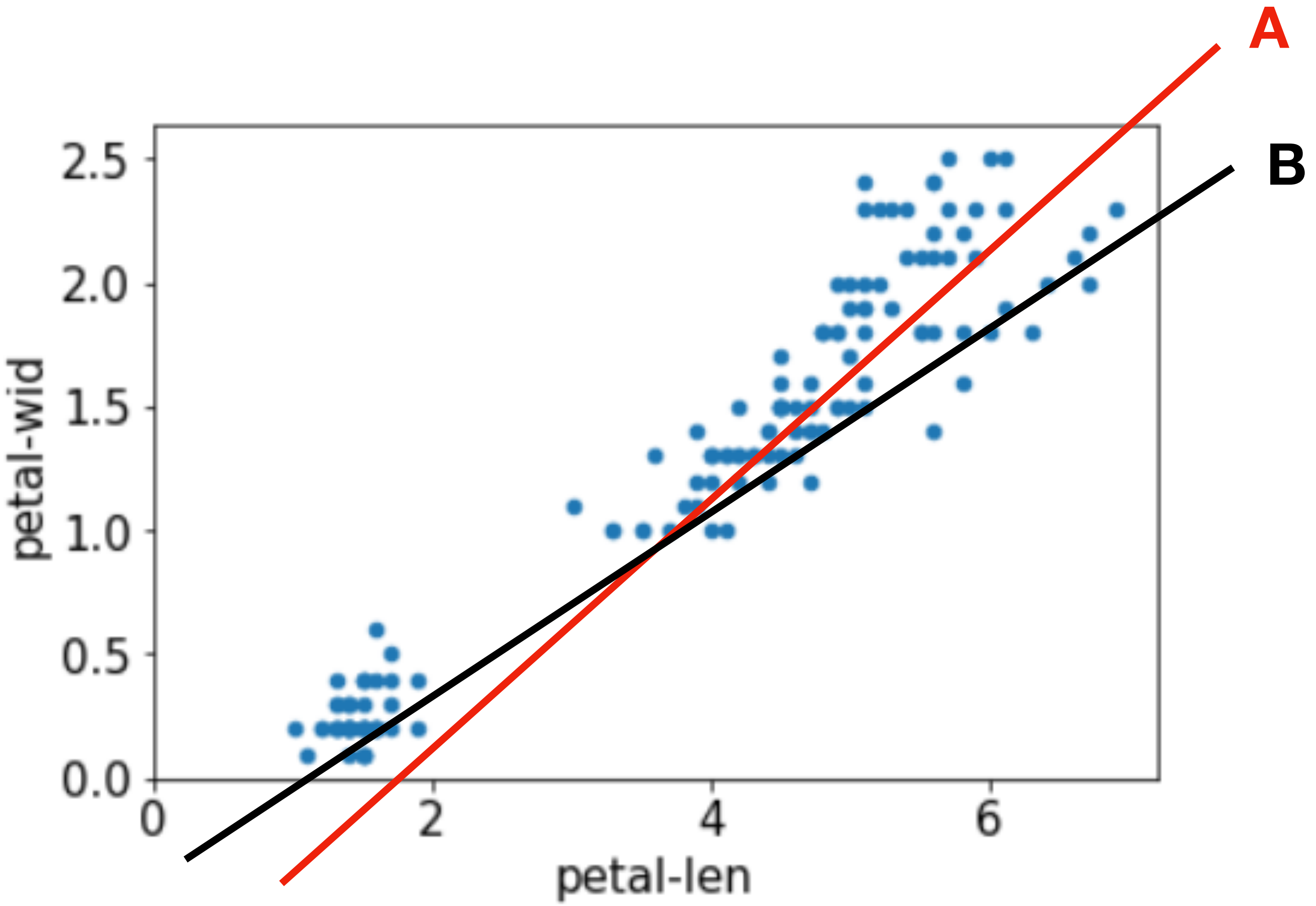
Drawing a fit line

Finding the slope/intercept w/ least-squares method

Numpy introduction

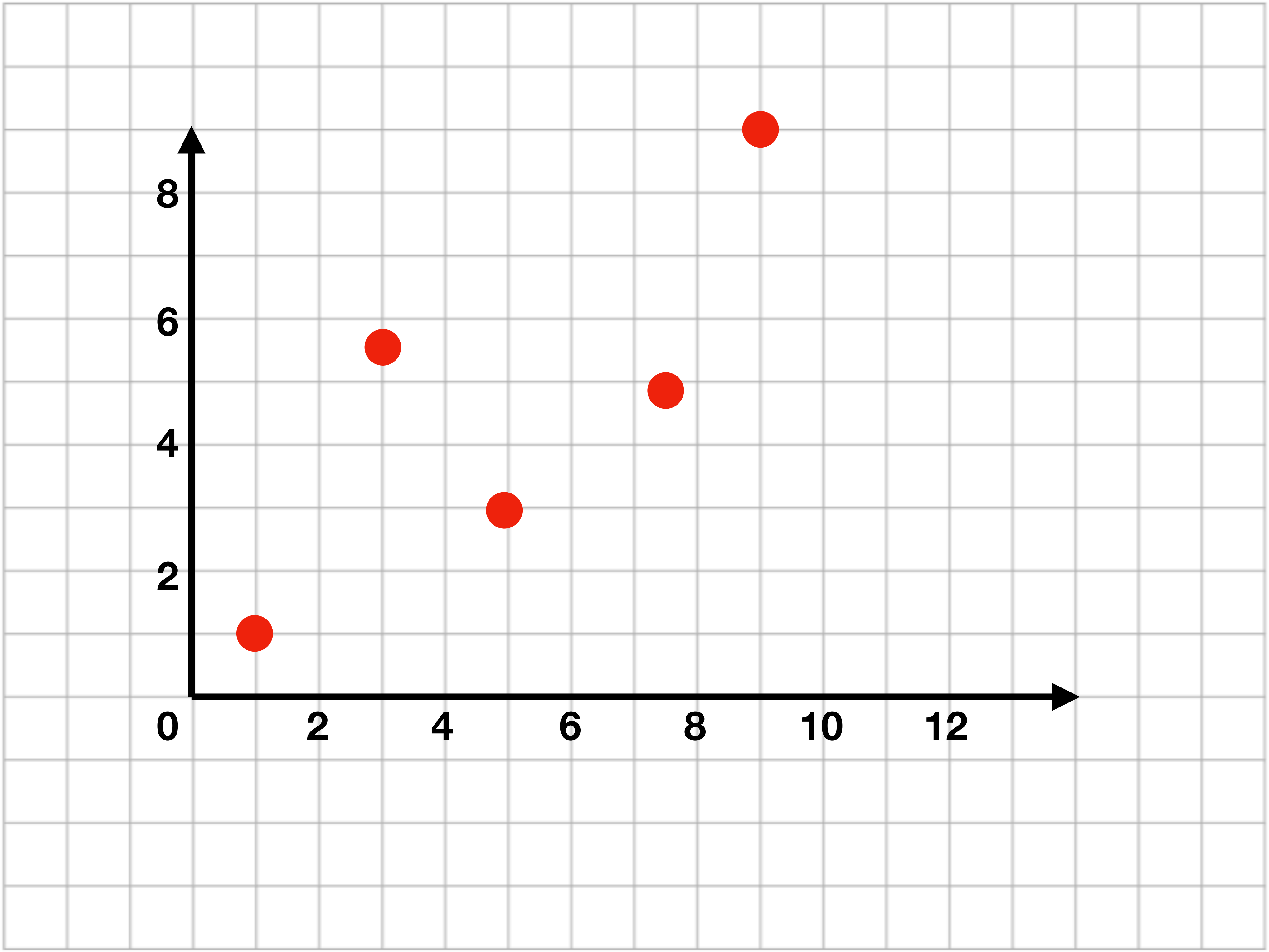
Using numpy.linalg.lstsq

Which fit line is better?

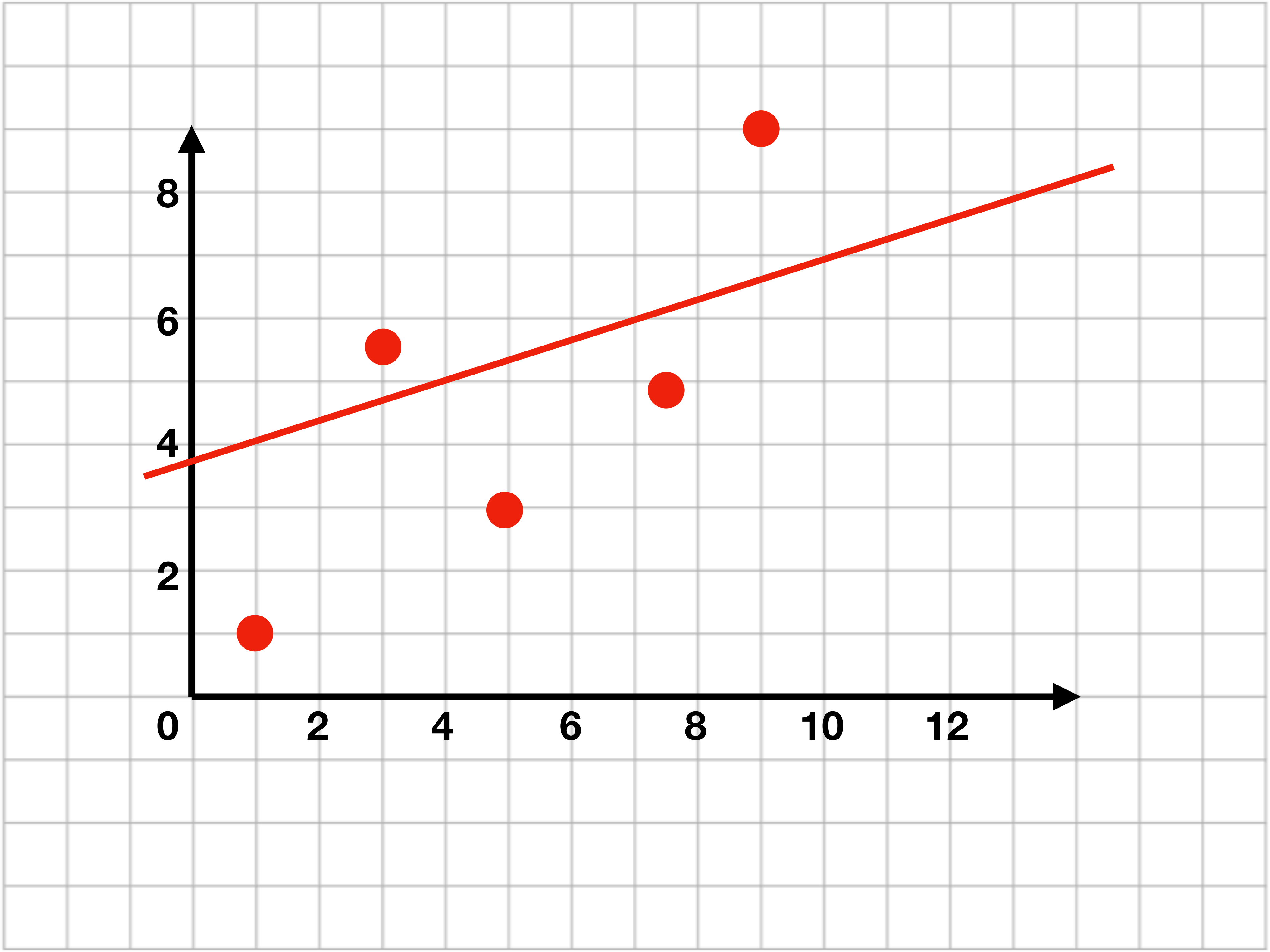


We need a metric to evaluate how good a fit is

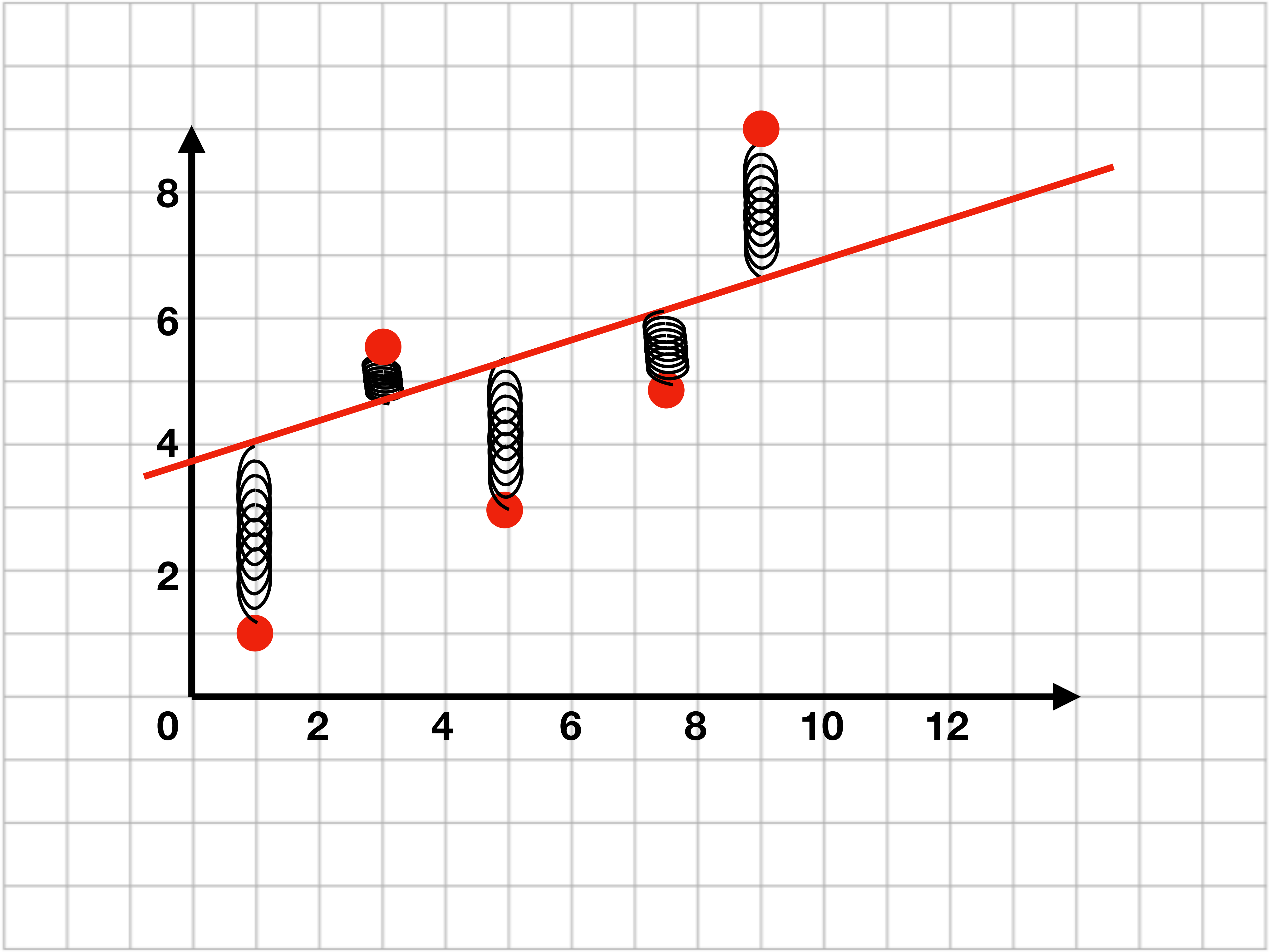
Intuition: Springs



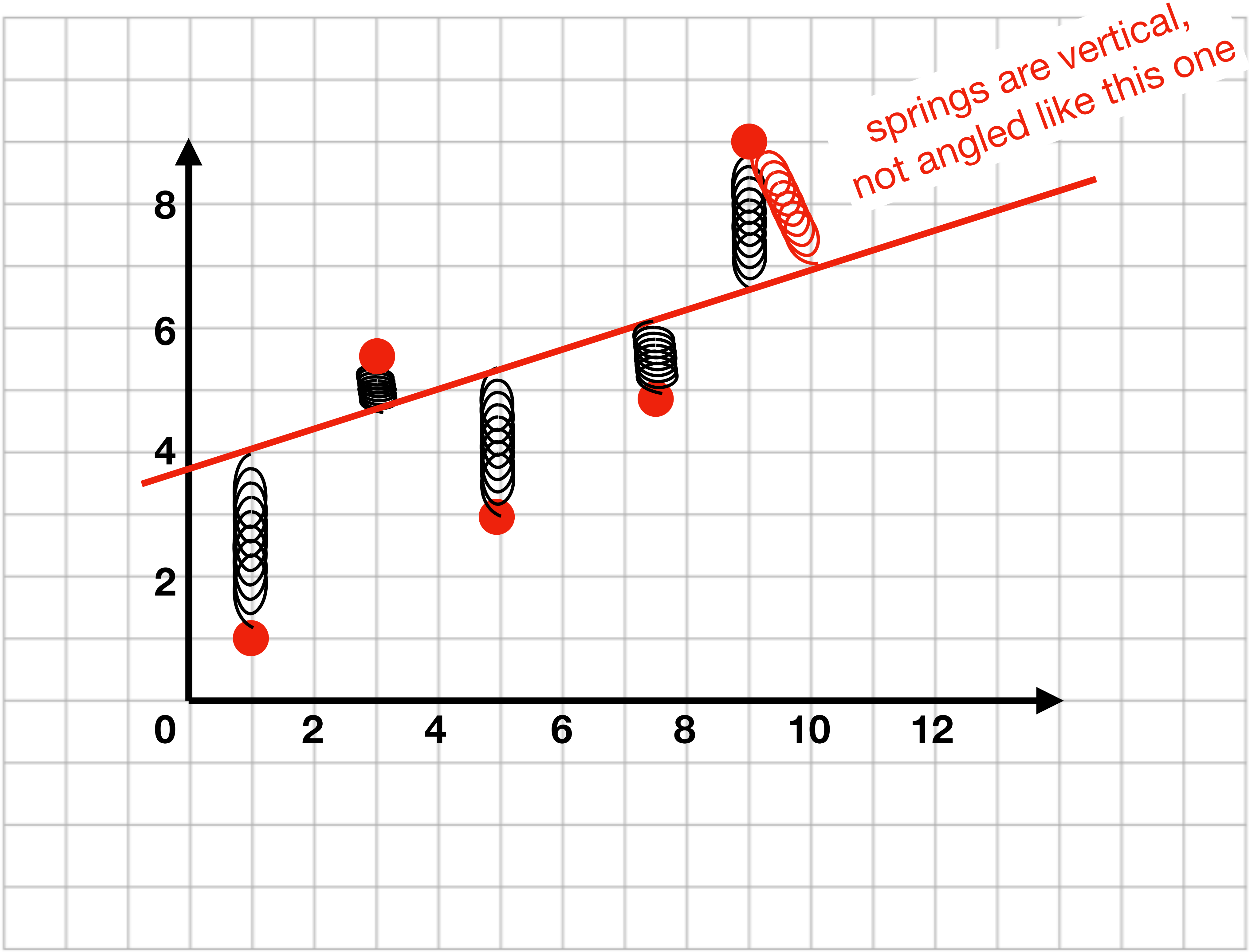
Intuition: Springs



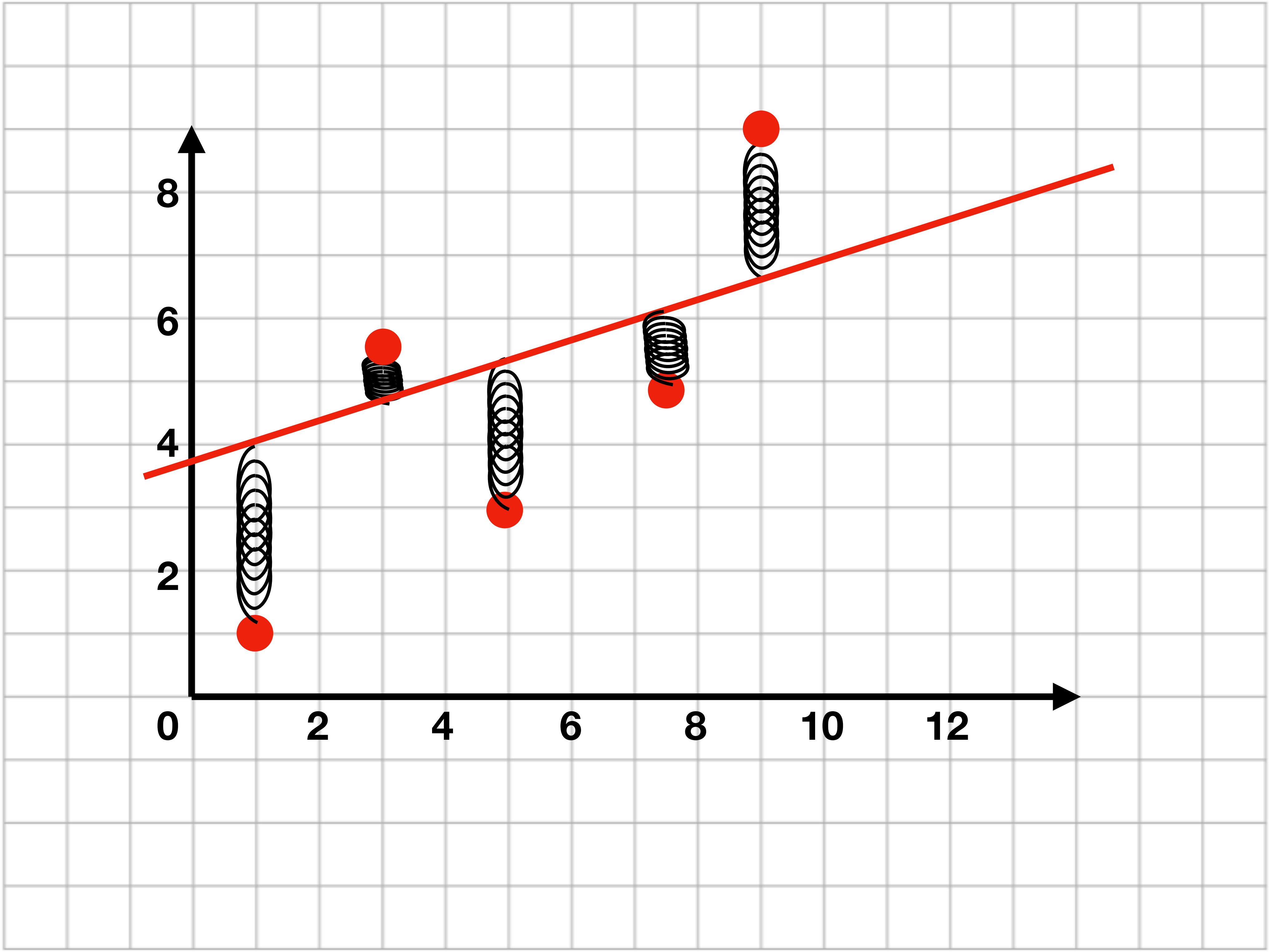
Intuition: Springs



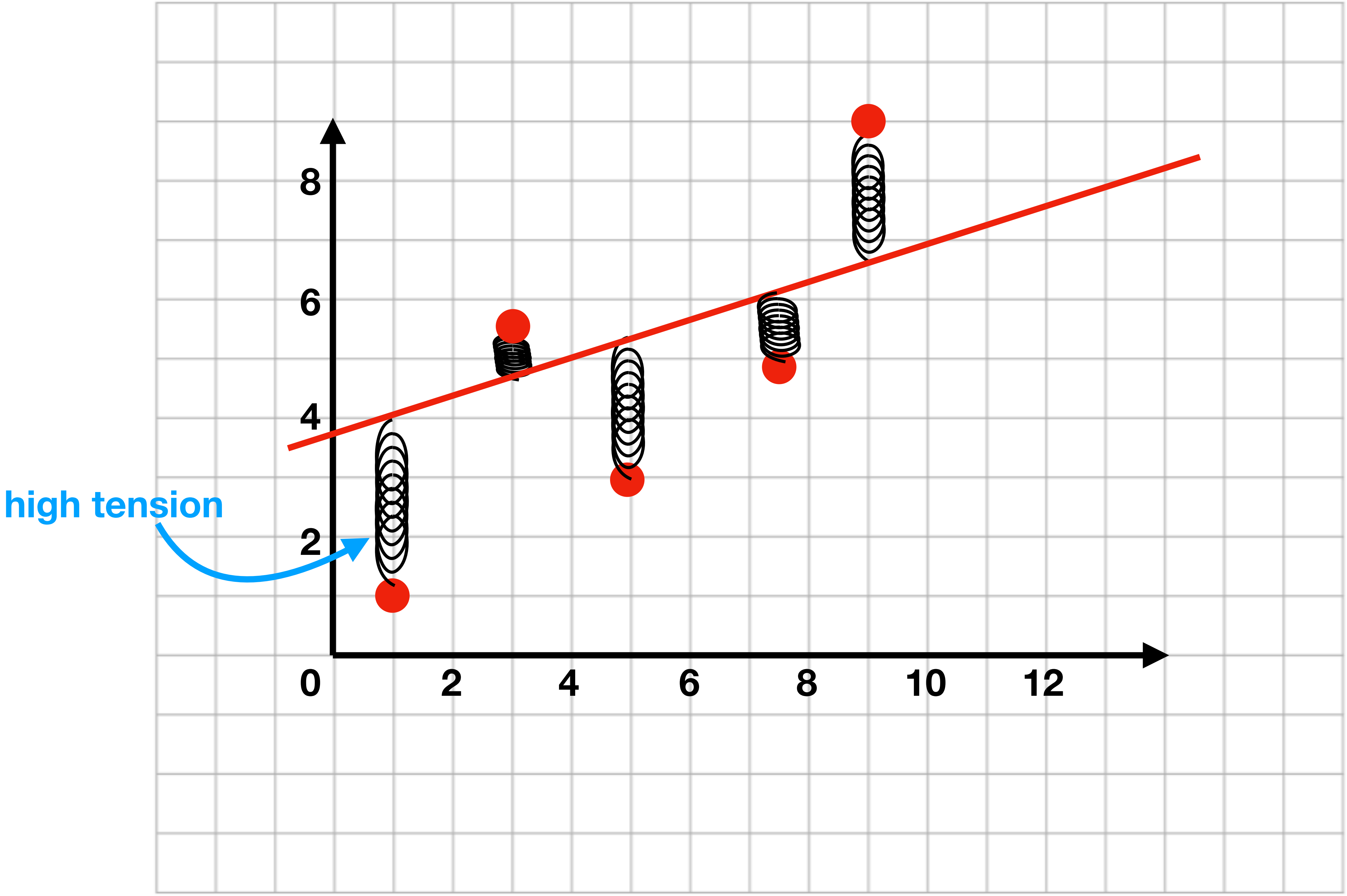
Intuition: Springs



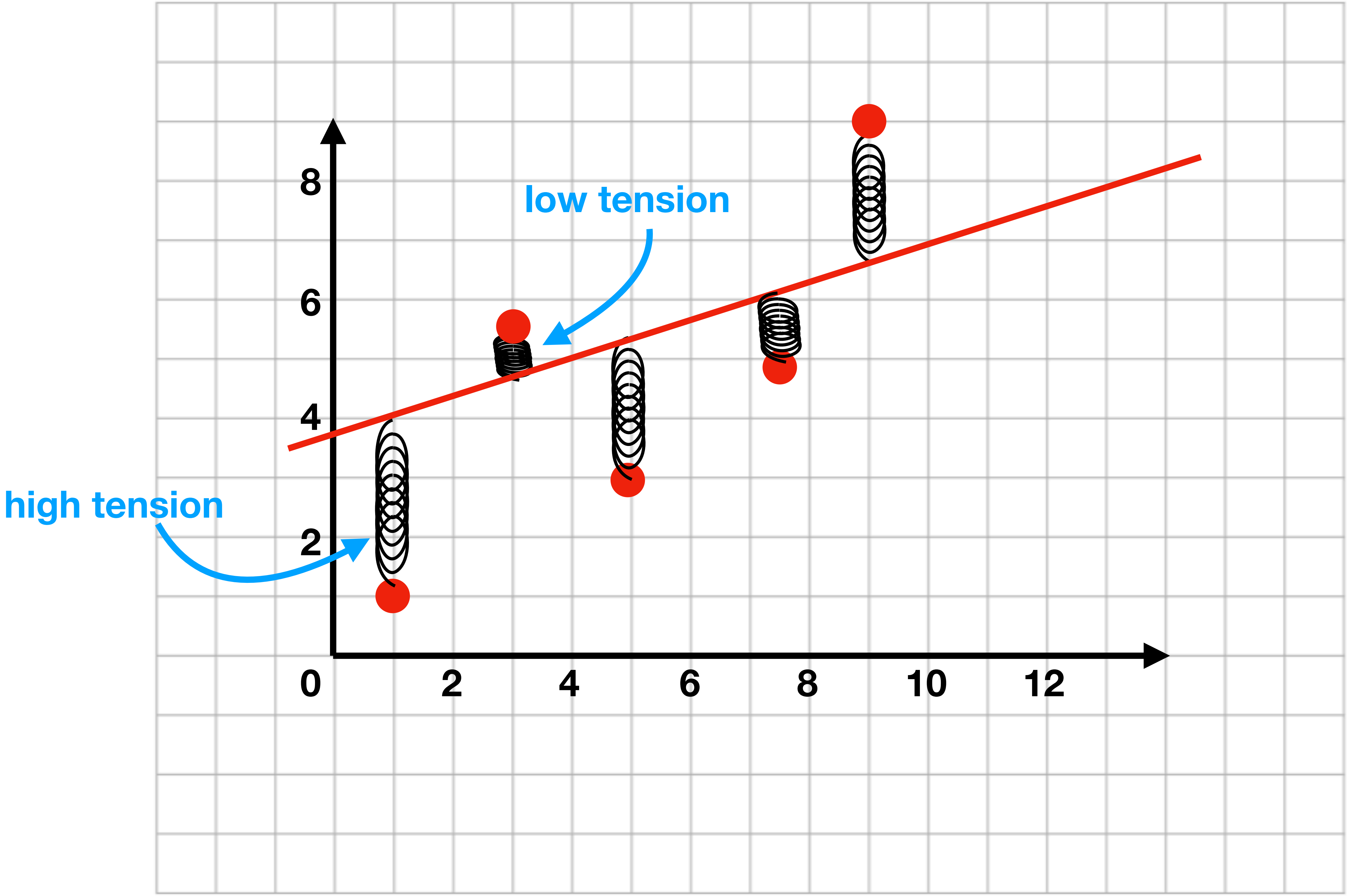
Intuition: Springs



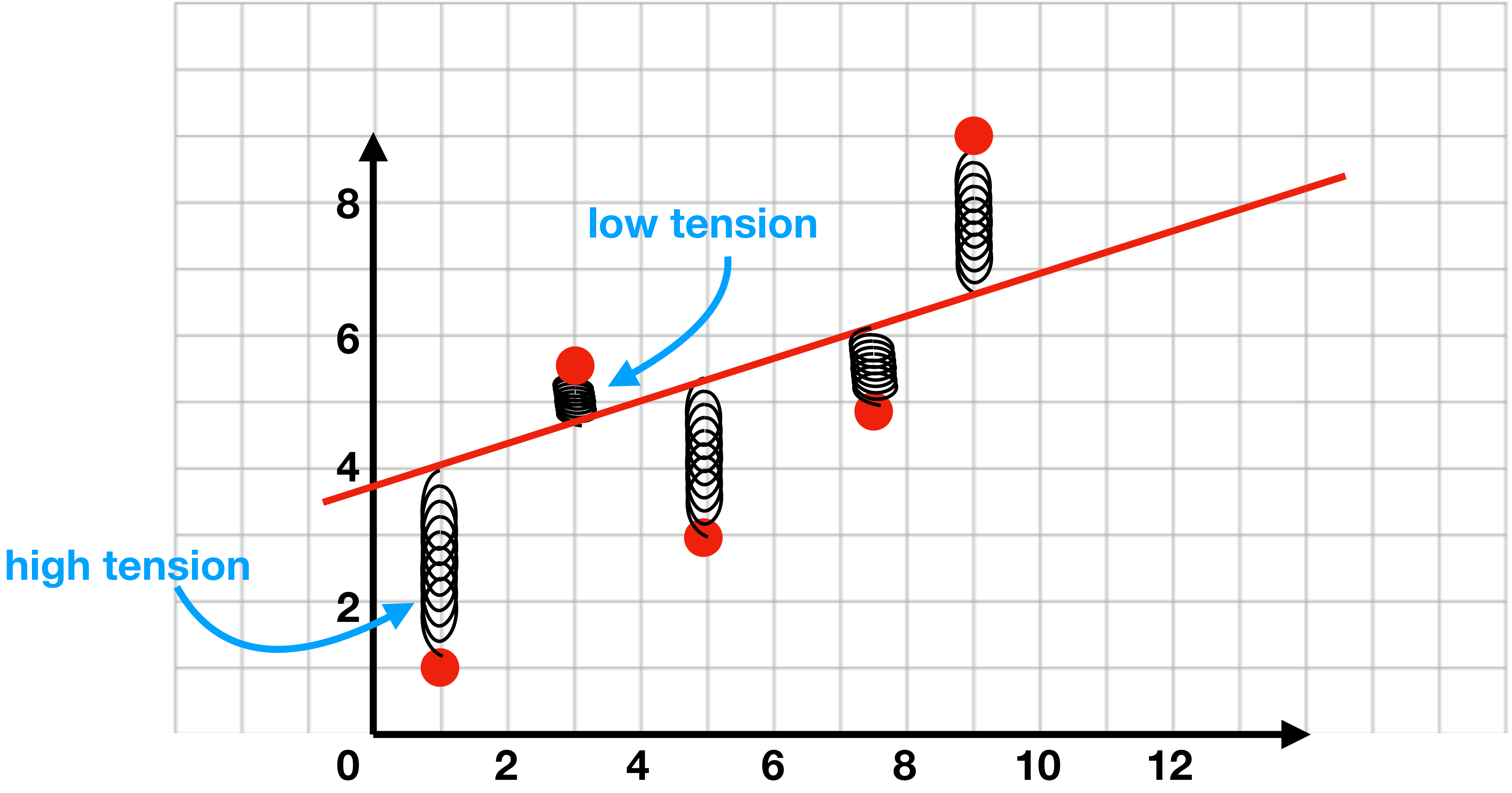
Intuition: Springs



Intuition: Springs

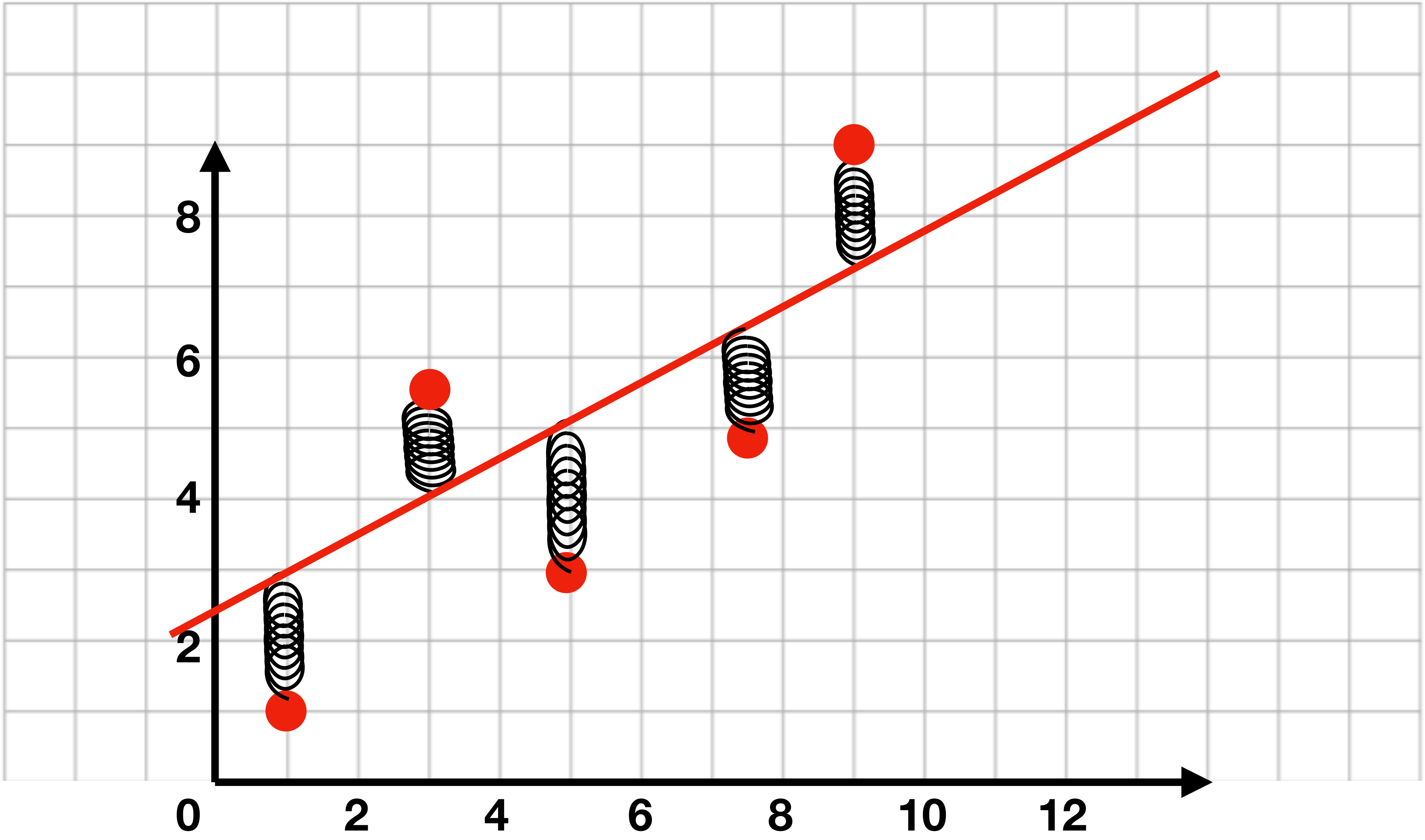


Intuition: Springs



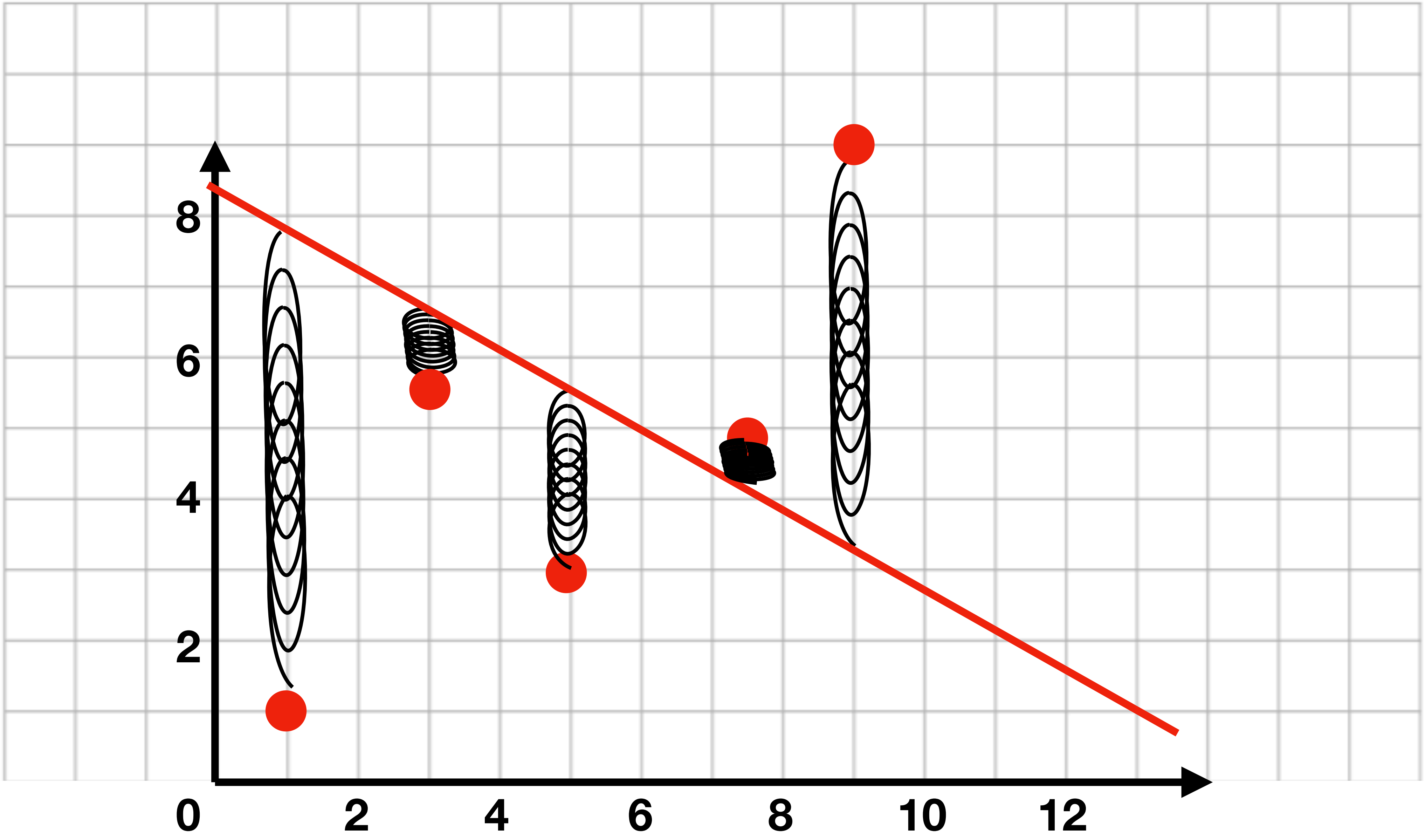
The best line minimizes total tension across springs

Intuition: Springs



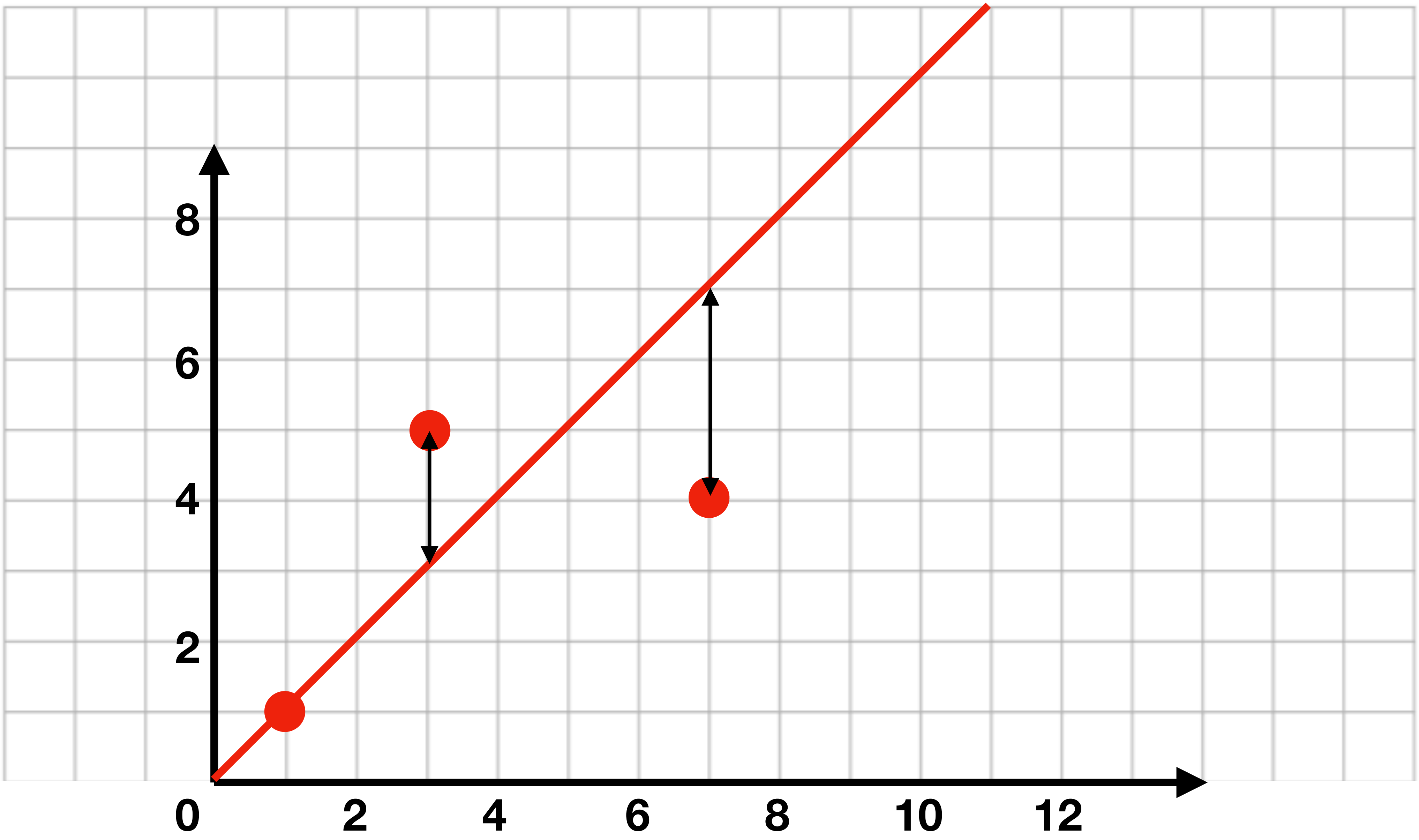
Good fit with low overall tension

Intuition: Springs



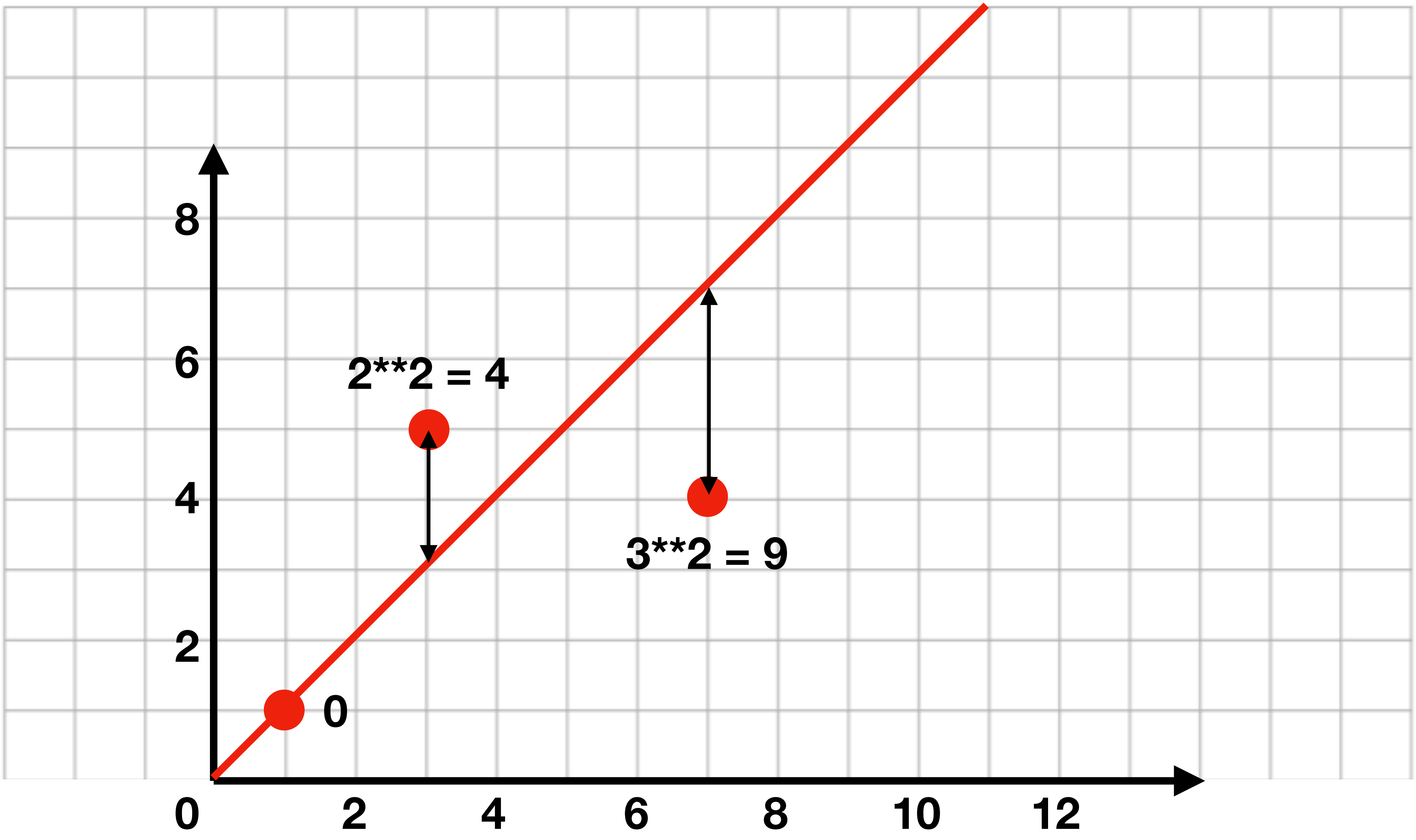
Bad fit with high overall tension

Intuition: Springs



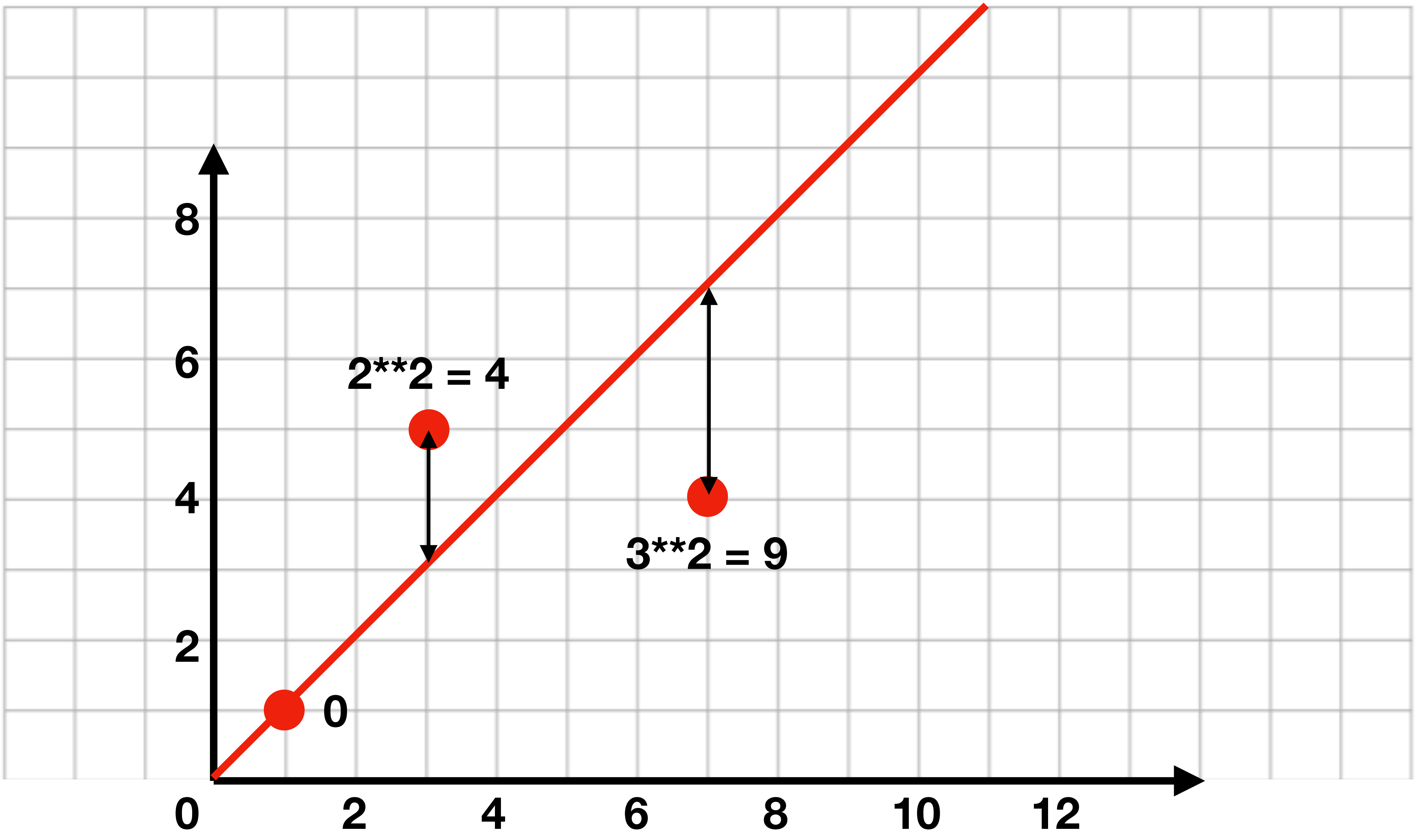
Tension is defined as distance squared

Intuition: Springs



Tension is defined as distance squared

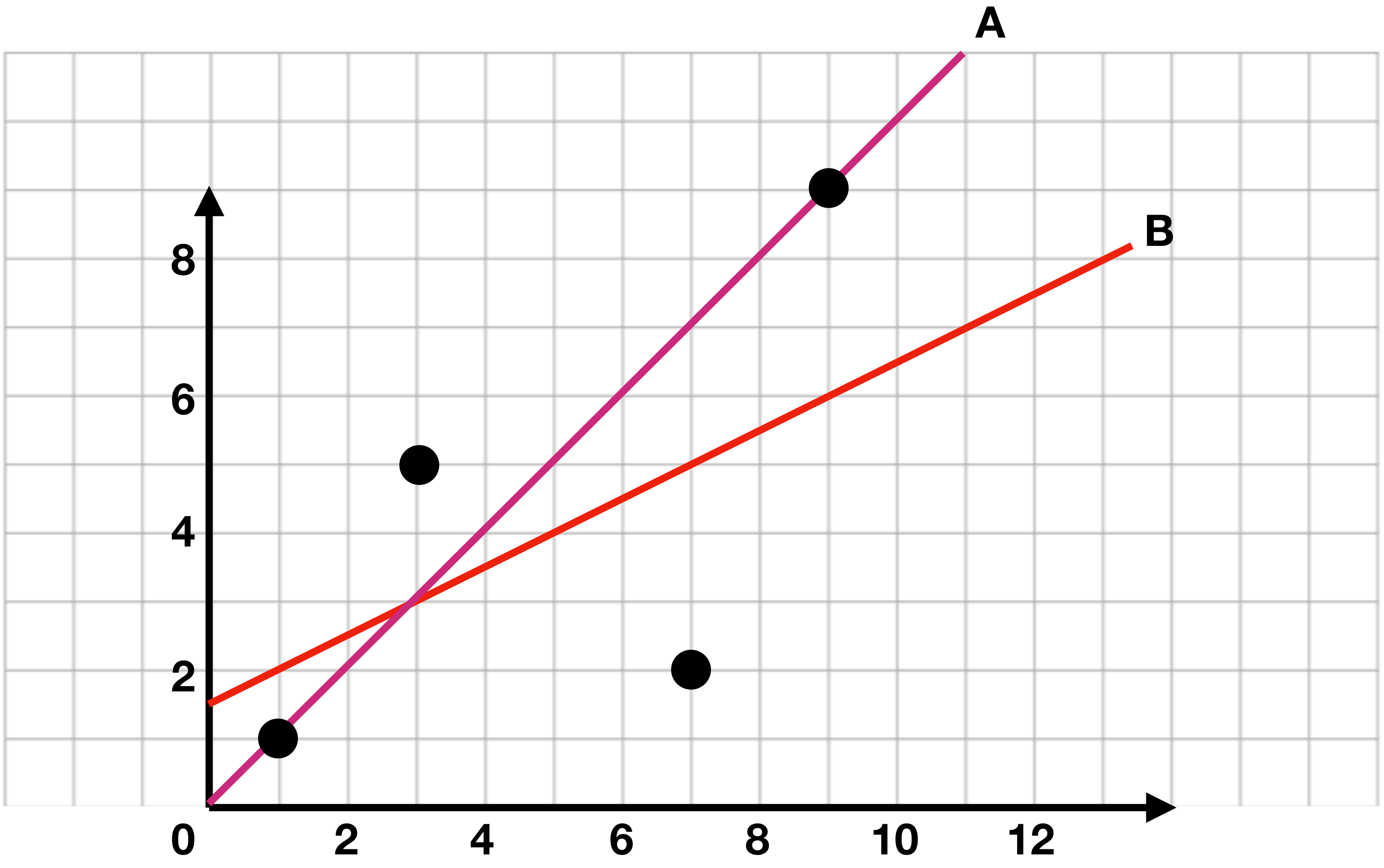
Intuition: Springs



Tension is defined as distance squared

Total: 4+9 = 13

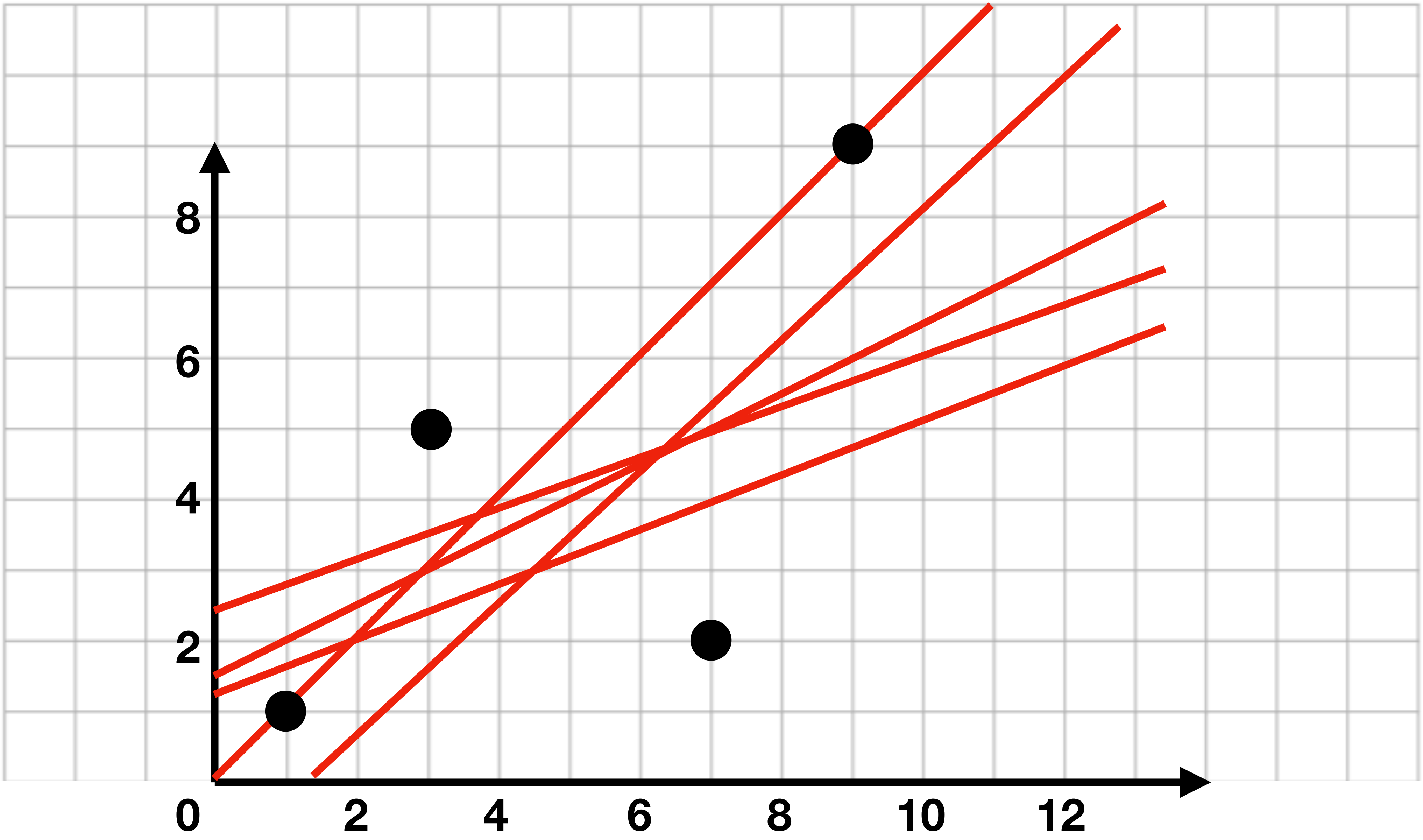
Practice



How much tension is in line A?

How much tension is in line B?

Optimization



There are many possible fit lines, but we want the one with the minimal tension.

Rather than crunch the numbers ourselves, we'll use a function from the numpy module

Learning Objectives Today

History of regression

Drawing a fit line

Finding the slope/intercept w/ least squares method

Numpy introduction

Using numpy.linalg.lstsq

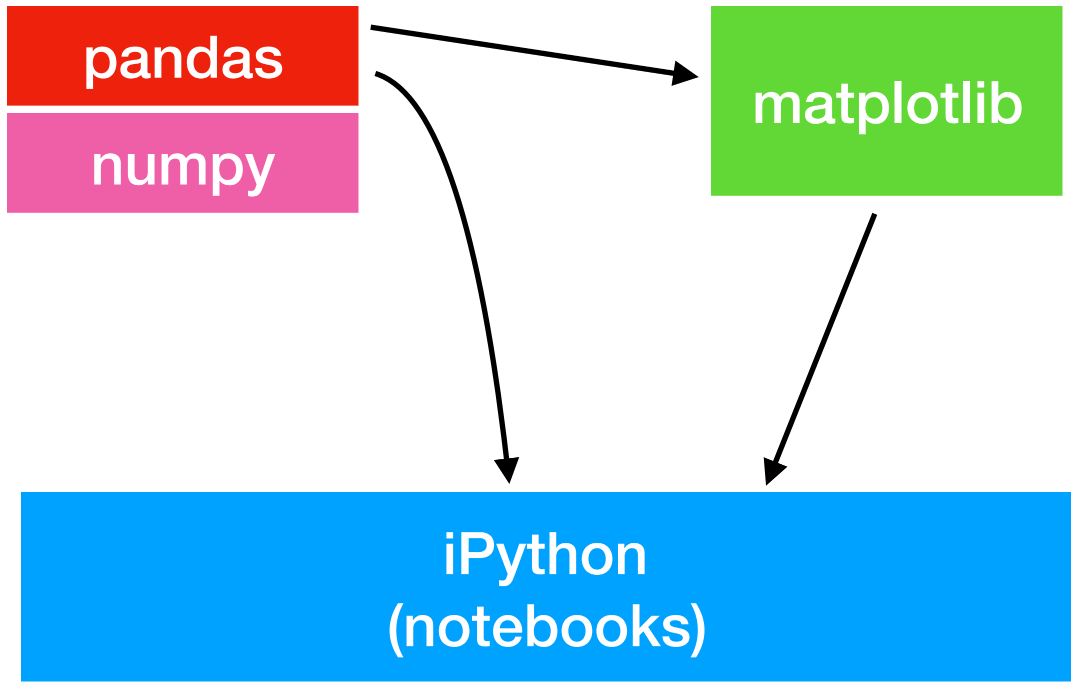
Modules we’ve learned this semester

- math
- collections
- json
- csv
- sys
- os
- copy
- recordclass
- requests
- bs4 (BeautifulSoup)
- pandas
- sqlite3
- matplotlib
- numpy ← today

numpy is the second most popular installed Python package after django (by some measures)

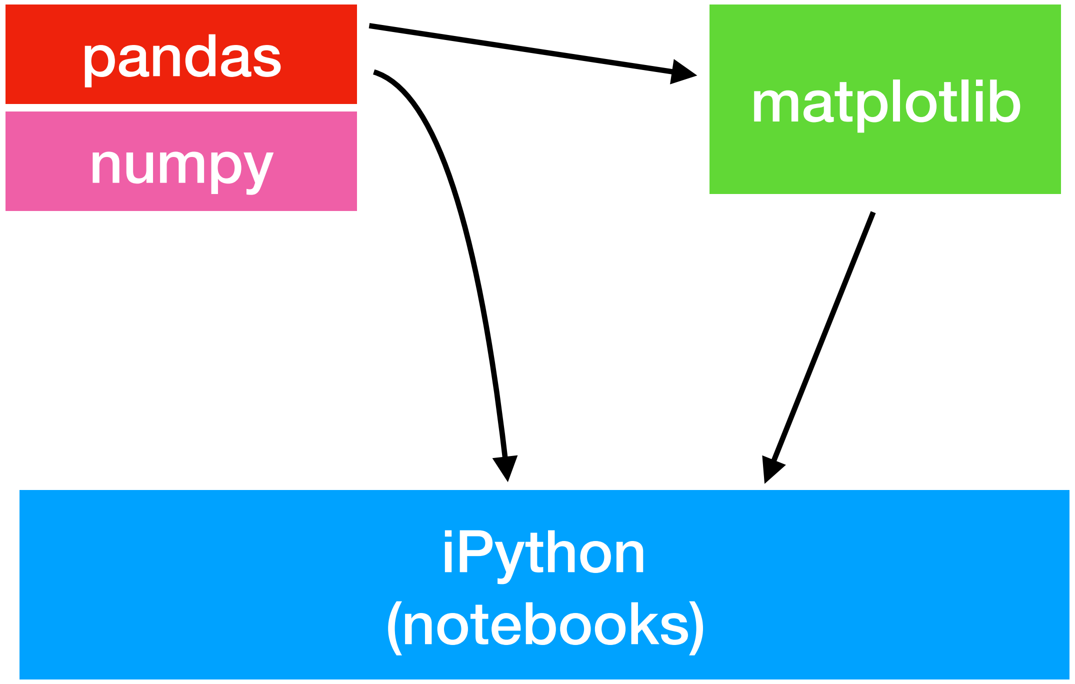
Modules we’ve learned this semester

- math
- collections
- json
- csv
- sys
- os
- copy
- recordclass
- requests
- bs4 (BeautifulSoup)
- pandas
- sqlite3
- matplotlib
- numpy ← today



Modules we've learned this semester

- math
- collections
- json
- csv
- sys
- os
- copy
- recordclass
- requests
- bs4 (BeautifulSoup)
- pandas
- sqlite3
- matplotlib
- numpy ← today



pandas Series and DataFrames use numpy, so you've been using it too without realizing it

numpy

```
import numpy as np
```

conventional alias

numpy

```
import numpy as np

a = np.array([10, 20, 30])
```

ndarray (created by this call)
is the core data structure of numpy

numpy

```
import numpy as np

a = np.array([10, 20, 30])
```

it can be initialized from a Python list

ndarray (created by this call) is the core data structure of numpy

numpy

```
import numpy as np

a = np.array([10, 20, 30])

a[1]
```

indexing

20

numpy

```
import numpy as np

a = np.array([10, 20, 30])

a[-1]
```

↓ **negative works**

30

numpy

```
import numpy as np

a = np.array([10, 20, 30])

a[1:]
```

↓ **slicing**

```
array([20, 30])
```

numpy

```
import numpy as np

a = np.array([10, 20, 30])

a + 1
```

element-wise ops

```
array([11, 21, 31])
```

numpy

```
import numpy as np

a = np.array([10, 20, 30])

a + a
```

element-wise ops

```
array([20, 40, 60])
```

numpy

```
import numpy as np

a = np.array([10, 20, 30])

a * a
```

↓ **element-wise ops**

```
array([100, 400, 900])
```

numpy

```
import numpy as np

a = np.array([10, 20, 30])

print(type(a))
```

↓

```
numpy.ndarray
```

numpy

```
import numpy as np

a = np.array([10, 20, 30])

print(type(a))
```

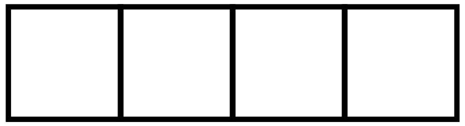
```
numpy.ndarray
```

why is it called an ndarray?

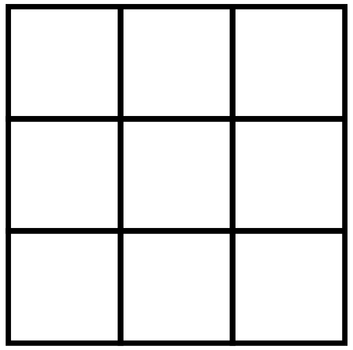
numpy

```
import numpy as np

a = np.array([10, 20, 30])
```

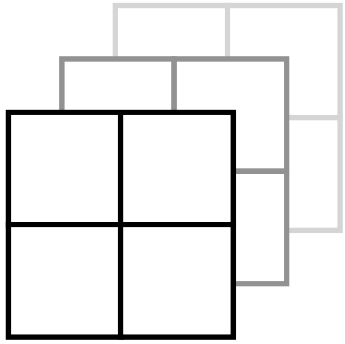
1-dimensional array



2-dimensional array



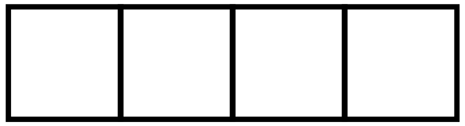
3-dimensional array



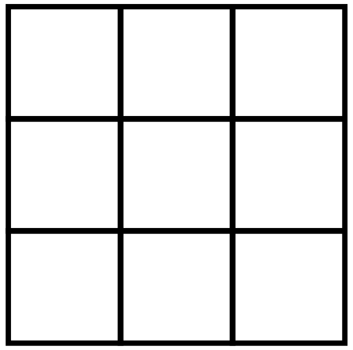
numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])
```

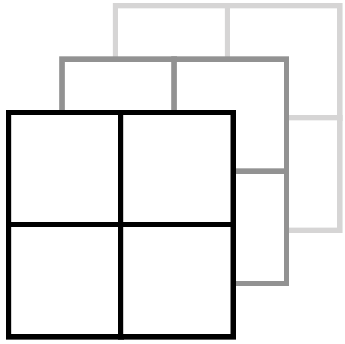
1-dimensional array



2-dimensional array



3-dimensional array

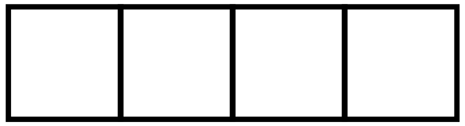


numpy

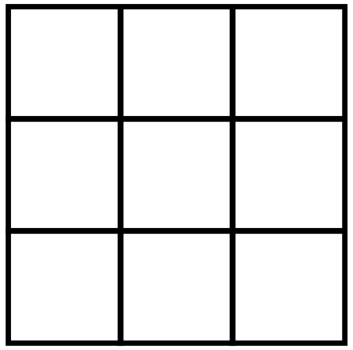
```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape(     )
```

shape tuple

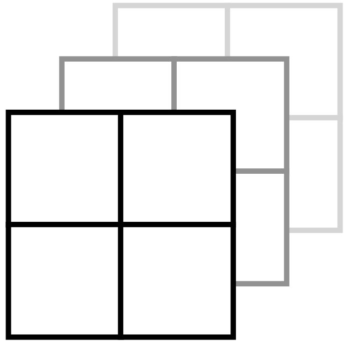
1-dimensional array



2-dimensional array

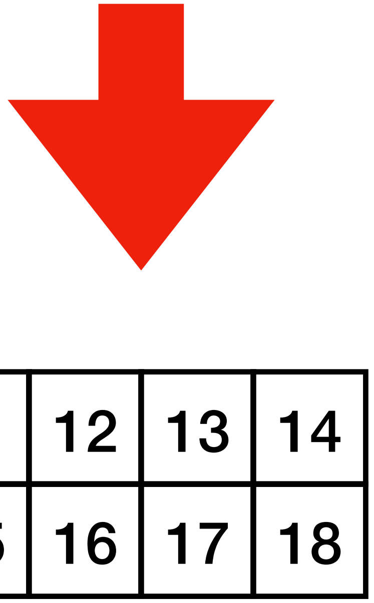


3-dimensional array



numpy

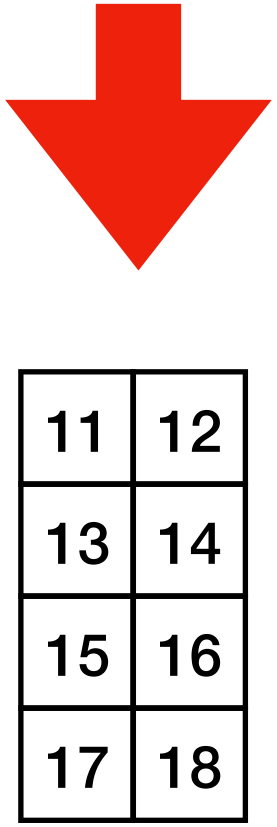
```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((2,4))
```



11	12	13	14
15	16	17	18

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((4,2))
```



11	12
13	14
15	16
17	18

numpy

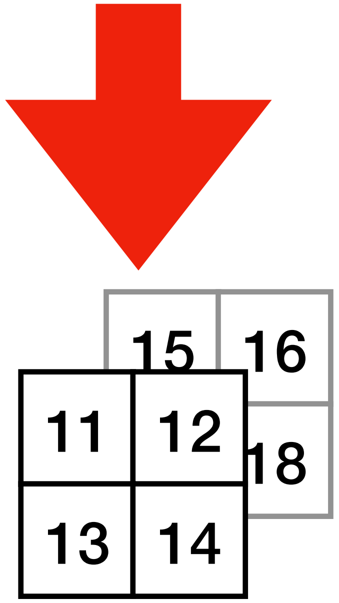
```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((4,2))
```

11	12
13	14
15	16
17	18

note that reshape fills in row-by-row first by default

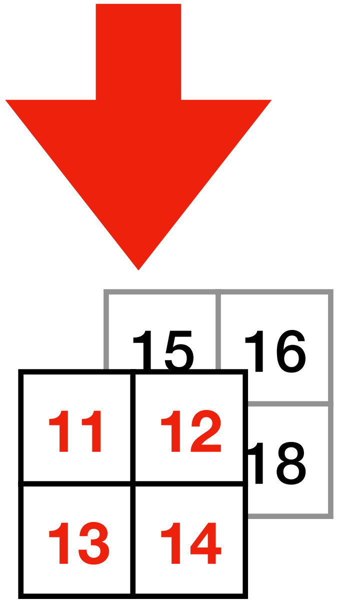
numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((2,2,2))
```



numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((2,2,2))
```

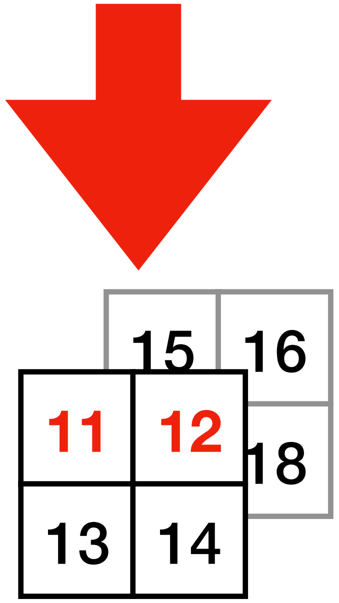


default fill order:

- layers
- rows
- columns

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((2,2,2))
```

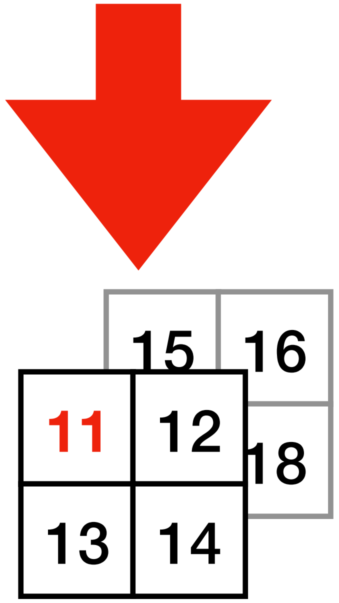


default fill order:

- layers
- rows
- columns

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((2,2,2))
```

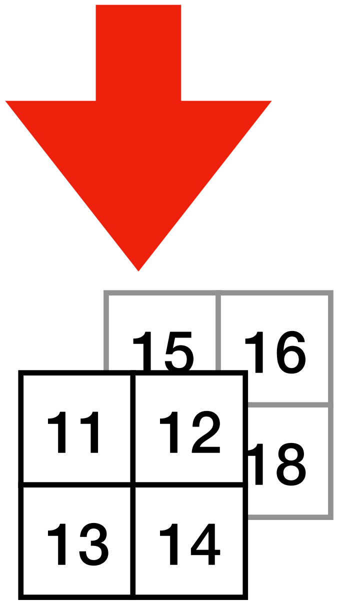


default fill order:

- layers
- rows
- columns

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

a.reshape((2,2,2))
```

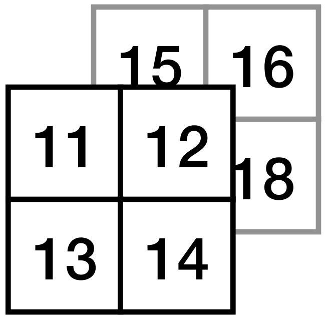


default fill order:

- layers
- rows
- columns

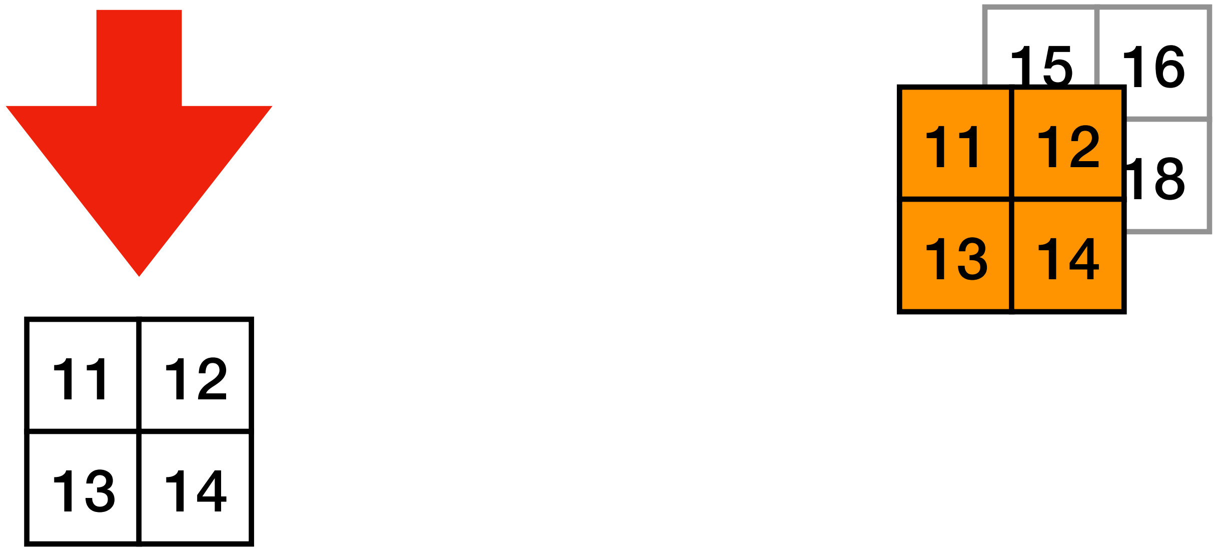
numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))
```



numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[0]
```



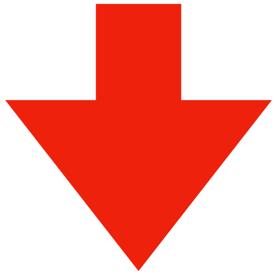
11	12
13	14

15	16
17	18

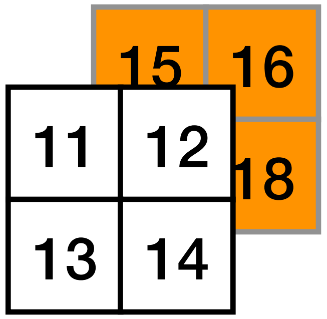
11	12
13	14

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[1]
```



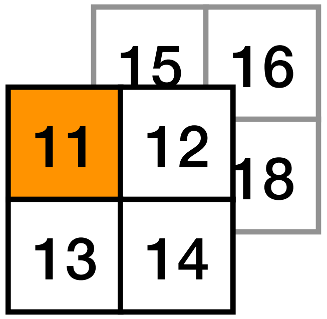
15	16
17	18



numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[0,0,0]
```

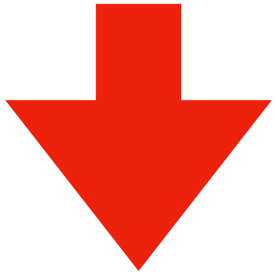
```
11
```



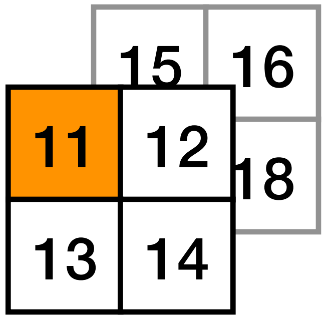
indexing: `ndarray[layer, row, col]`

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[0,0,0]
```



11



indexing: `ndarray[layer, row, col]`

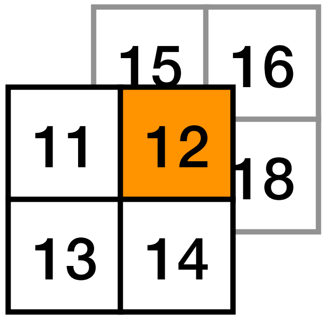
contrast with indexing into a list of lists of lists:

`data[layer][row][col]`

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[0,0,1]
```

12

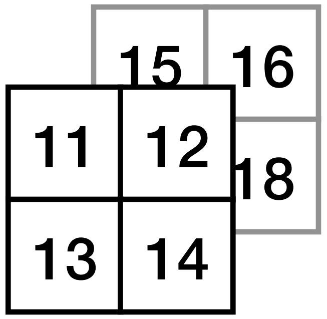


indexing: `ndarray[layer, row, col]`

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[0,1,1]
```

???

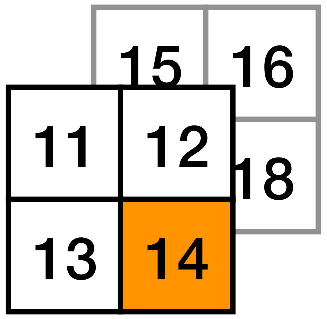


indexing: `ndarray[layer, row, col]`

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[0,1,1]
```

14



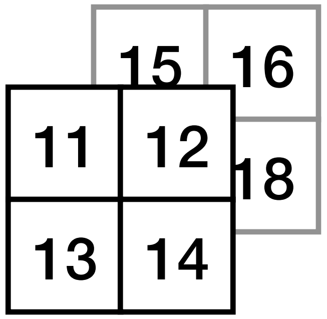
indexing: `ndarray[layer, row, col]`

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[1,1,1]
```

↓

???

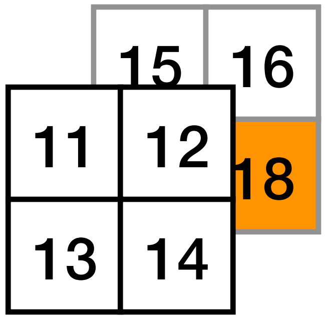


indexing: `ndarray[layer, row, col]`

numpy

```
import numpy as np

a = np.array([11,12,13,14,15,16,17,18])

b = a.reshape((2,2,2))

b[1,1,1]
```

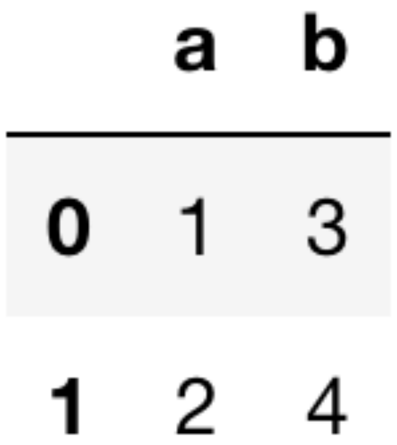
18



indexing: `ndarray[layer, row, col]`

Pandas and numpy

```
df = DataFrame({"a":[1,2], "b":[3,4]})
```

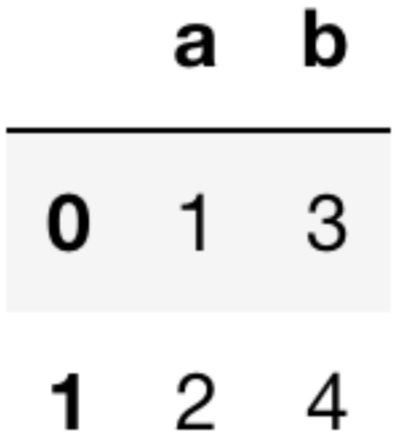


	a	b
0	1	3
1	2	4

```
s = df[“a”]
```

Pandas and numpy

```
df = DataFrame({"a":[1,2], "b":[3,4]})
```



	a	b
0	1	3
1	2	4

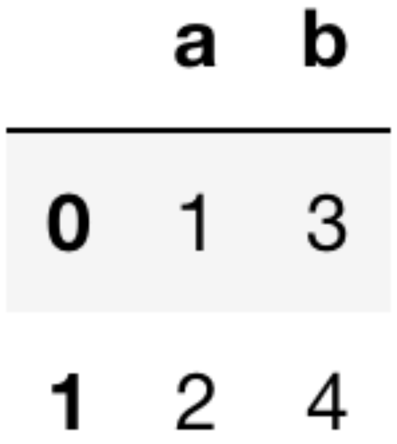
```
s = df[“a”]
```

TODO: fix a vs b

```
df.values   →   array([[1, 3],
                       [2, 4]])

s.values    →   array([3, 4])
```

Pandas and numpy

```
df = DataFrame({"a":[1,2], "b":[3,4]})
```



	a	b
0	1	3
1	2	4

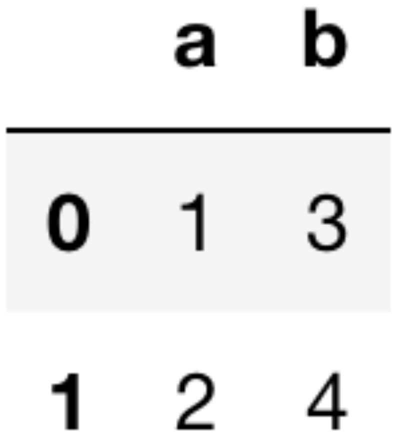
you’ve been using numpy arrays without knowing it!

```
s = df[“a”]
```

```
type(df.values)  →  numpy.ndarray

type(s.values)   →  numpy.ndarray
```

Pandas and numpy

```
df = DataFrame({"a":[1,2], "b":[3,4]})
```



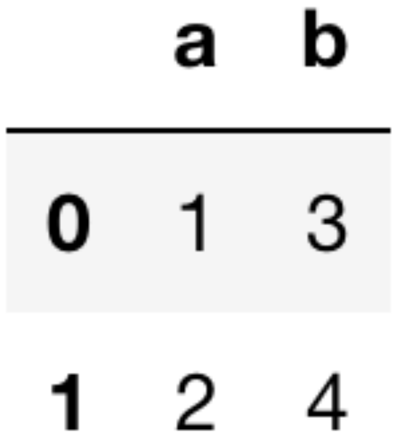
	a	b
0	1	3
1	2	4

```
s = df[“a”]
```

```
df.shape    →    (2,2)

s.shape     →    (2,)
```

Pandas and numpy

```
df = DataFrame({"a":[1,2], "b":[3,4]})
```



	a	b
0	1	3
1	2	4

```
s = df[“a”]
```

```
df.shape → (2,2)

s.shape  → (2,)
```

(2,) is a tuple with one number in it

Learning Objectives Today

History of regression

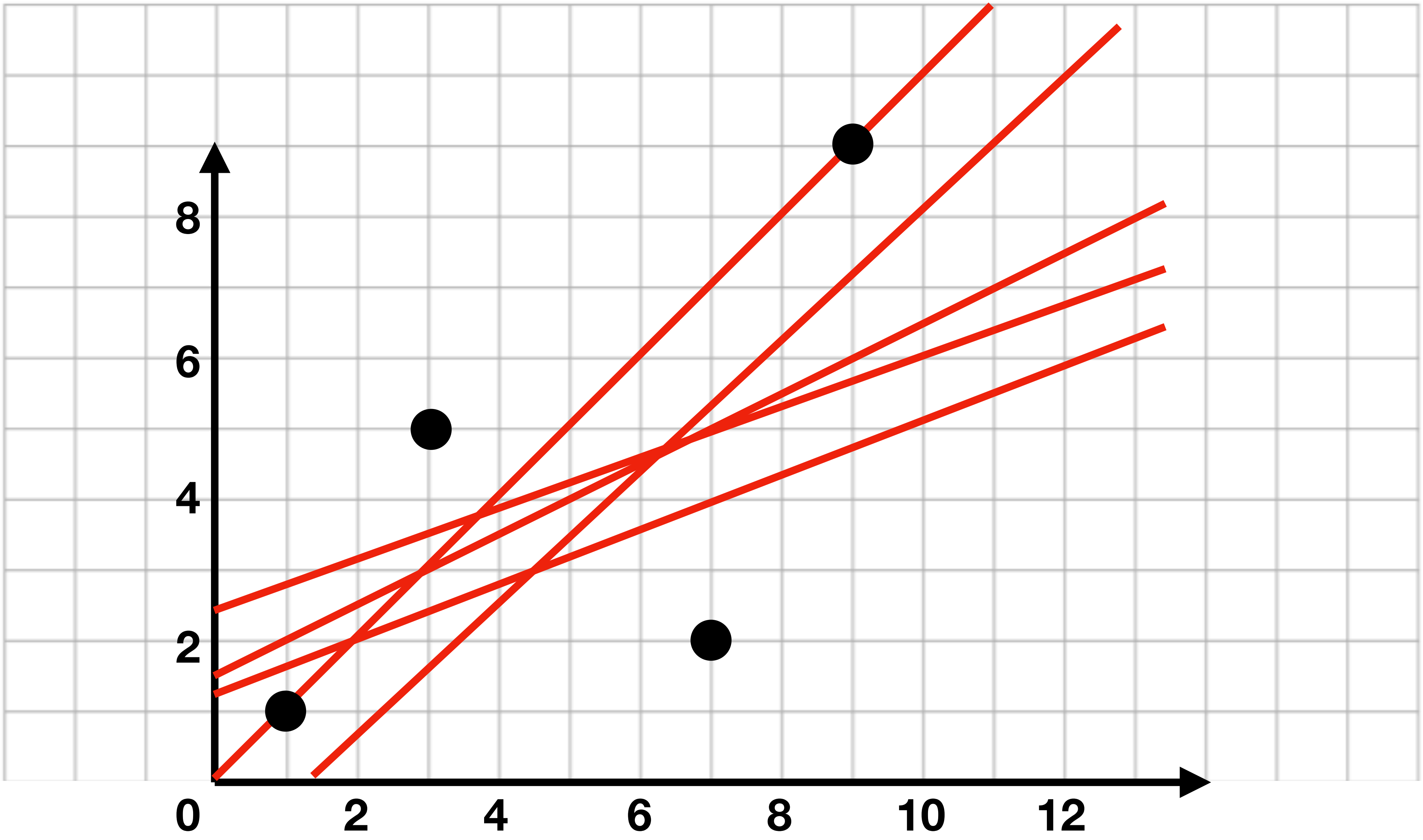
Drawing a fit line

Finding the slope/intercept w/ least squares method

Numpy introduction

Using numpy.linalg.lstsq

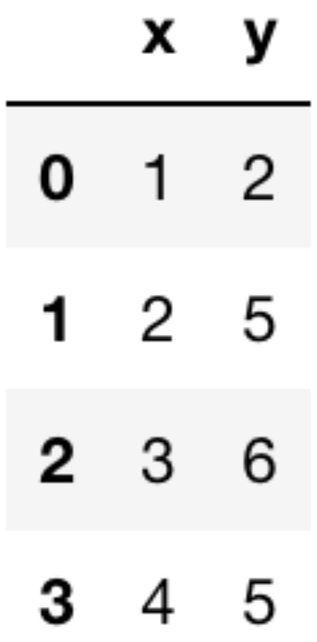
Use numpy to solve this!



There are many possible fit lines, but we want the one with the minimal tension.

Example data

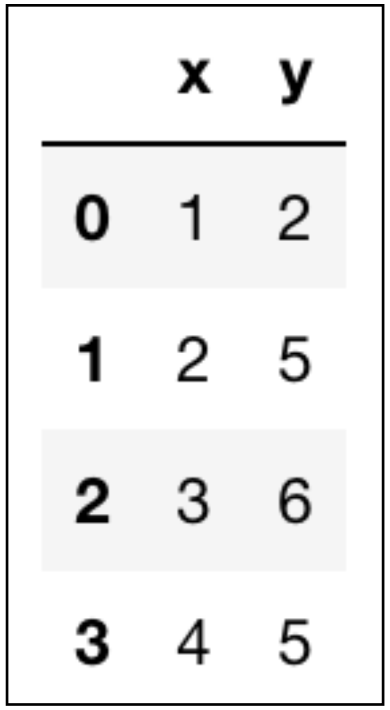
```
df = DataFrame({
        "x": [1,2,3,4],
        "y": [2,5,6,5]
})
```



	x	y
0	1	2
1	2	5
2	3	6
3	4	5

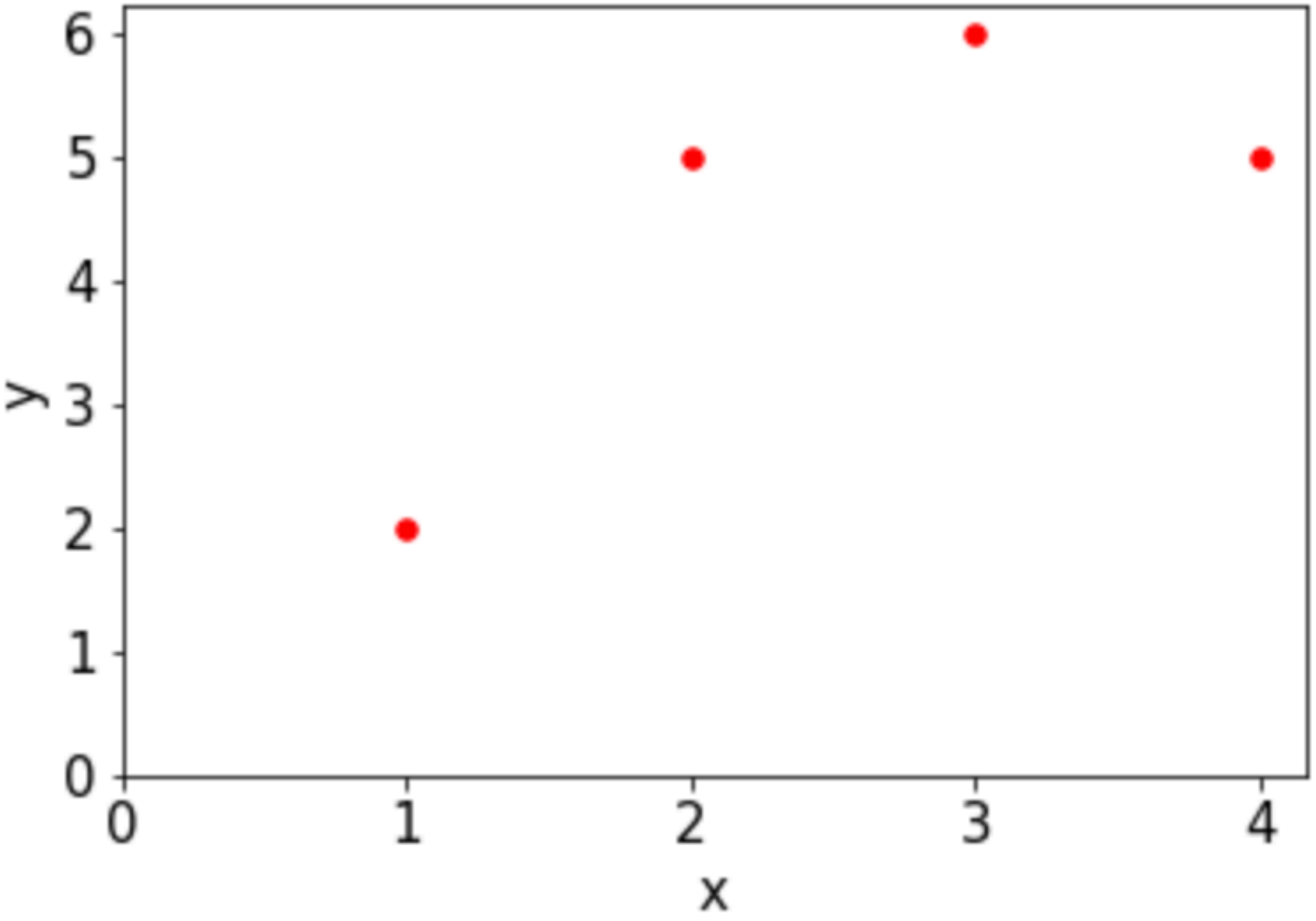
Example data

```
df = DataFrame({
    "x": [1,2,3,4],
    "y": [2,5,6,5]
})
```



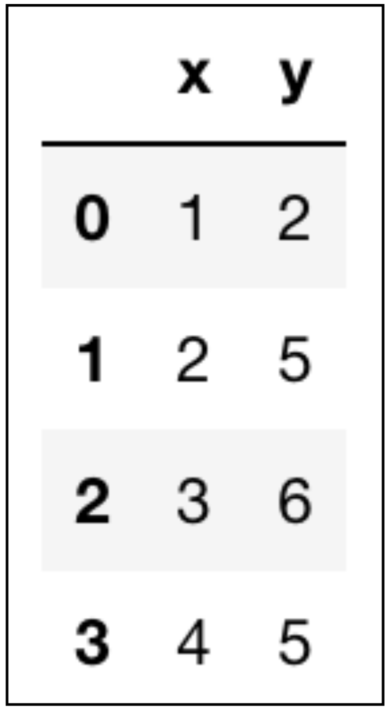
	x	y
0	1	2
1	2	5
2	3	6
3	4	5

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```



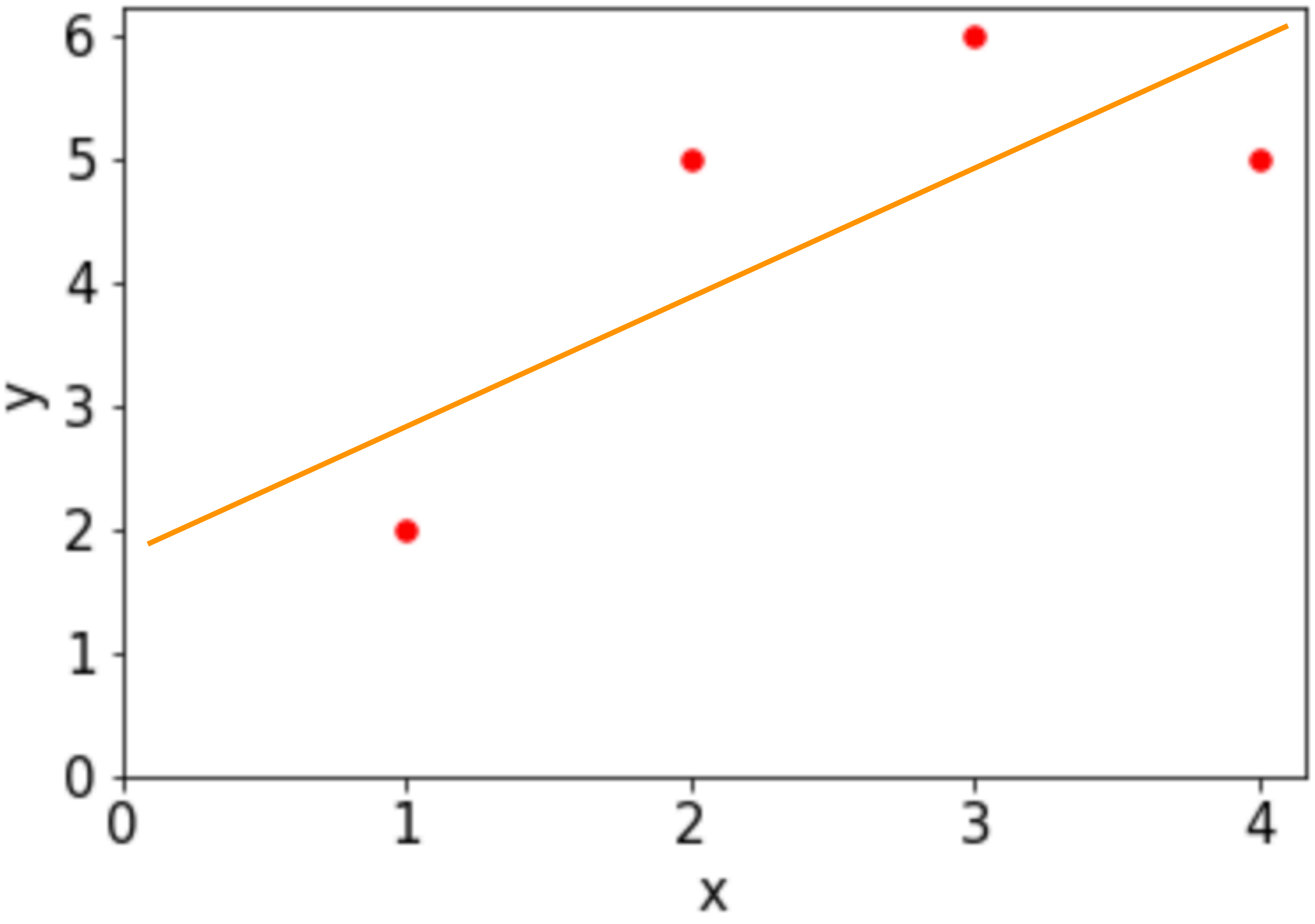
Example data

```
df = DataFrame({
     "x": [1,2,3,4],
     "y": [2,5,6,5]
})
```



	x	y
0	1	2
1	2	5
2	3	6
3	4	5

df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

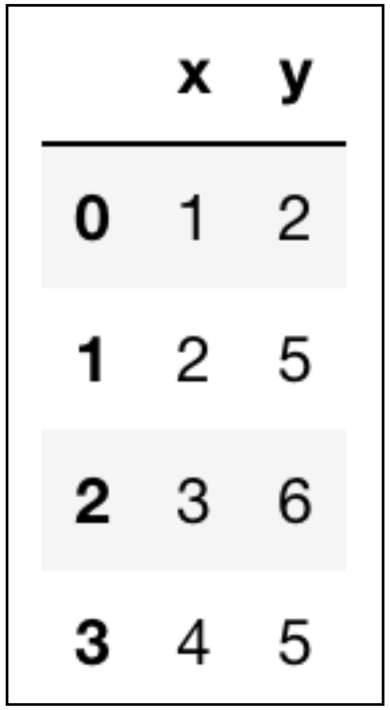


we want a formula like this:

m*x + n = y

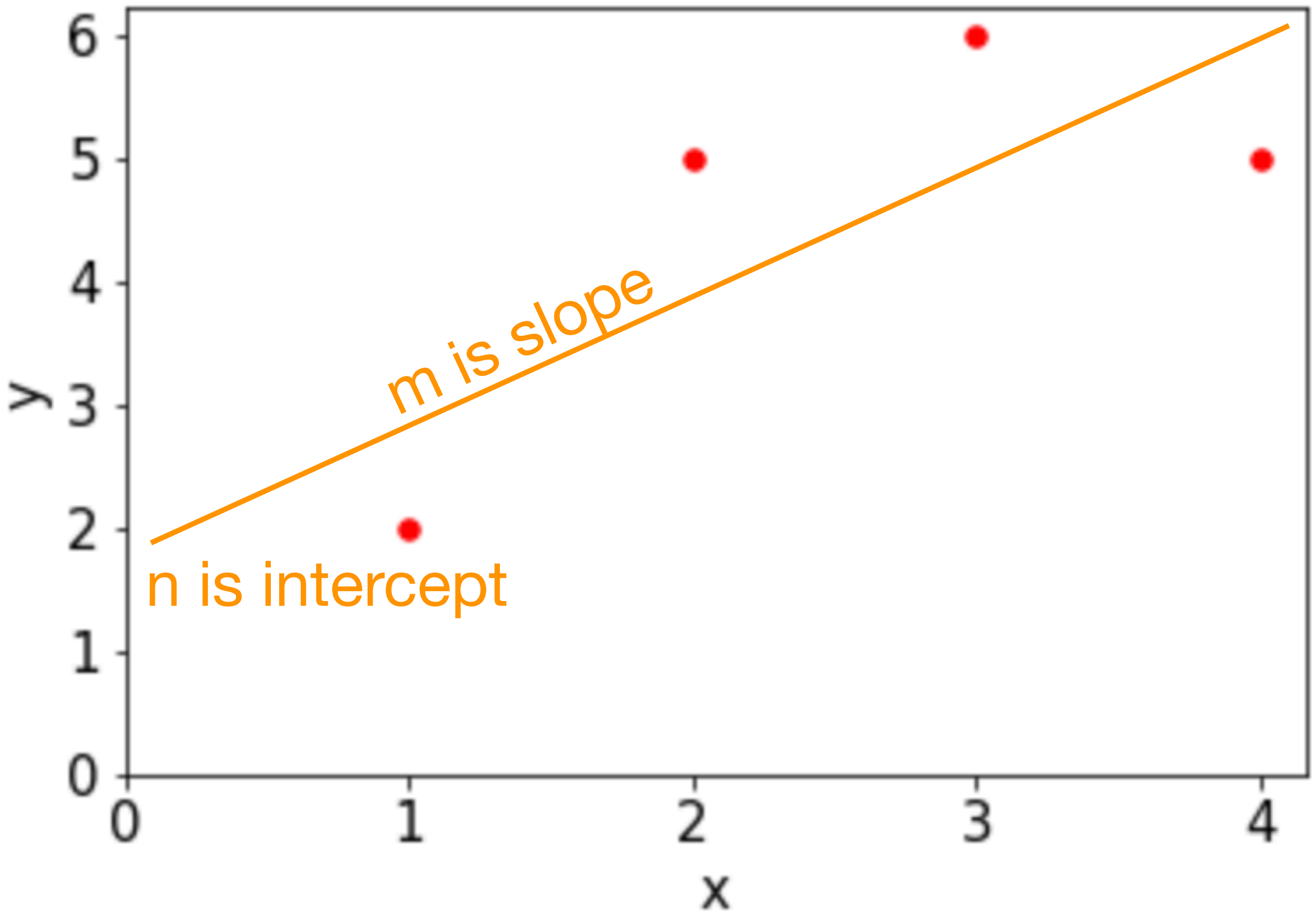
Example data

```
df = DataFrame({
     "x": [1,2,3,4],
     "y": [2,5,6,5]
})
```



	x	y
0	1	2
1	2	5
2	3	6
3	4	5

df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)



m is slope

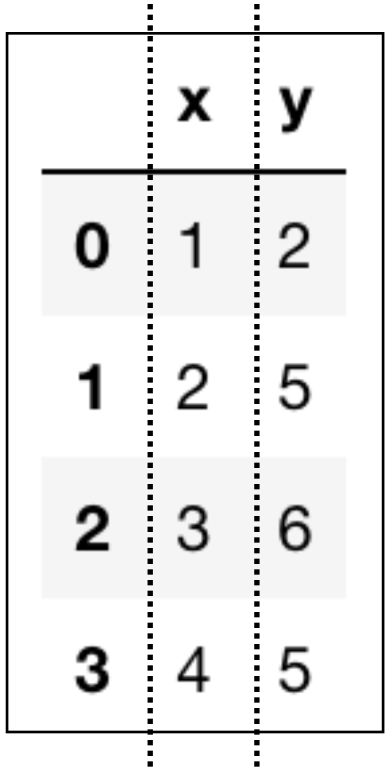
n is intercept

we want a formula like this:

$$m*x + n = y$$

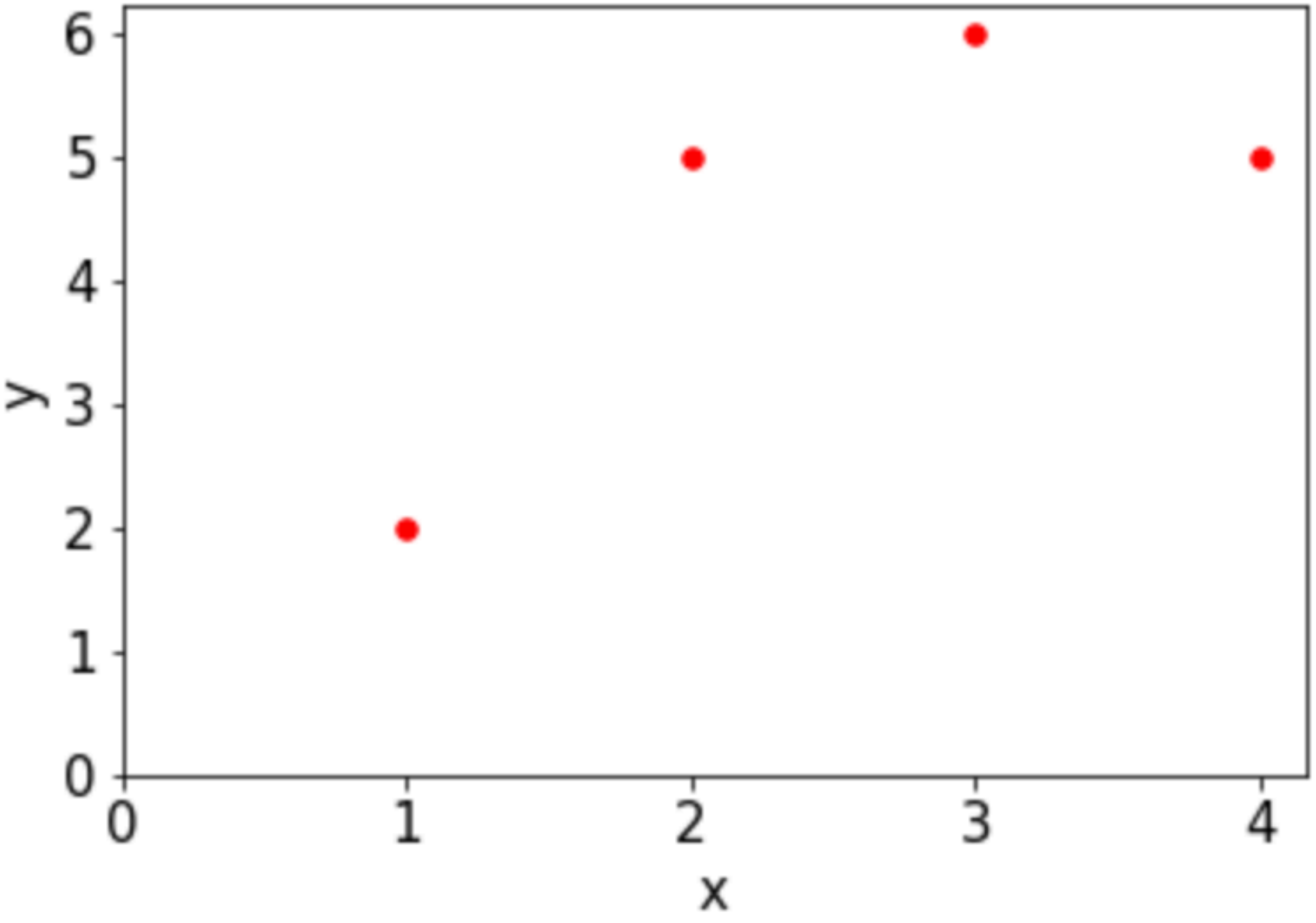
Example data

```
df = DataFrame({
    "x": [1,2,3,4],
    "y": [2,5,6,5]
})
```



	x	y
0	1	2
1	2	5
2	3	6
3	4	5

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```

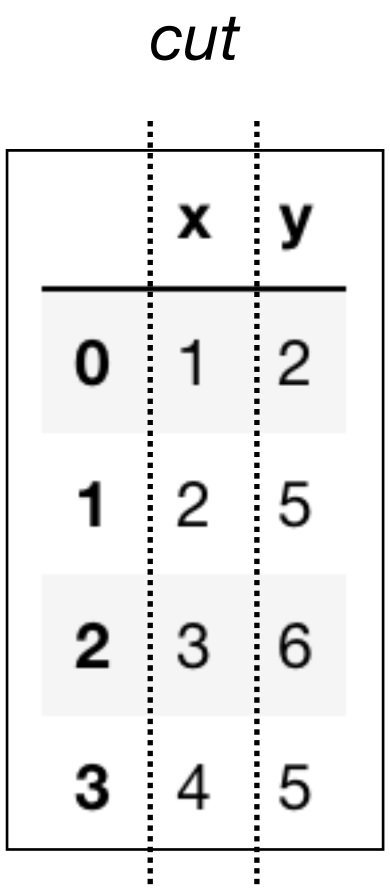


we want a formula like this:

m*x + n = y

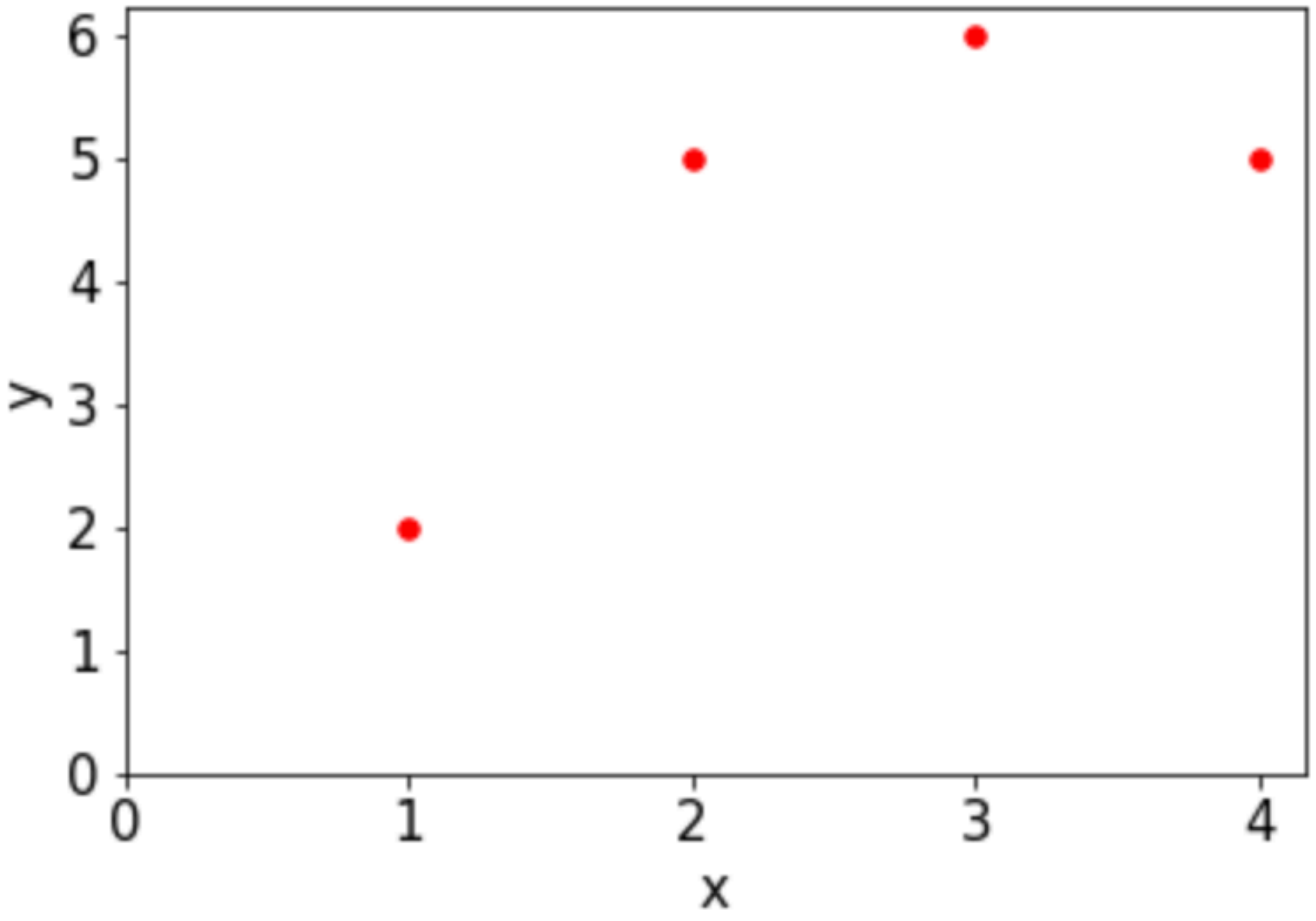
Example data

```
df = DataFrame({
    "x": [1,2,3,4],
    "y": [2,5,6,5]
})
```



	x	y
0	1	2
1	2	5
2	3	6
3	4	5

df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)



we want a formula like this:

$$m*x + n = y$$

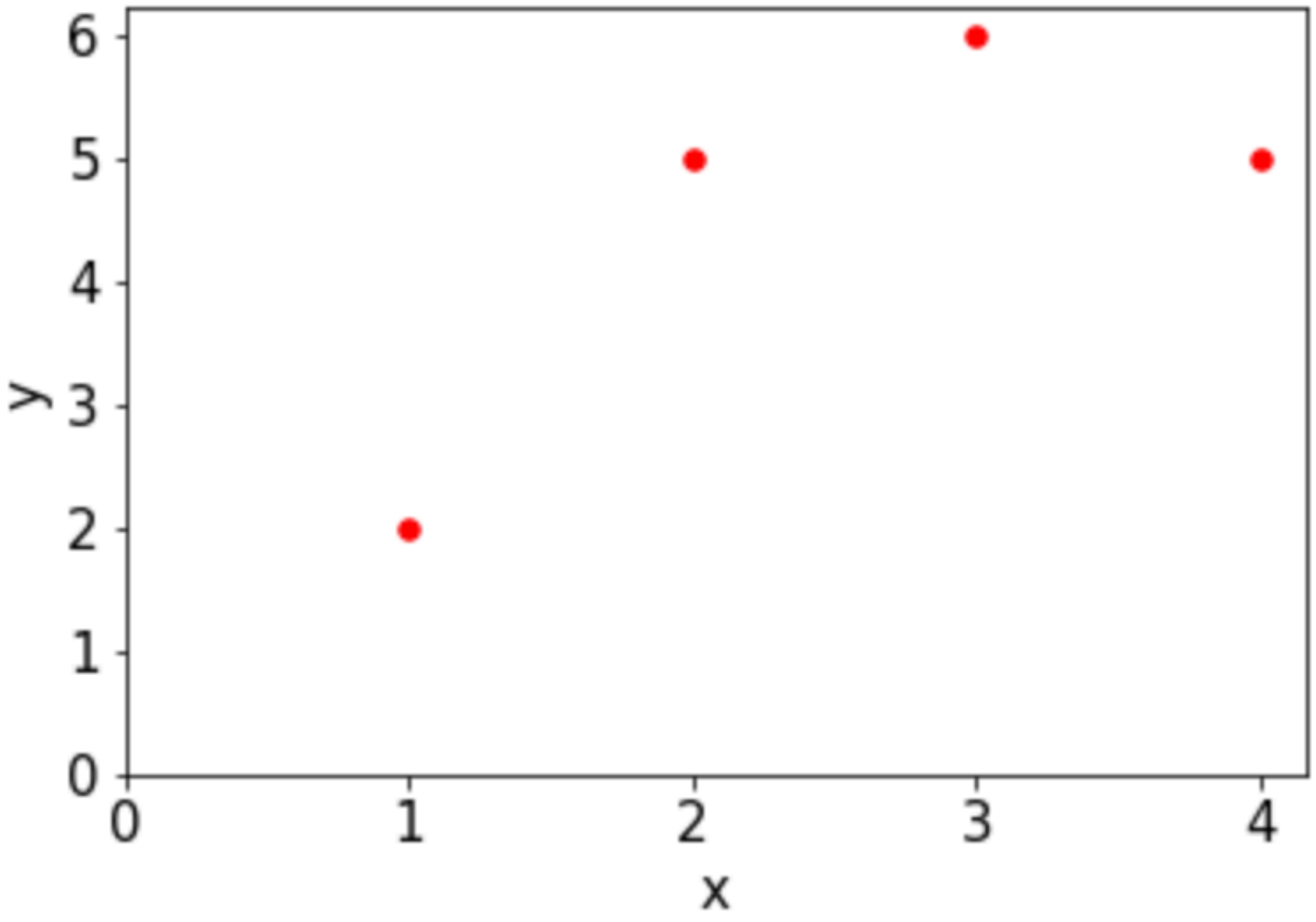
Example data

```
df = DataFrame({
    "x": [1,2,3,4],
    "y": [2,5,6,5]
})
```

x
1
2
3
4

y
2
5
6
5

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```



we want a formula like this:

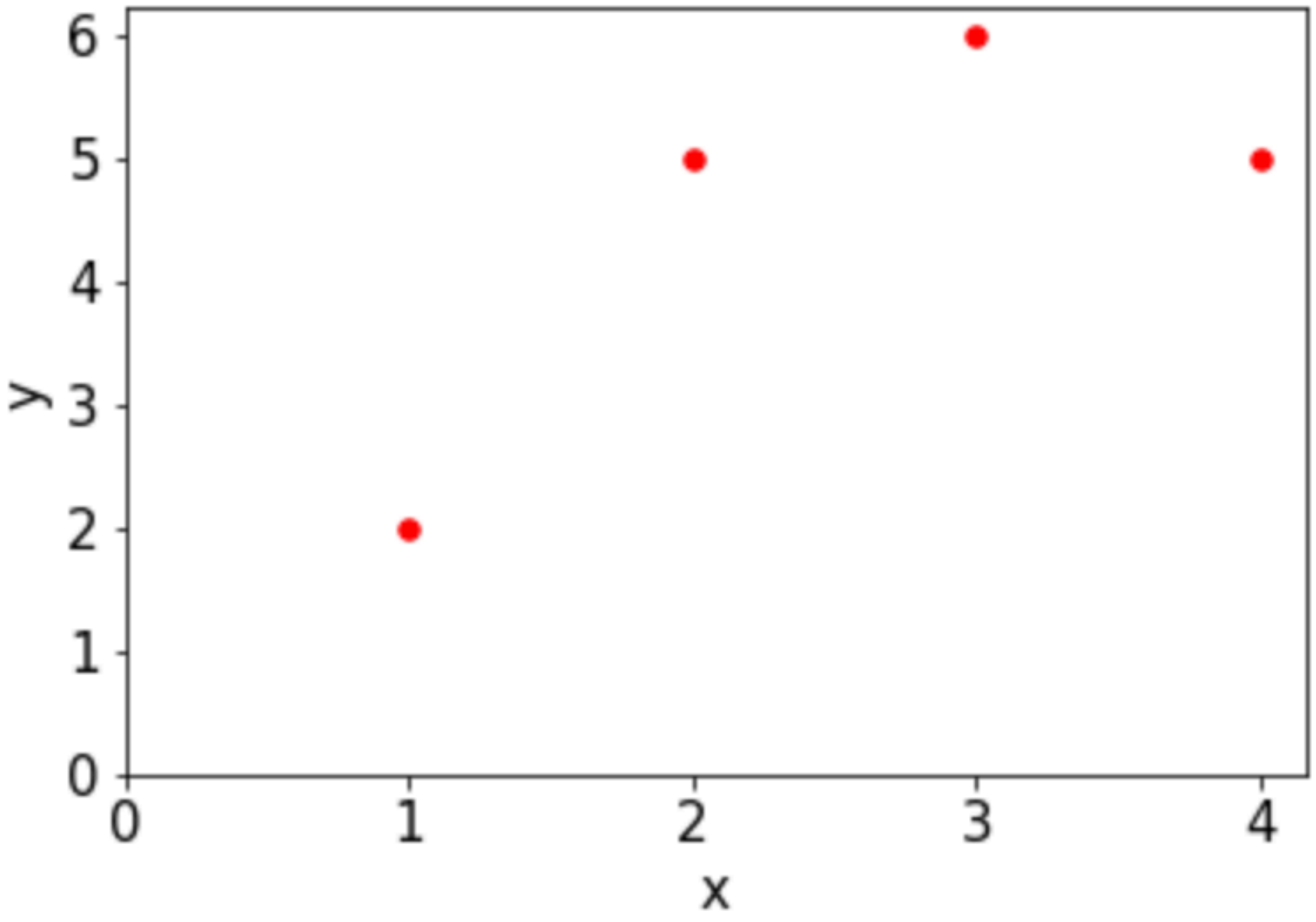
m*x + n = y

Example data

```
df = DataFrame({
     "x": [1,2,3,4],
     "y": [2,5,6,5]
})
```

m *	x	+ n ≈	y
	1		2
	2		5
	3		6
	4		5

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```



we want a formula like this:

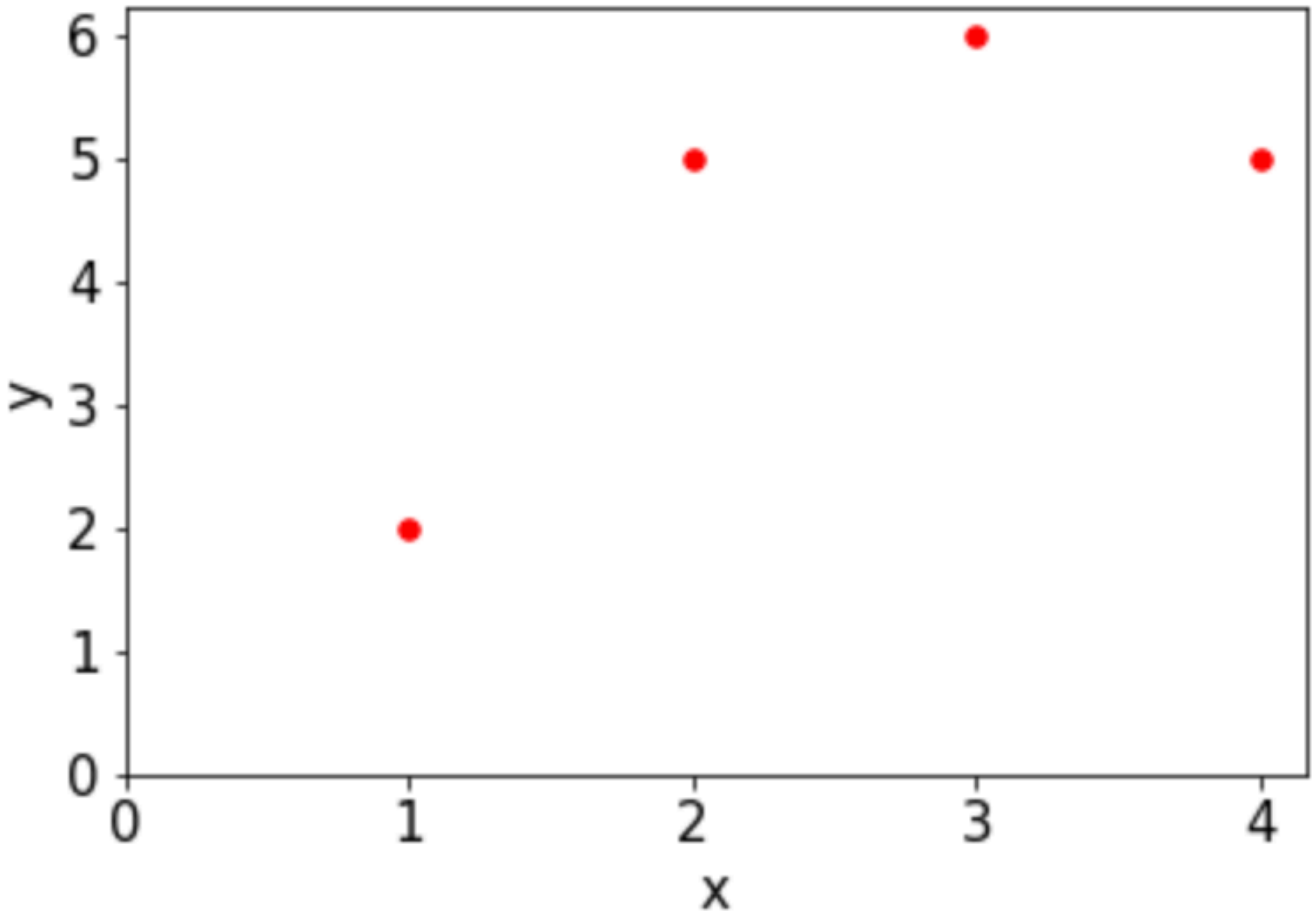
$$m^*x + n = y$$

Example data

```
df = DataFrame({
     "x": [1,2,3,4],
     "y": [2,5,6,5]
})
```

m *	x	+ n ≈	y
	1		2
	2		5
	3		6
	4		5

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```



we want a formula like this:

m*x + n = y

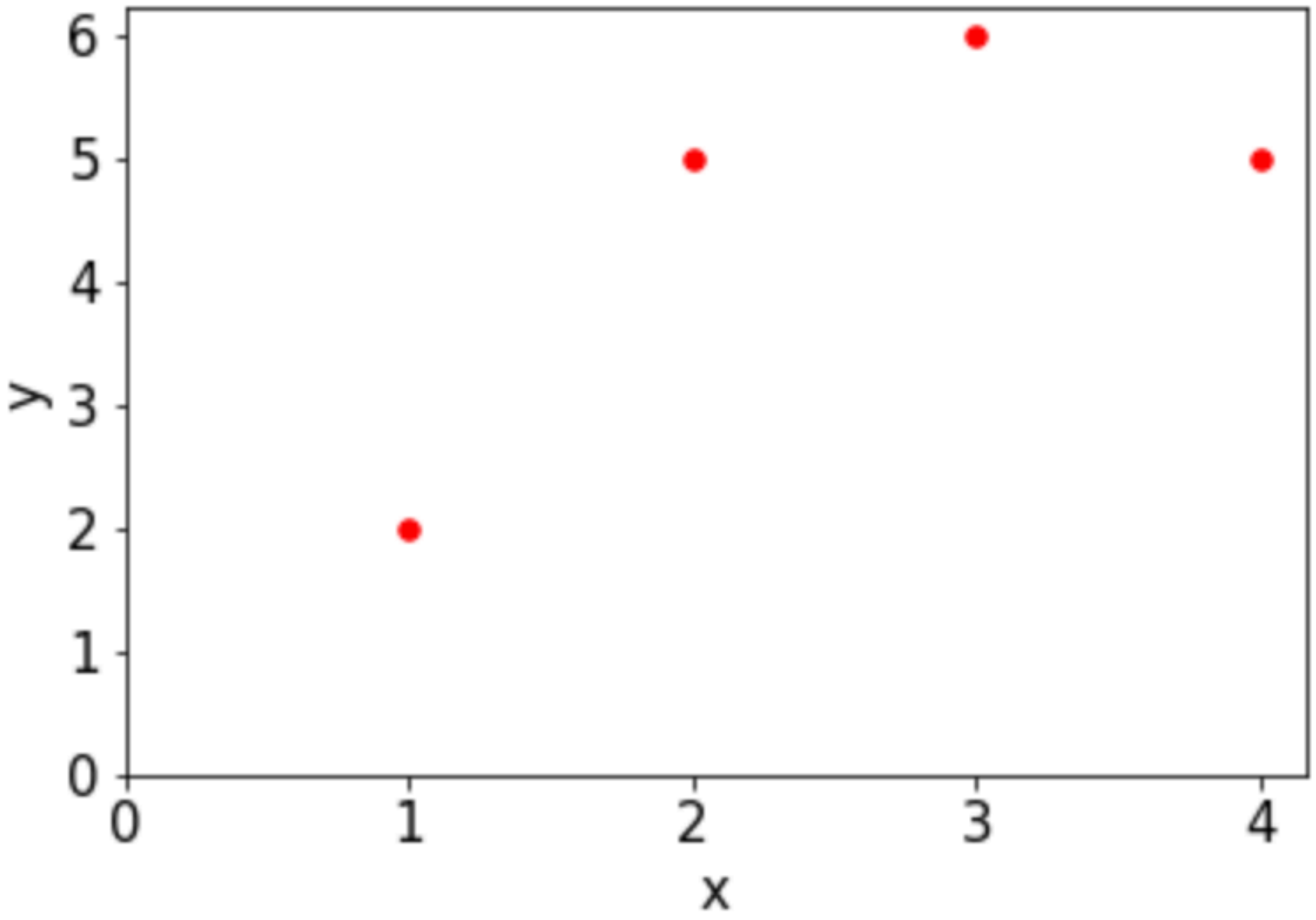
when numpy solves for a coefficient, it needs to be multiplied by some column in our data

Example data

```
df = DataFrame({
     "x": [1,2,3,4],
     "1": [1,1,1,1],
     "y": [2,5,6,5]
})
```

$$m * \begin{array}{|c|} \hline x \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array} + n * \begin{array}{|c|} \hline 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline \end{array} \approx \begin{array}{|c|} \hline y \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline \end{array}$$

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```



we want a formula like this:

$$m * x + n * 1 = y$$

when numpy solves for a coefficient, it needs to be multiplied by some column in our data

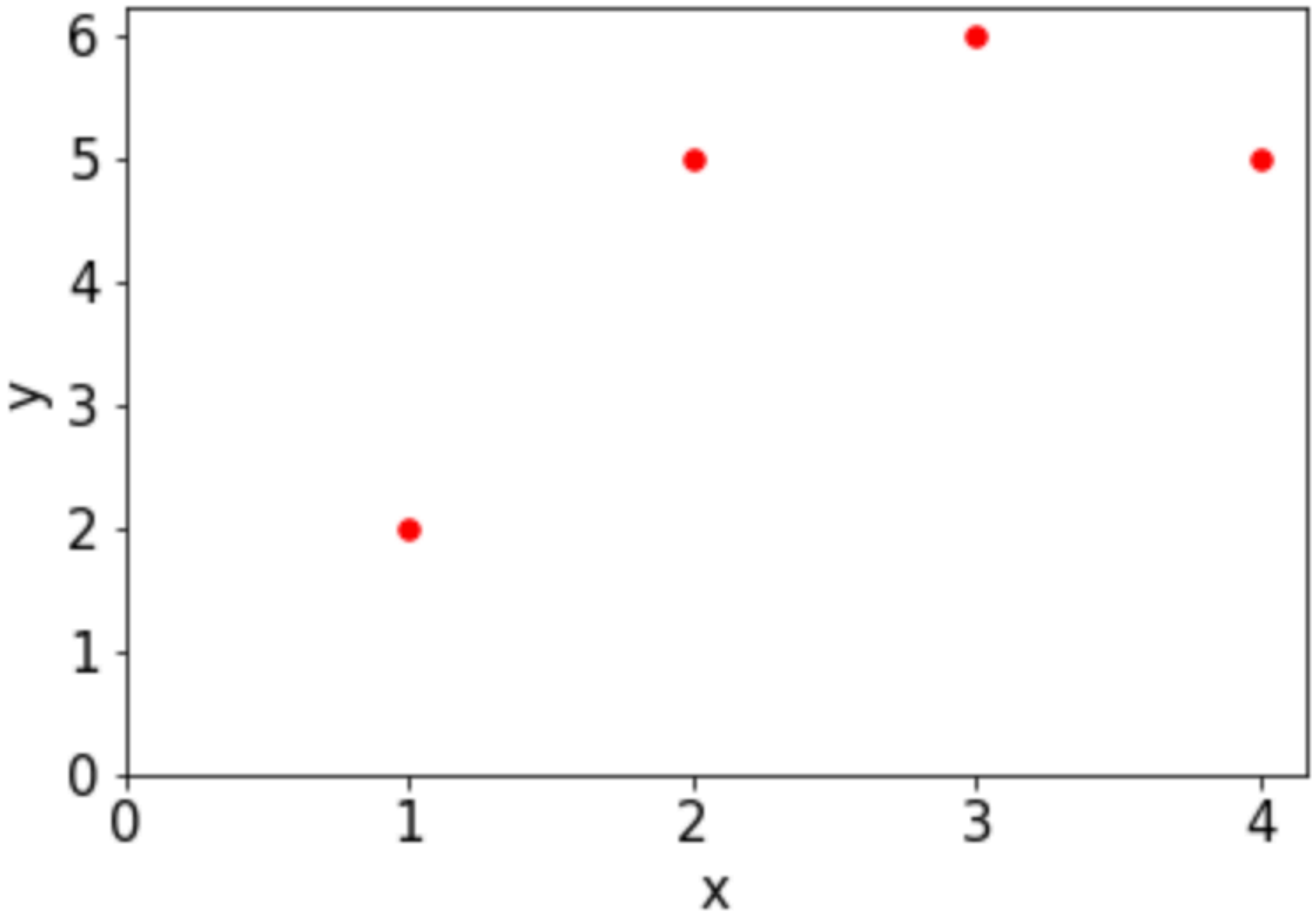
Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```

→

$$m * \begin{array}{|c|}\hline x \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline\end{array} + n * \begin{array}{|c|}\hline 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline\end{array} \approx \begin{array}{|c|}\hline y \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline\end{array}$$

df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)



we want a formula like this:

$$m*x + n*1 = y$$

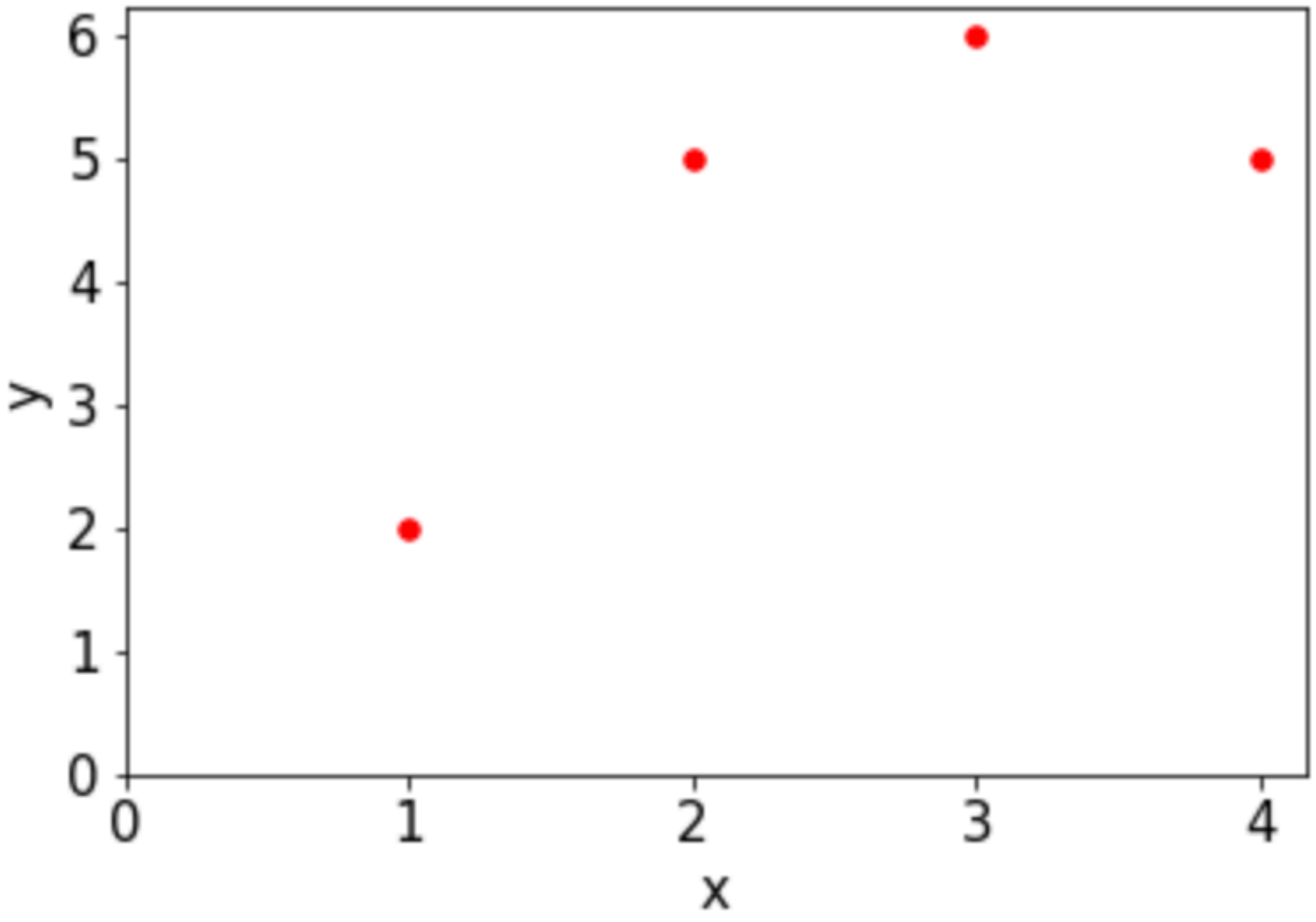
when numpy solves for a coefficient, it needs to be multiplied by some column in our data

Example data

```
df = DataFrame({
    "x": [1,2,3,4],
    "1": np.ones(4),
    "y": [2,5,6,5]
})
```

$$m * \begin{array}{|c|}\hline \mathbf{x} \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array} + n * \begin{array}{|c|}\hline \mathbf{1} \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline \end{array} \approx \begin{array}{|c|}\hline \mathbf{y} \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline \end{array}$$

df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

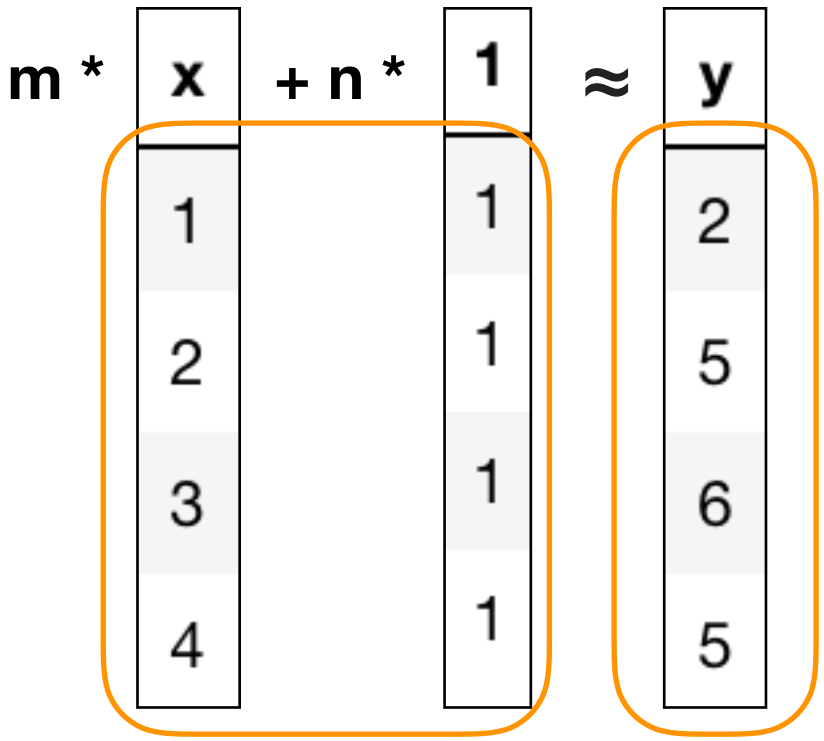


we want a formula like this:

m*x + n*1 = y

Example data

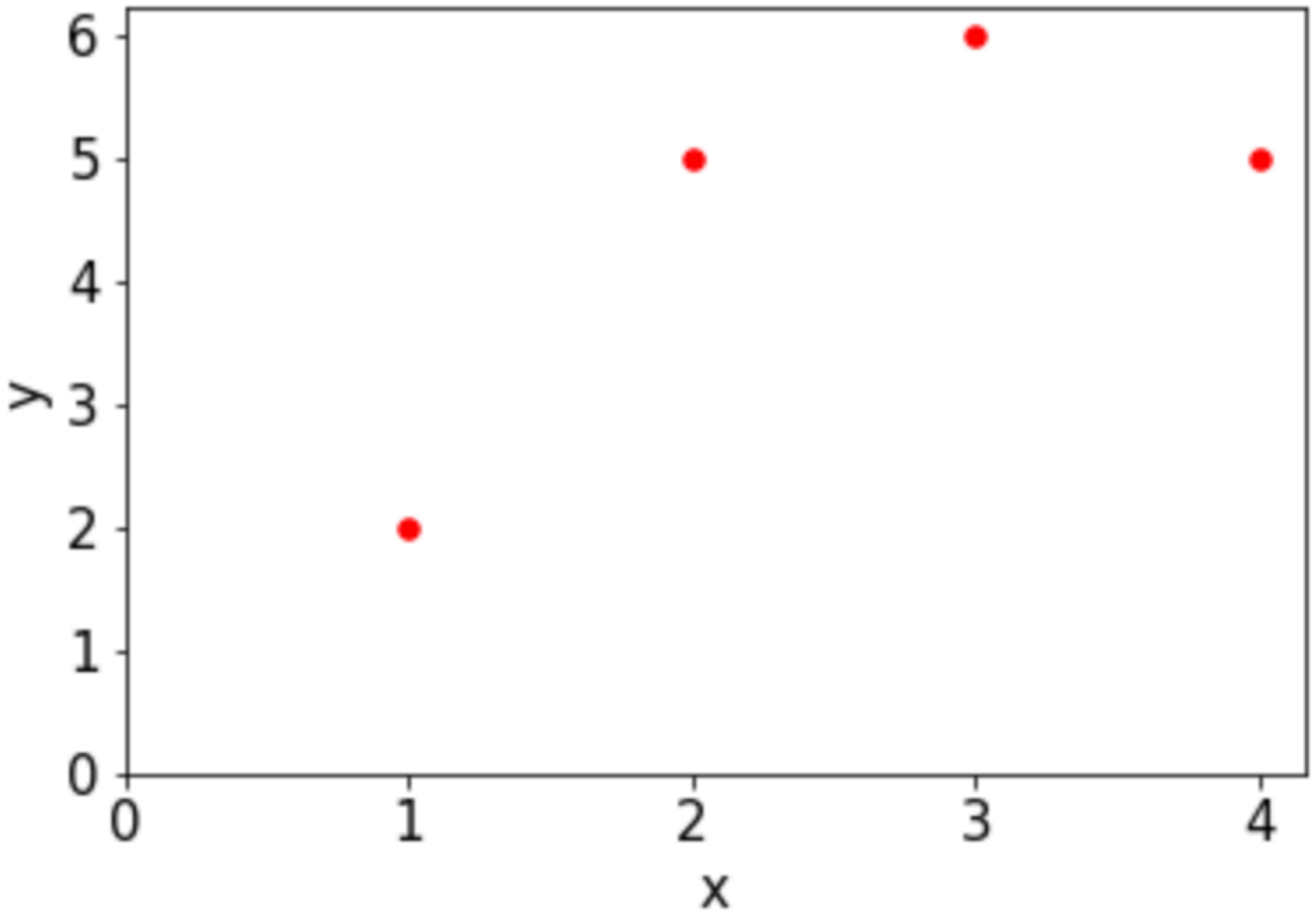
```
df = DataFrame({
     "x": [1,2,3,4],
     "1": np.ones(4),
     "y": [2,5,6,5]
})
```



$$m * \begin{array}{|c|}\hline \textbf{x} \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array} + n * \begin{array}{|c|}\hline \textbf{1} \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline \end{array} \approx \begin{array}{|c|}\hline \textbf{y} \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline \end{array}$$

given these inputs, as ndarrays…

```
df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```



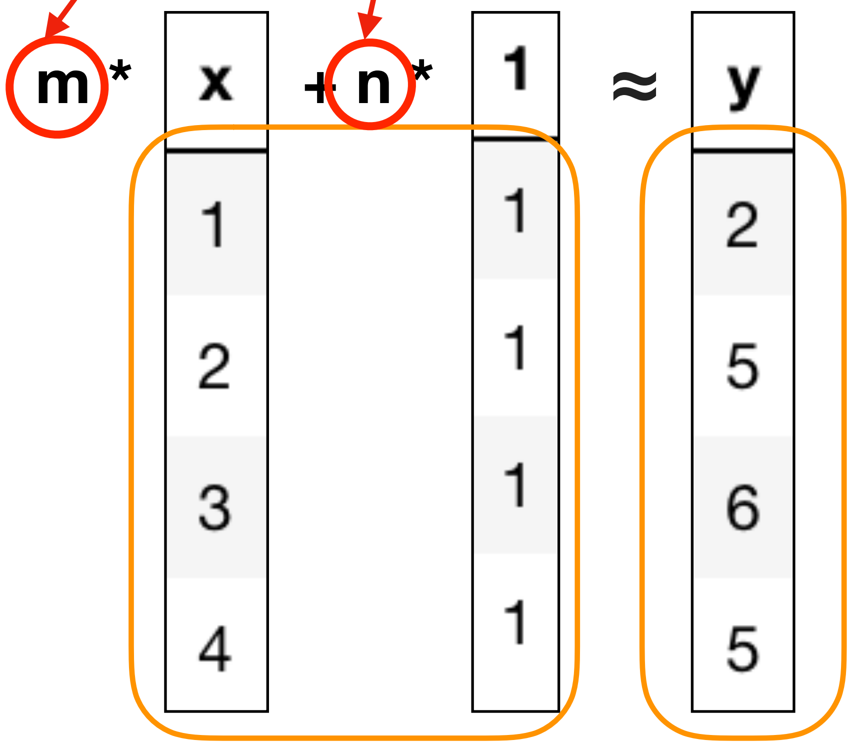
we want a formula like this:

$$m*x + n*1 = y$$

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```

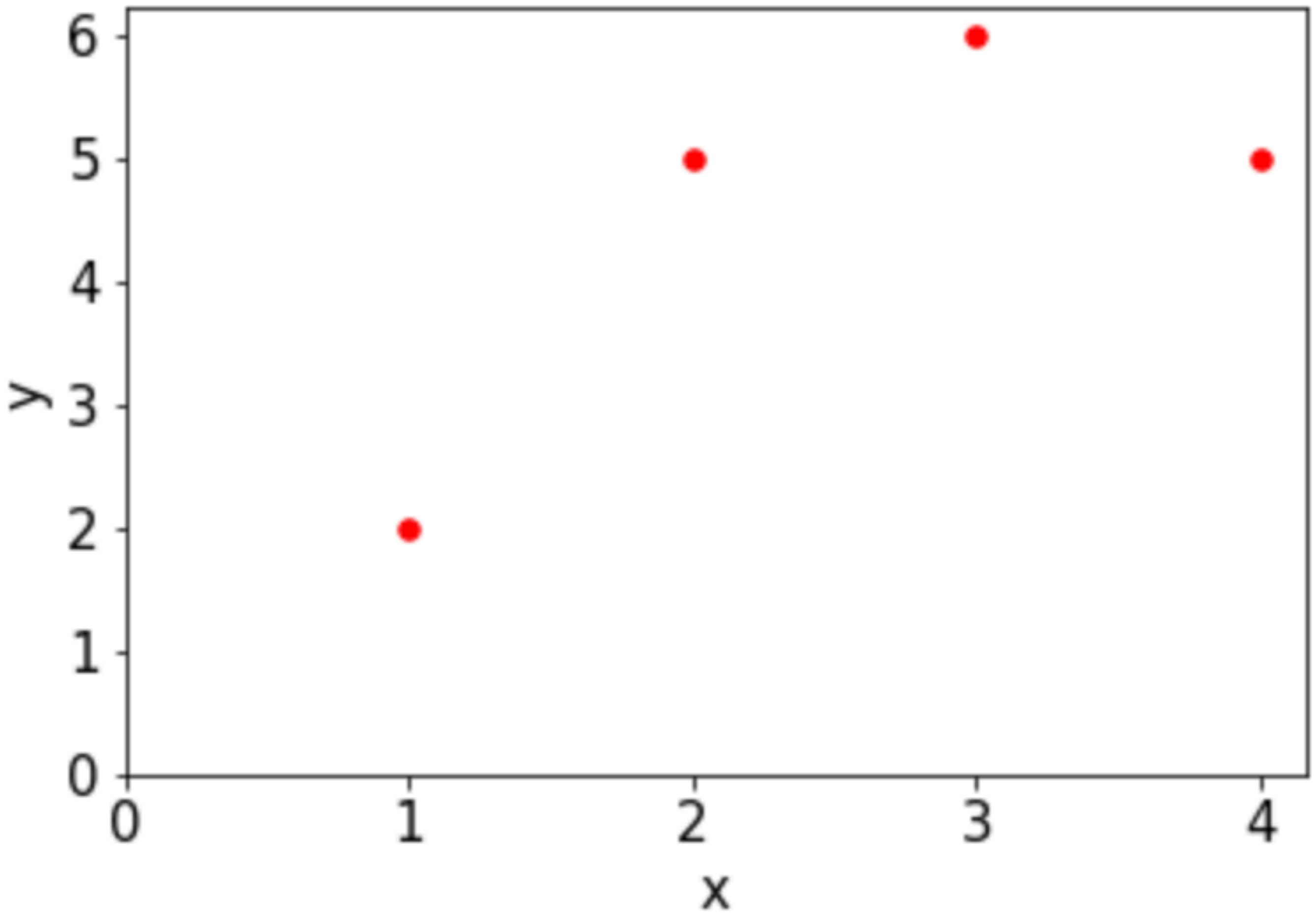
…numpy will give us these coefficients



$$m * \begin{array}{|c|}\hline x \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline\end{array} + n * \begin{array}{|c|}\hline 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline\end{array} \approx \begin{array}{|c|}\hline y \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline\end{array}$$

given these inputs, as ndarrays…

df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

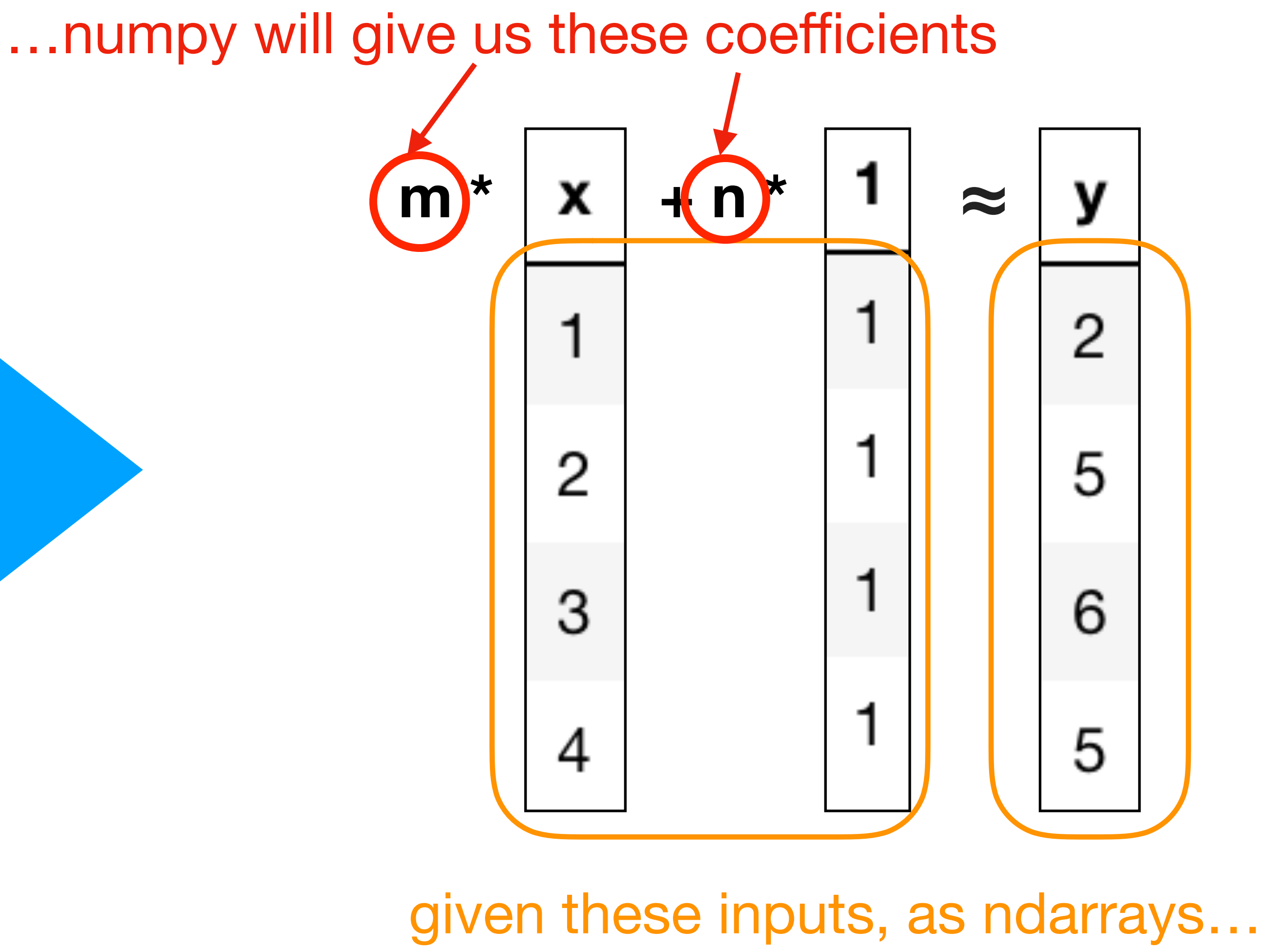


we want a formula like this:

m*x + n*1 = y

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```



…numpy will give us these coefficients

$$m * \begin{array}{|c|}\hline \mathbf{x} \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array} + n * \begin{array}{|c|}\hline \mathbf{1} \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline \end{array} \approx \begin{array}{|c|}\hline \mathbf{y} \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline \end{array}$$

given these inputs, as ndarrays…

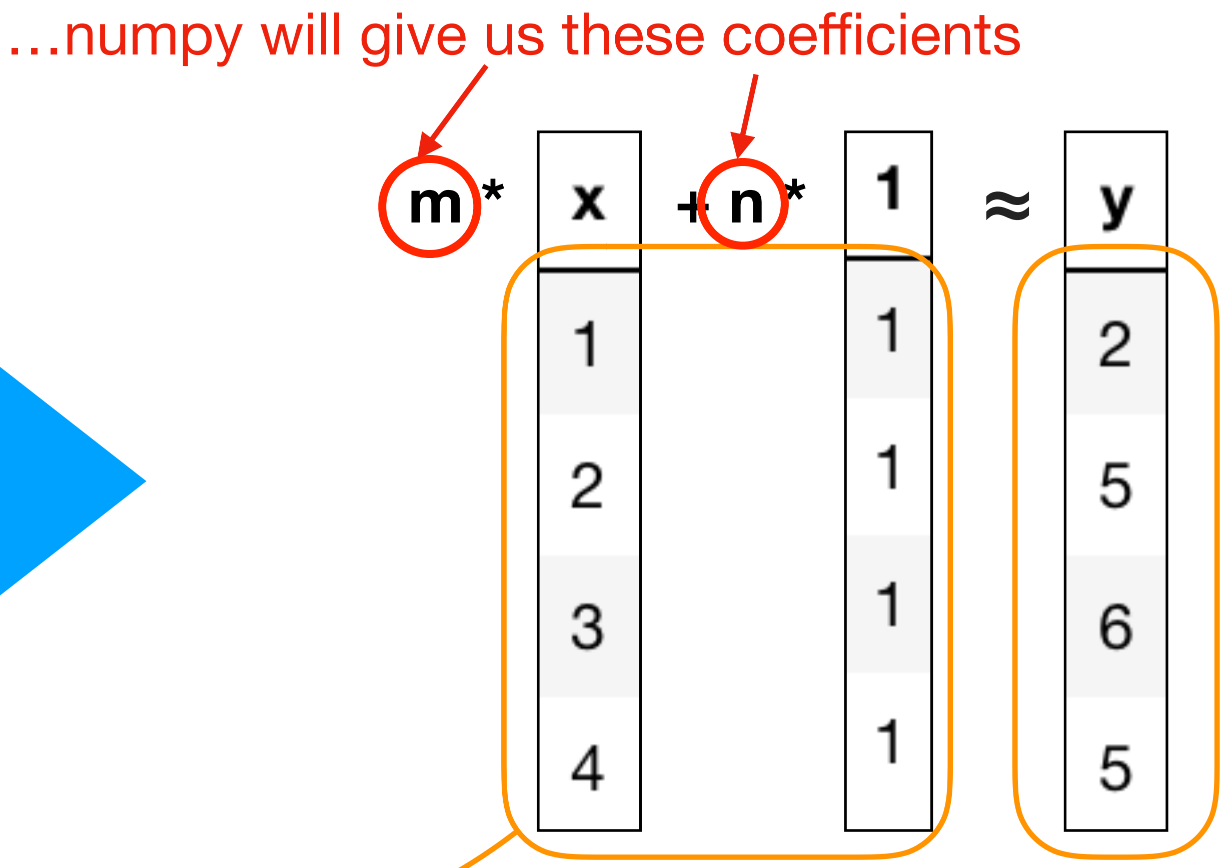
```
np.linalg.lstsq(          )
```

we want a formula like this:

$$m*x + n*1 = y$$

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```



…numpy will give us these coefficients

$m * x + n * 1 \approx y$

x	1	y
1	1	2
2	1	5
3	1	6
4	1	5

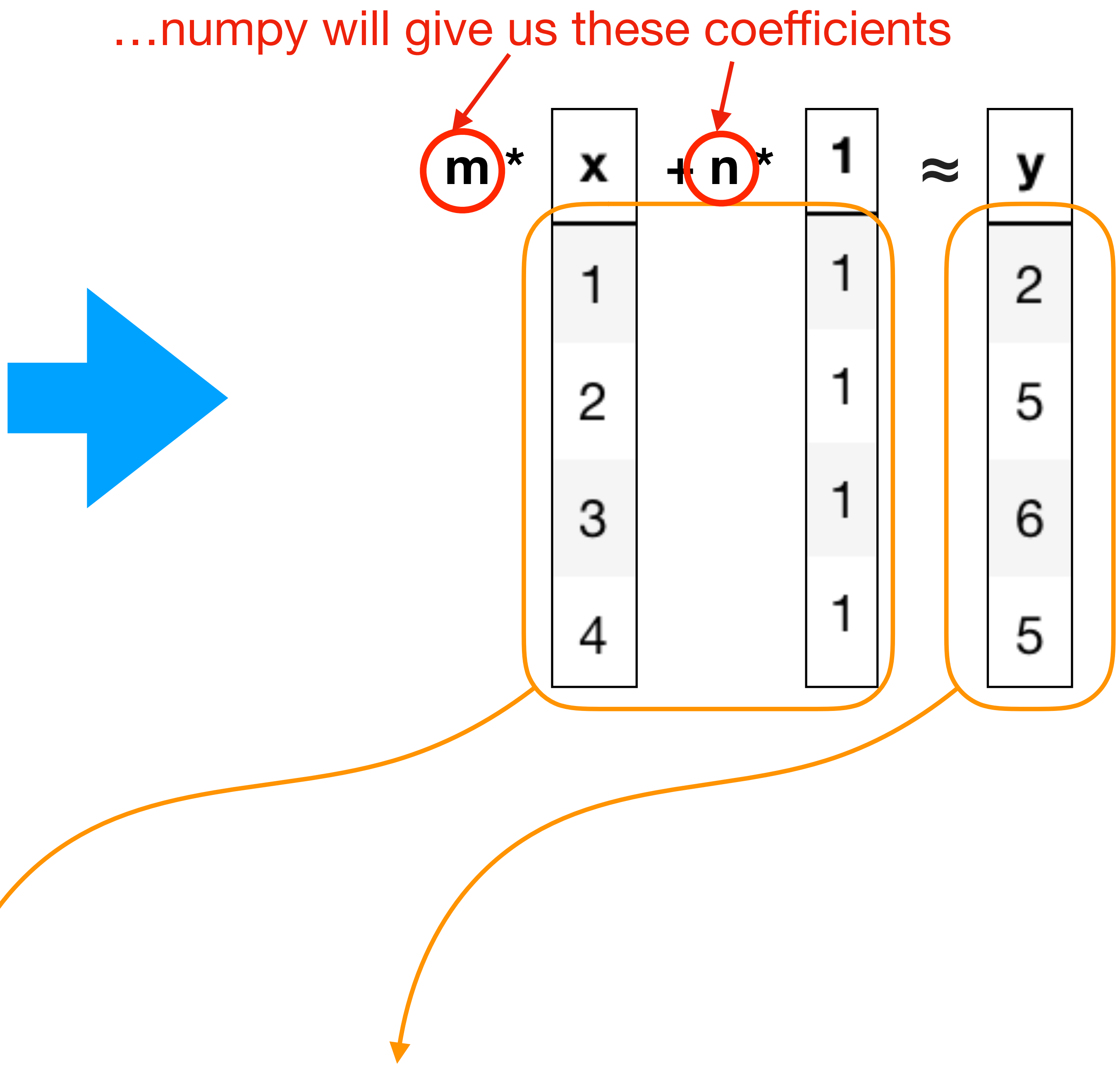
```
np.linalg.lstsq(df[["x", “1”]],   )
```

we want a formula like this:

$$m*x + n*1 = y$$

Example data

```
df = DataFrame({
         "x": [1,2,3,4],
         "1": np.ones(4),
         "y": [2,5,6,5]
})
```



…numpy will give us these coefficients

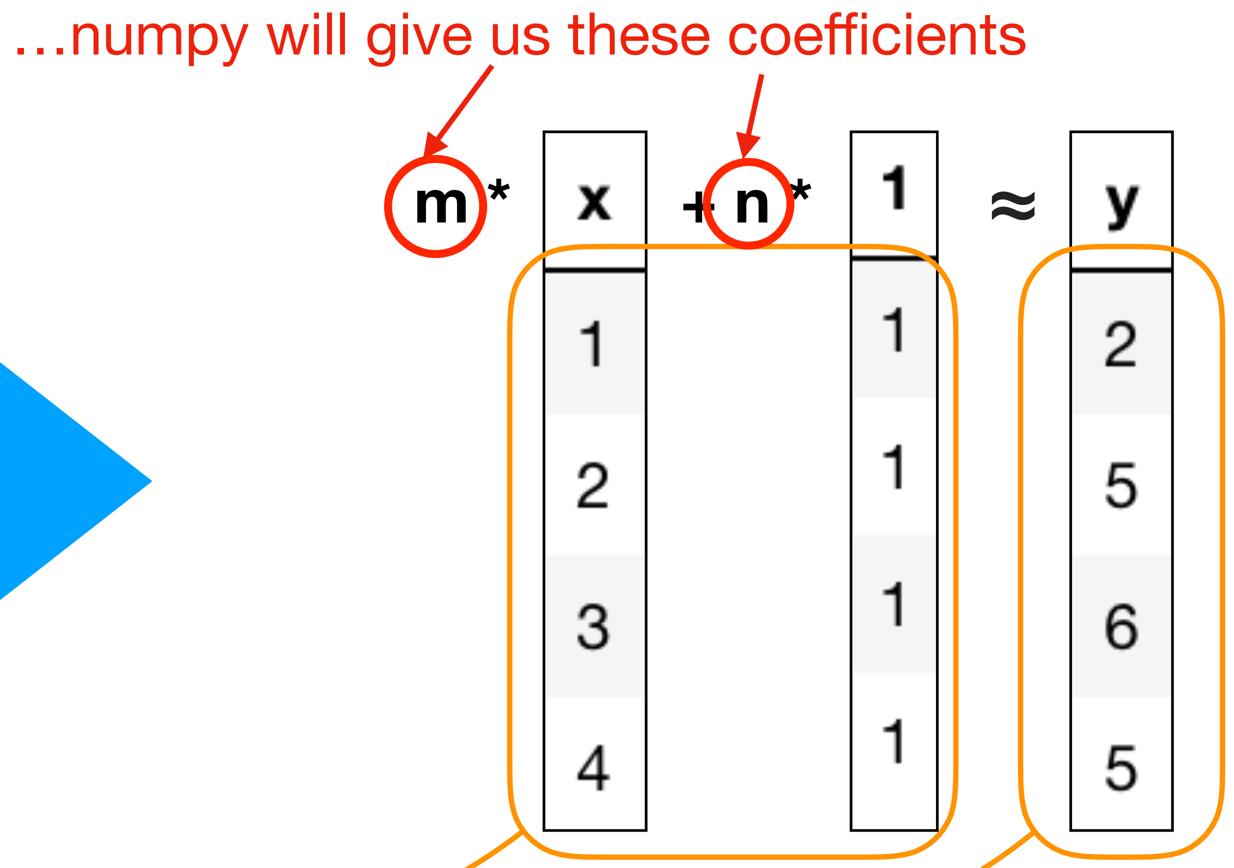
m * x + n * 1 ≈ y

x	1	y
1	1	2
2	1	5
3	1	6
4	1	5

```
np.linalg.lstsq(df[["x", “1”]], df[“y”],    )
```

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```



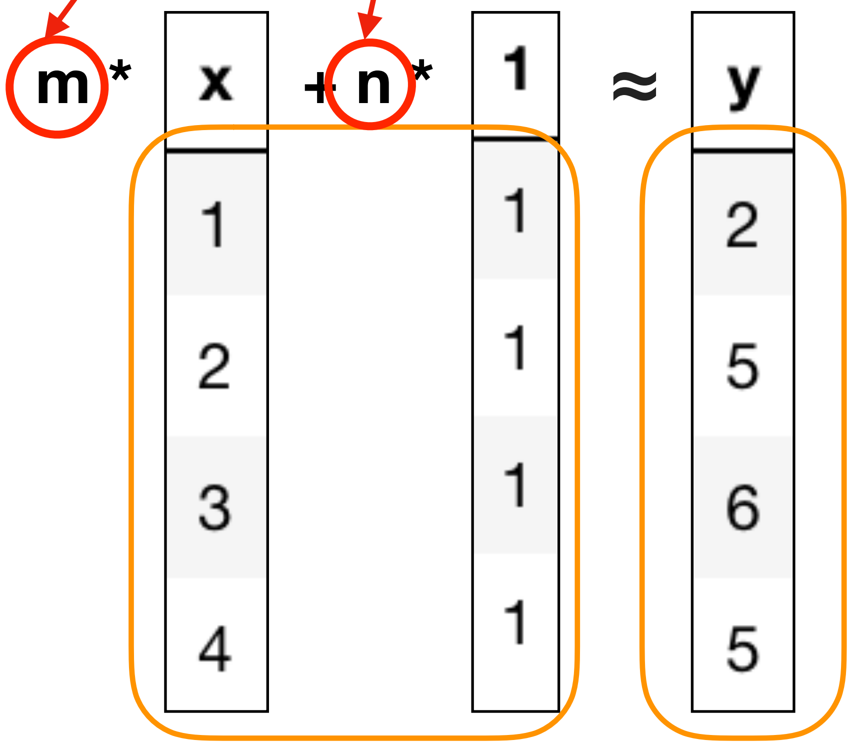
```
np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)
```

rcond is required, but not important for us

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```

…numpy will give us these coefficients



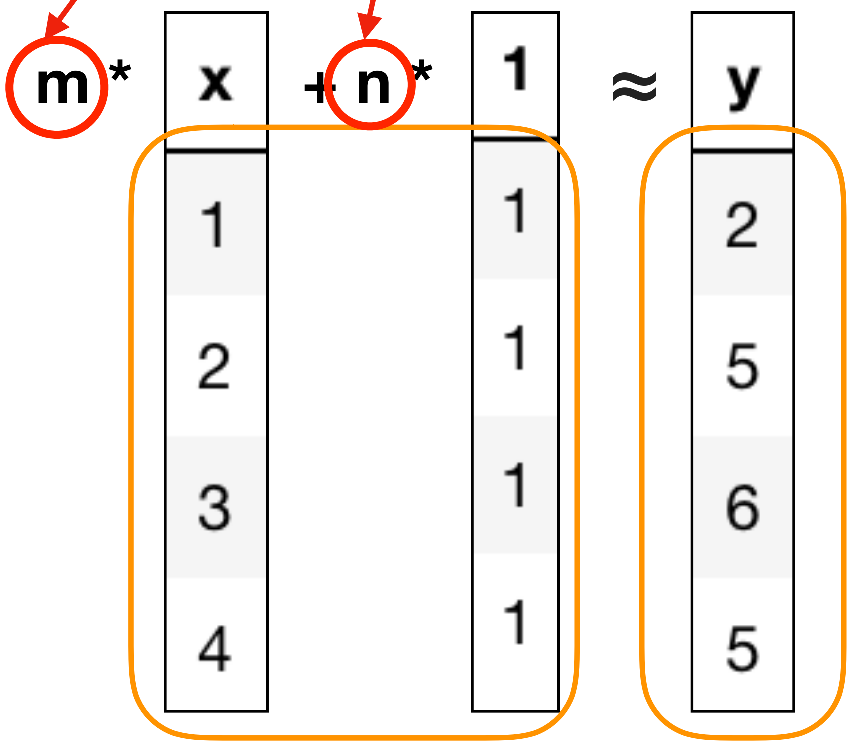
```
(array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))
```

```
res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)
```

Example data

```
df = DataFrame({
         "x": [1,2,3,4],
         "1": np.ones(4),
         "y": [2,5,6,5]
})
```

…numpy will give us these coefficients



$$m * \begin{array}{|c|} x \\ \hline 1 \\ 2 \\ 3 \\ 4 \end{array} + n * \begin{array}{|c|} 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \end{array} \approx \begin{array}{|c|} y \\ \hline 2 \\ 5 \\ 6 \\ 5 \end{array}$$

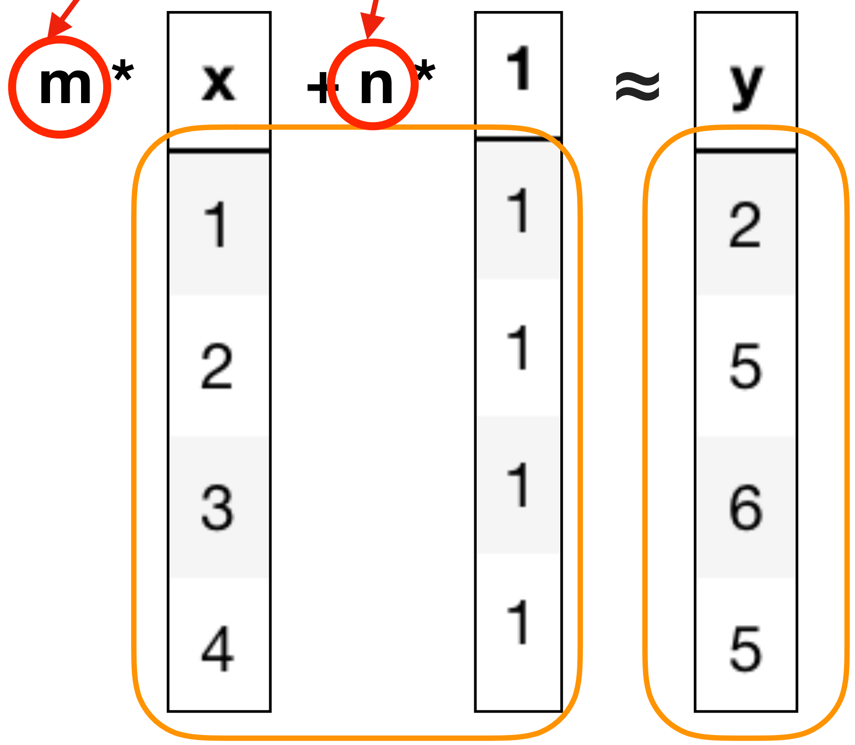
```
(array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))
```

```
res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
```

Example data

```
df = DataFrame({
         "x": [1,2,3,4],
         "1": np.ones(4),
         "y": [2,5,6,5]
})
```

…numpy will give us these coefficients



$m * x + n * 1 \approx y$

x	1	y
1	1	2
2	1	5
3	1	6
4	1	5

```
(array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))
```

```
res = np.linalg.lstsq(df[["x", "1"]], df["y"], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
```

Example data

```
df = DataFrame({
     "x": [1,2,3,4],
     "1": np.ones(4),
     "y": [2,5,6,5]
})
```

…numpy will give us these coefficients

$$m * \begin{array}{|c|} x \\ \hline 1 \\ 2 \\ 3 \\ 4 \end{array} + n * \begin{array}{|c|} 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \end{array} \approx \begin{array}{|c|} y \\ \hline 2 \\ 5 \\ 6 \\ 5 \end{array}$$

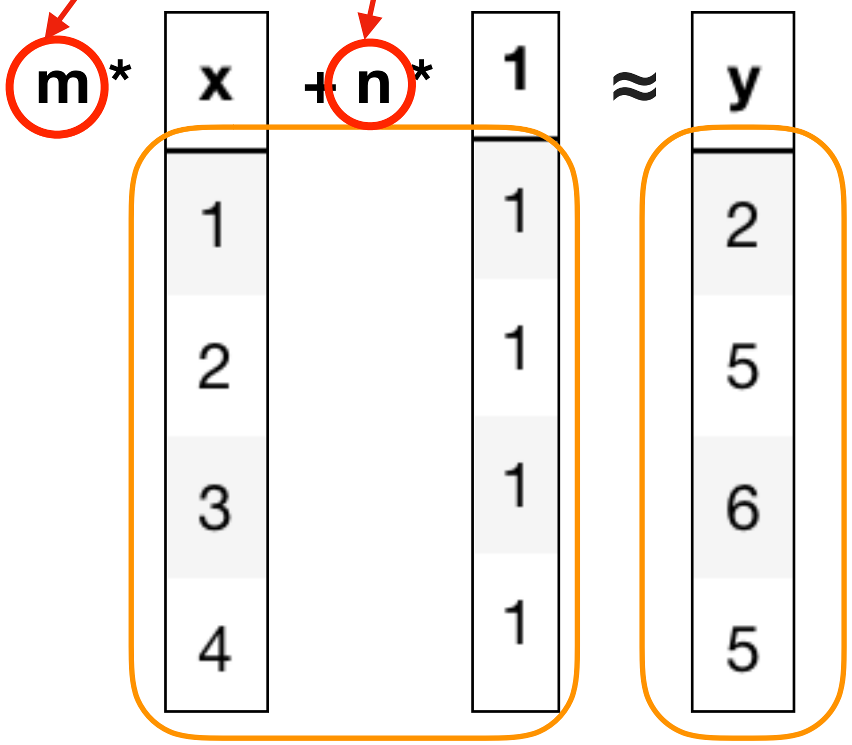
```
(array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))
```

```
res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0]
n = coefficients[1]
```

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```

…numpy will give us these coefficients



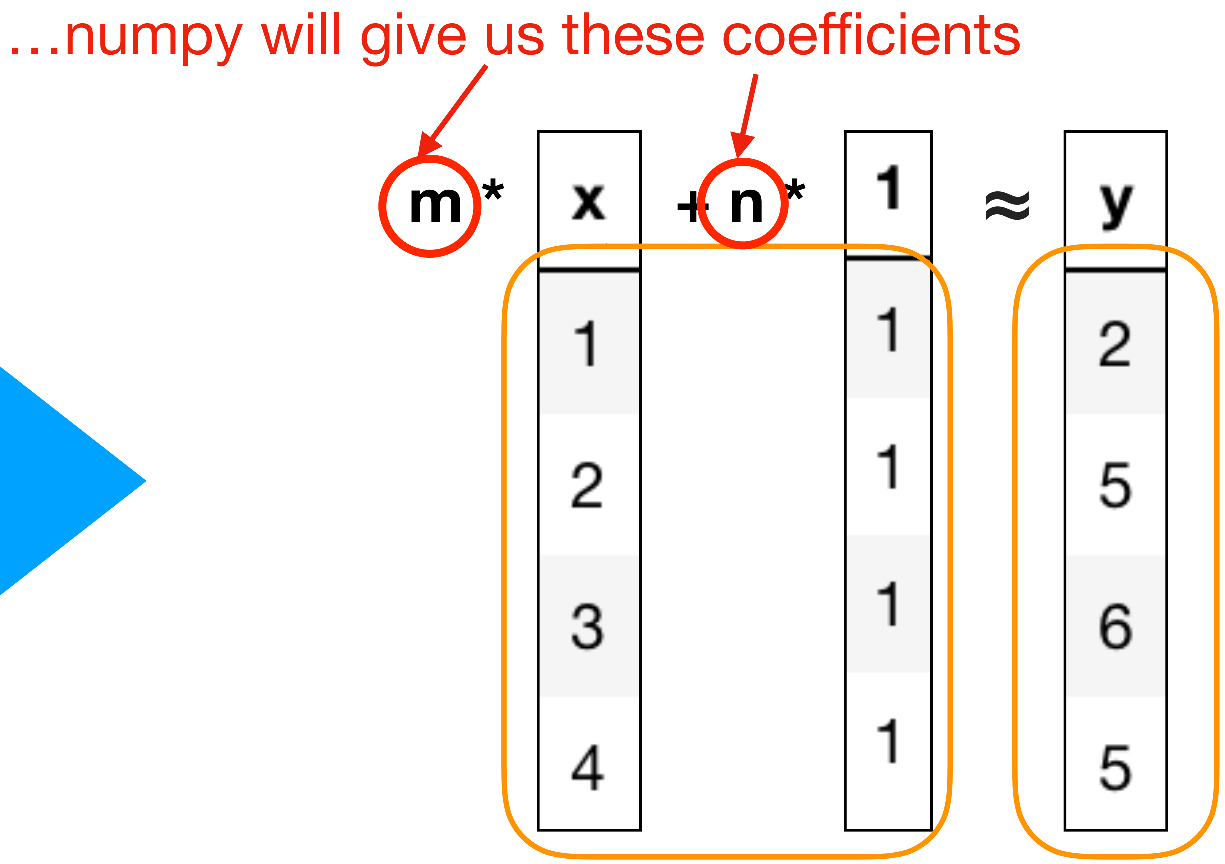
$$m * \begin{array}{|c|}\hline x \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array} + n * \begin{array}{|c|}\hline 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ \hline \end{array} \approx \begin{array}{|c|}\hline y \\ \hline 2 \\ 5 \\ 6 \\ 5 \\ \hline \end{array}$$

```
(array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))
```

```
res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
```

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```



…numpy will give us these coefficients

$$m * \begin{array}{|c|}\mathbf{x}\\\hline 1\\2\\3\\4\end{array} + n * \begin{array}{|c|}\mathbf{1}\\\hline 1\\1\\1\\1\end{array} \approx \begin{array}{|c|}\mathbf{y}\\\hline 2\\5\\6\\5\end{array}$$

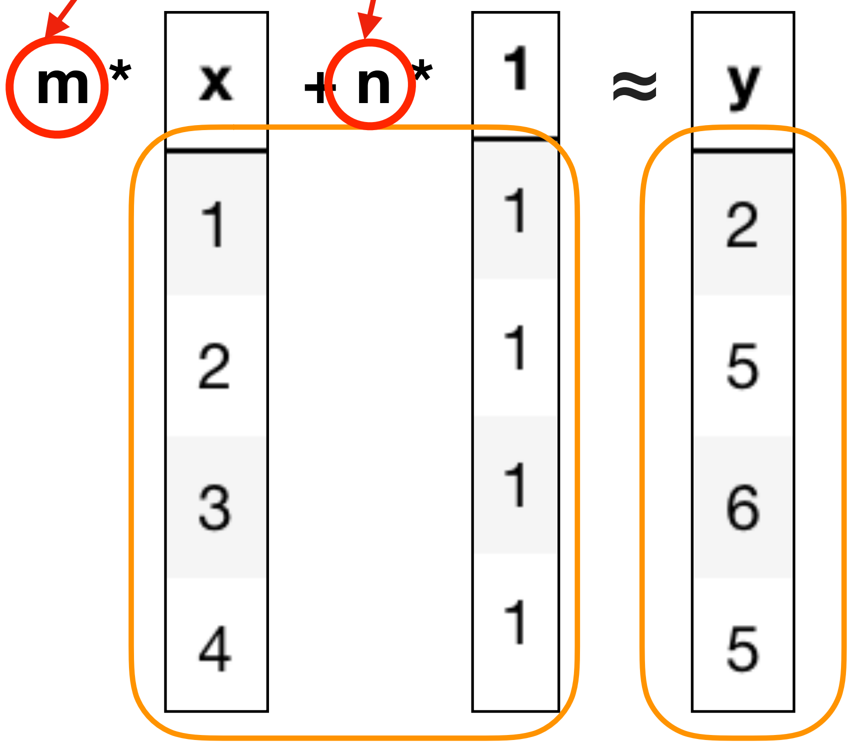
y = 1 * x + 2 * 1

```
      (array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))

res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
```

Example data

```
df = DataFrame({
        "x": [1,2,3,4],
        "1": np.ones(4),
        "y": [2,5,6,5]
})
```

…numpy will give us these coefficients



$$m * \begin{array}{|c|} x \\ \hline 1 \\ 2 \\ 3 \\ 4 \end{array} + n * \begin{array}{|c|} 1 \\ \hline 1 \\ 1 \\ 1 \\ 1 \end{array} \approx \begin{array}{|c|} y \\ \hline 2 \\ 5 \\ 6 \\ 5 \end{array}$$

```
(array([1., 2.]), array([4.]), 2, array([5.77937881, 0.77380911]))
```

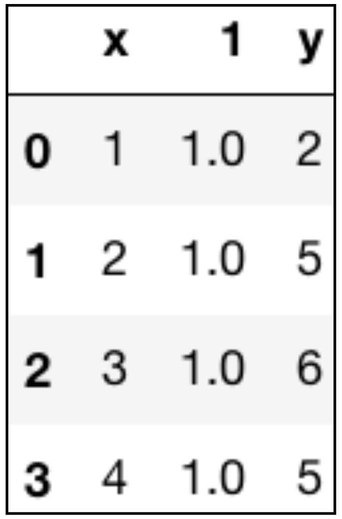
```
res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
```

```
df = DataFrame({
    "x": [1,2,3,4],
    "1": np.ones(4),
    "y": [2,5,6,5]
})
```

```
res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
```

```
df = DataFrame({
     "x": [1,2,3,4],
     "1": np.ones(4),
     "y": [2,5,6,5]
})

res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
```

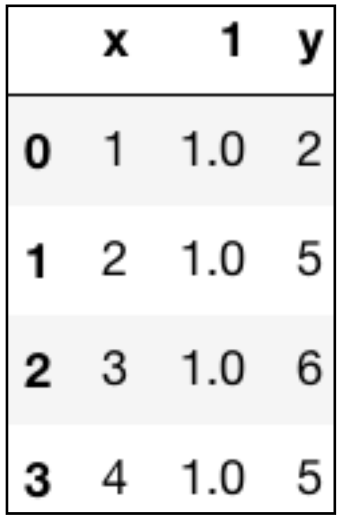
df:



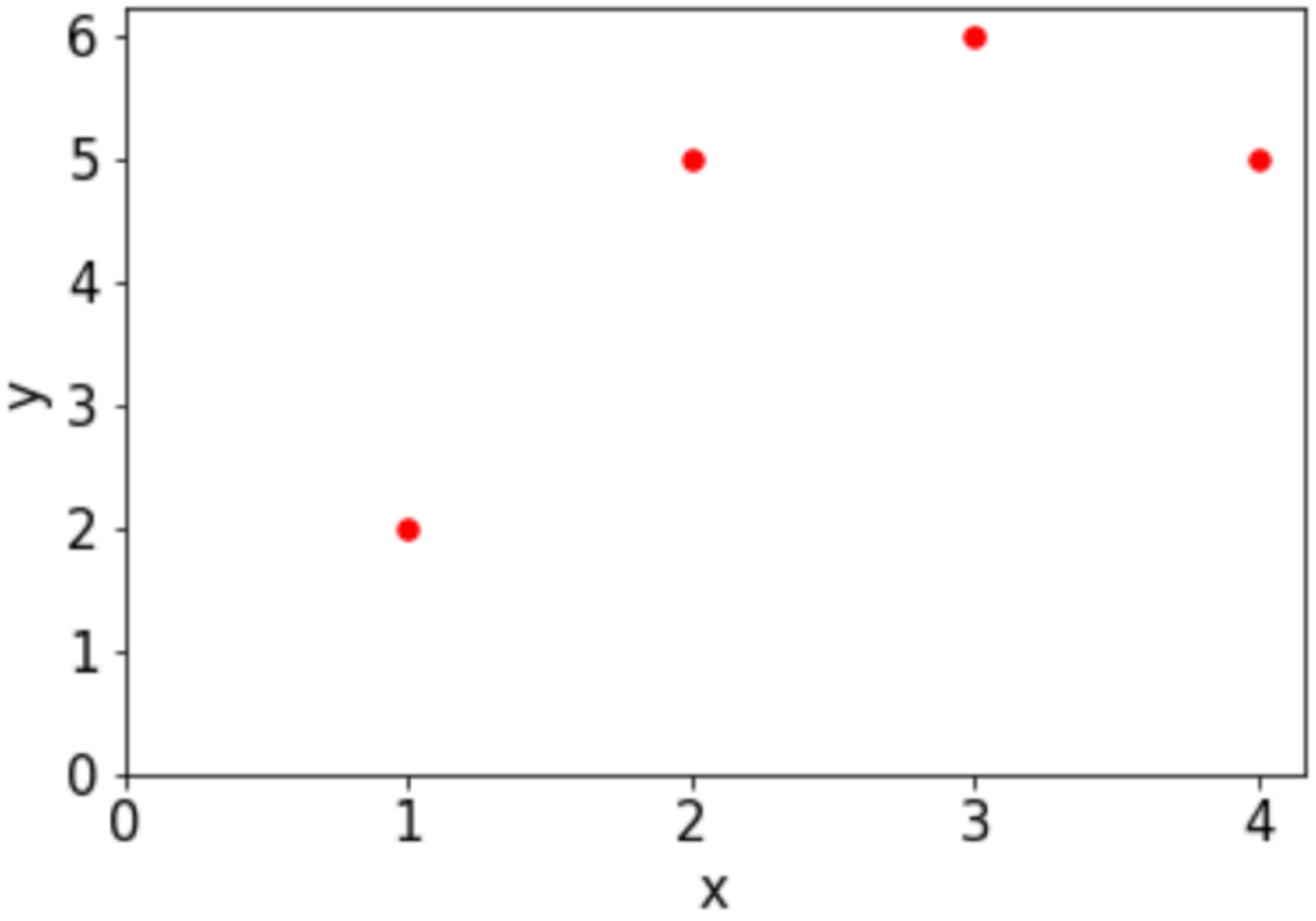
	x	1	y
0	1	1.0	2
1	2	1.0	5
2	3	1.0	6
3	4	1.0	5

```
df = DataFrame({
     "x": [1,2,3,4],
     "1": np.ones(4),
     "y": [2,5,6,5]
})

res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
ax = df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)
```

df:

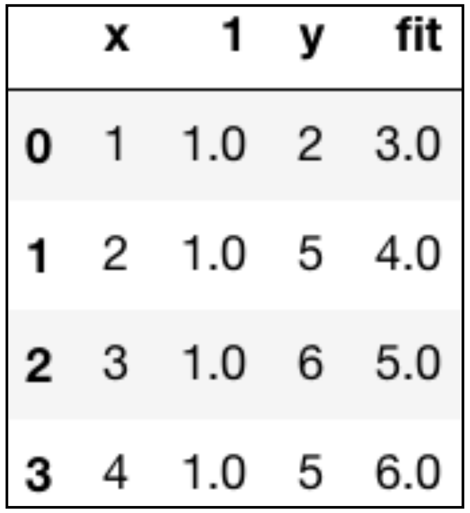


	x	1	y
0	1	1.0	2
1	2	1.0	5
2	3	1.0	6
3	4	1.0	5

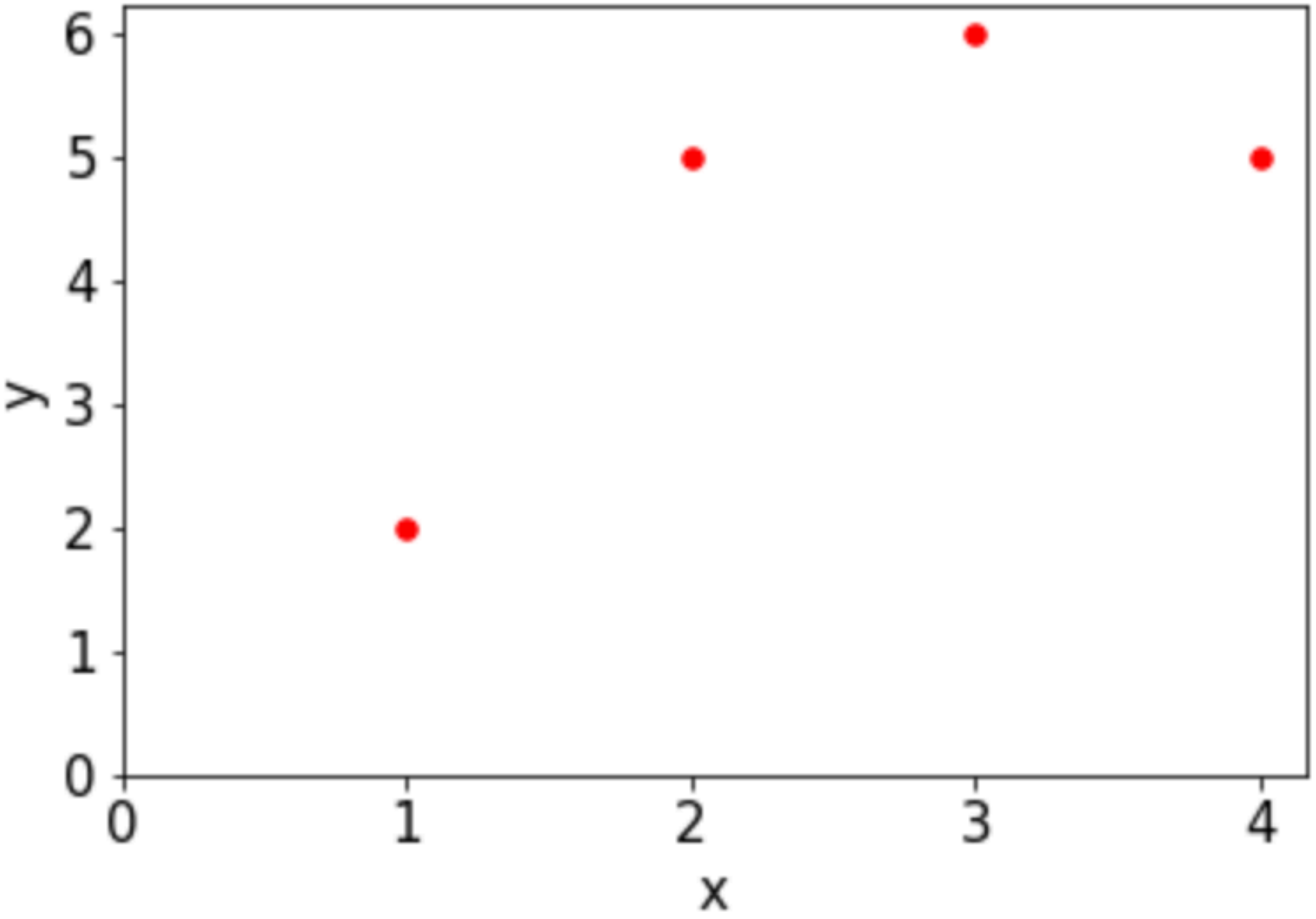


```
df = DataFrame({
     "x": [1,2,3,4],
     "1": np.ones(4),
     "y": [2,5,6,5]
})

res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
ax = df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

df[“fit”] = df[“x”] * m + n
```

df:



	x	1	y	fit
0	1	1.0	2	3.0
1	2	1.0	5	4.0
2	3	1.0	6	5.0
3	4	1.0	5	6.0

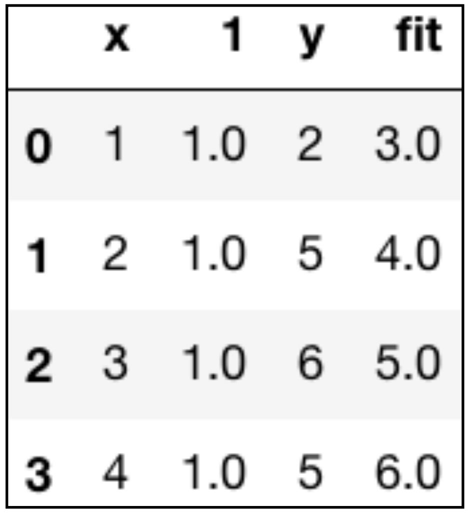


```
df = DataFrame({
    "x": [1,2,3,4],
    "1": np.ones(4),
    "y": [2,5,6,5]
})

res = np.linalg.lstsq(df[["x", “1”]], df[“y”], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
ax = df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

df[“fit”] = df[“x”] * m + n
```

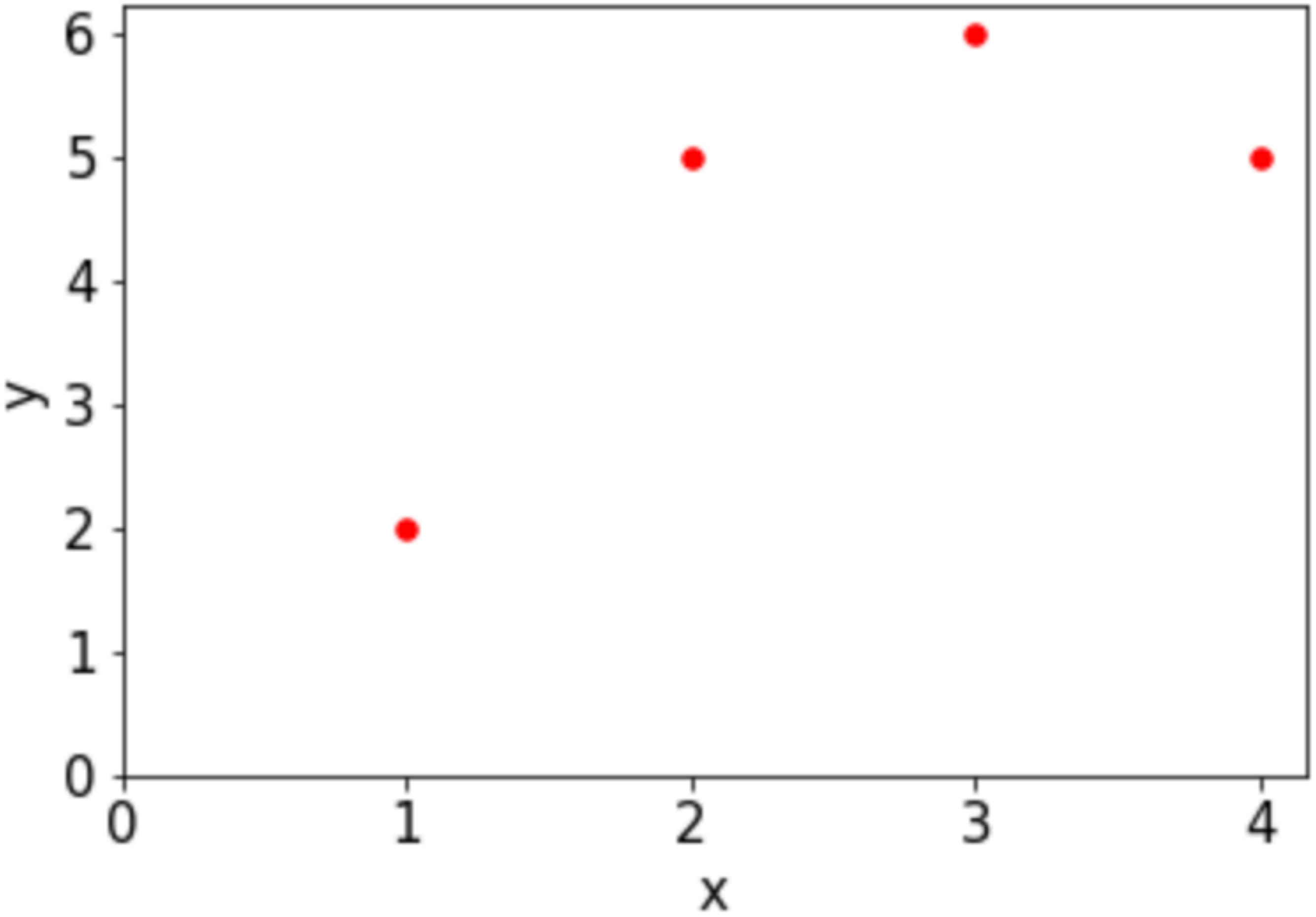
scatter data

fit line

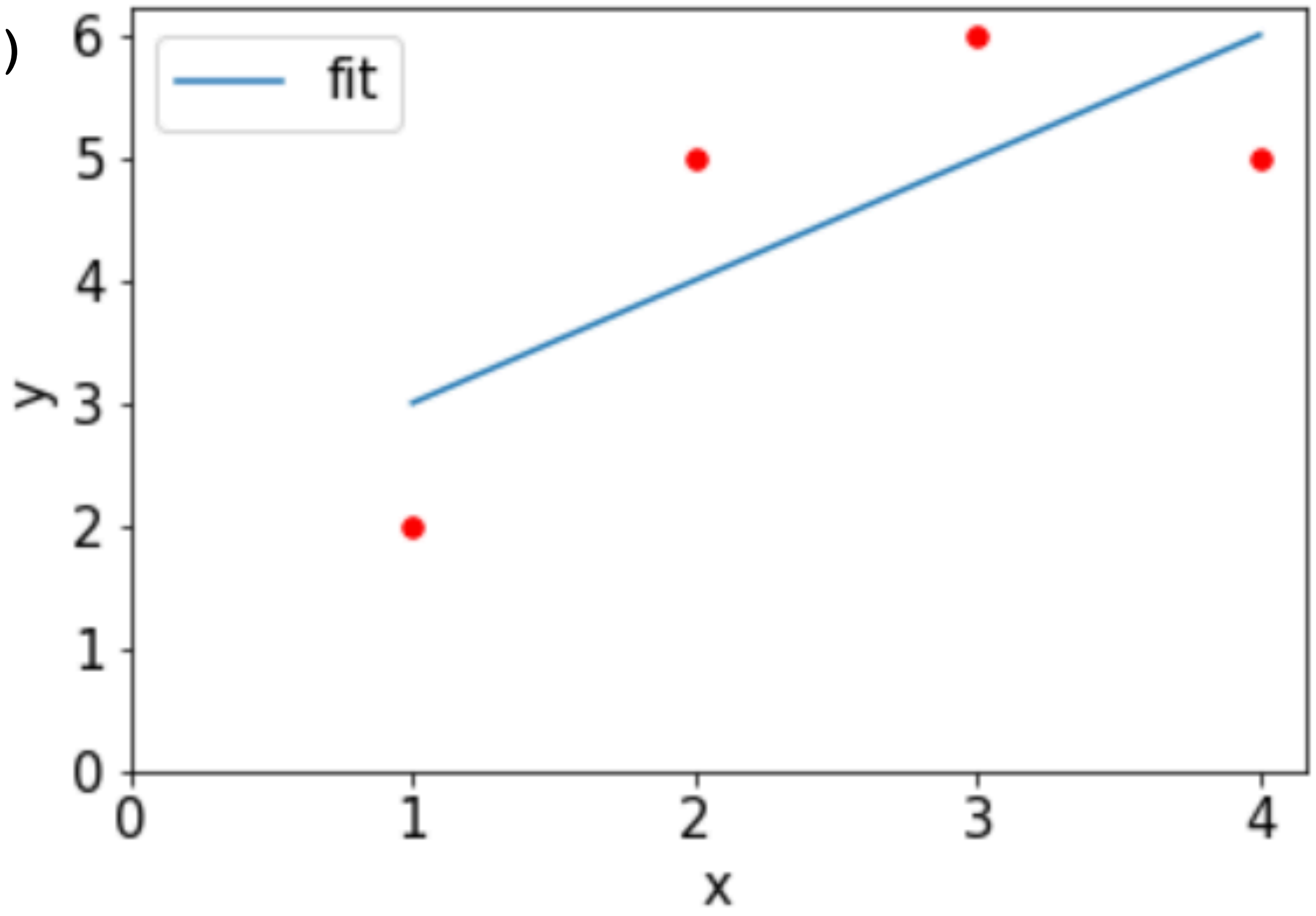
df:



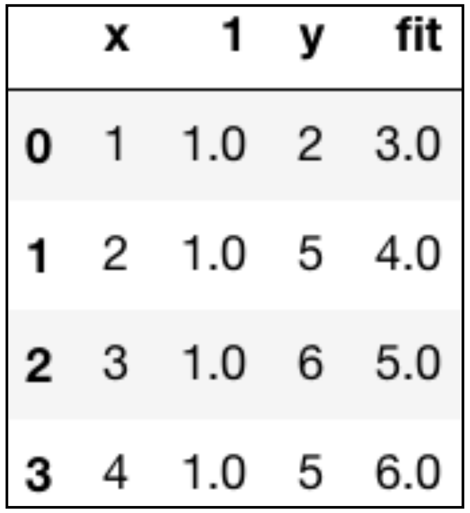
	x	1	y	fit
0	1	1.0	2	3.0
1	2	1.0	5	4.0
2	3	1.0	6	5.0
3	4	1.0	5	6.0



```
df = DataFrame({
     "x": [1,2,3,4],
     "1": np.ones(4),
     "y": [2,5,6,5]
})

res = np.linalg.lstsq(df[["x", "1"]], df["y"], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
ax = df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

df["fit"] = df["x"] * m + n
df.plot.line(x='x', y='fit', ax=ax)
```



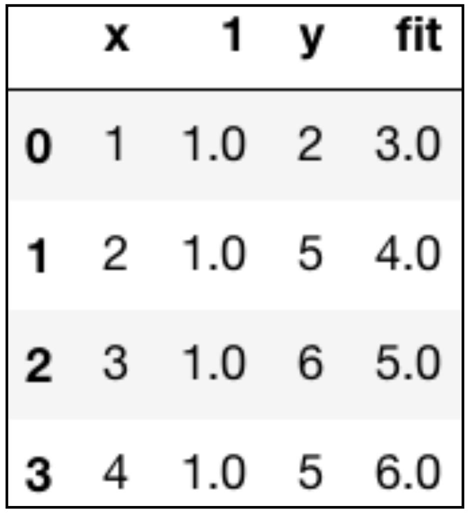
df:



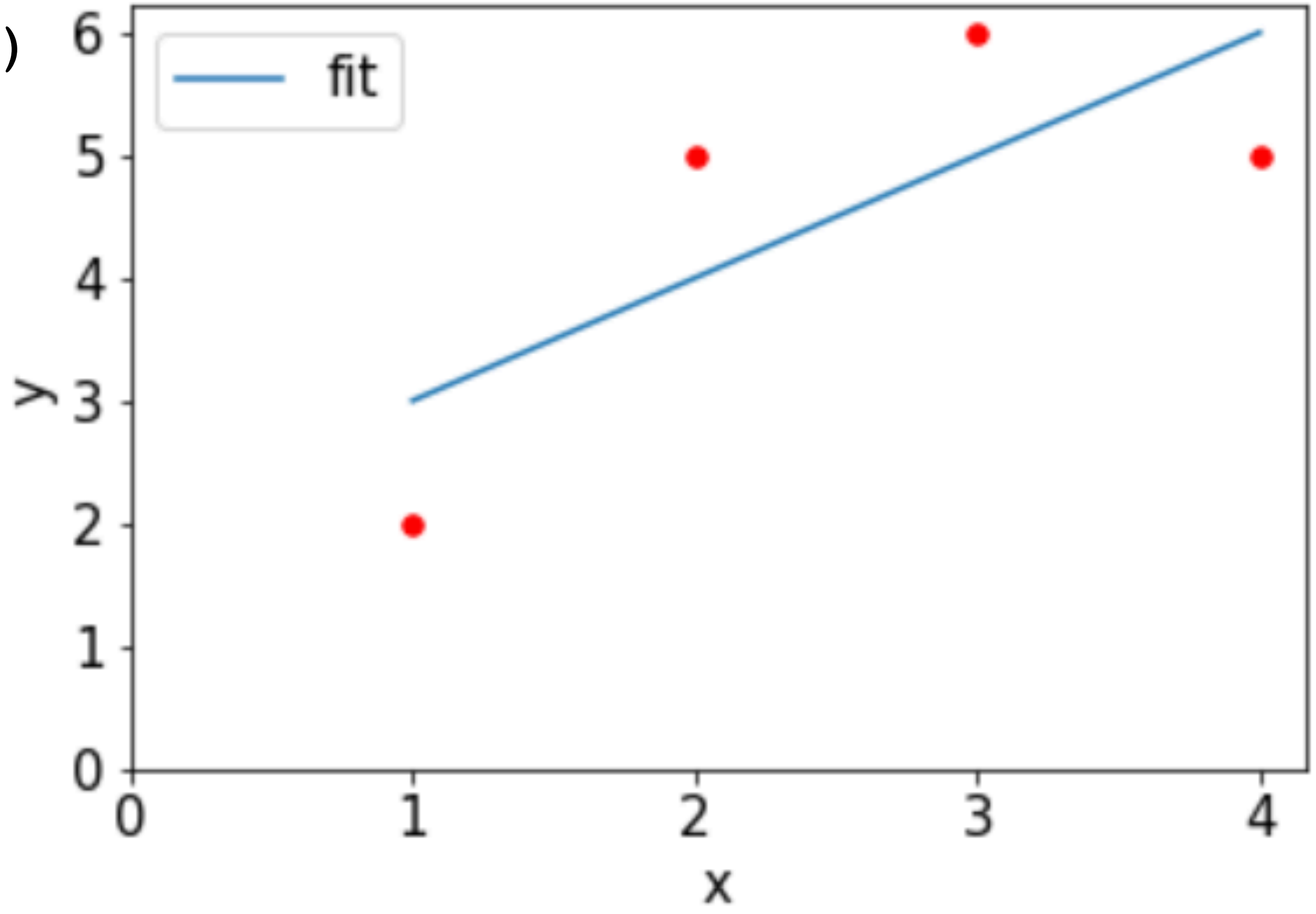
	x	1	y	fit
0	1	1.0	2	3.0
1	2	1.0	5	4.0
2	3	1.0	6	5.0
3	4	1.0	5	6.0

```
df = DataFrame({
     "x": [1,2,3,4],
     "y": [2,5,6,5]
})
df["1"] = 1

res = np.linalg.lstsq(df[["x", "1"]], df["y"], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
ax = df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

df["fit"] = df["x"] * m + n
df.plot.line(x='x', y='fit', ax=ax)
```

df:



	x	1	y	fit
0	1	1.0	2	3.0
1	2	1.0	5	4.0
2	3	1.0	6	5.0
3	4	1.0	5	6.0

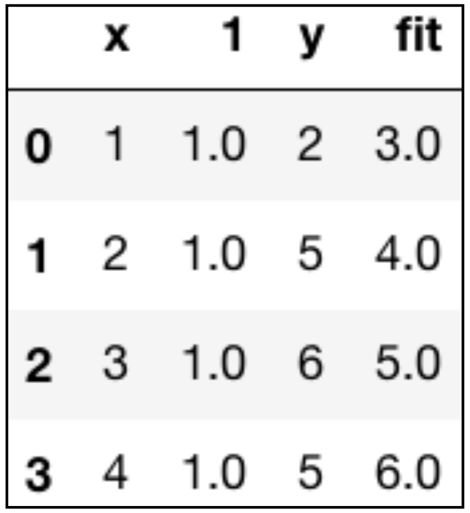


```
df = DataFrame({
    "x": [1,2,3,4],
    "y": [2,5,6,5]
})
df["1"] = 1
```

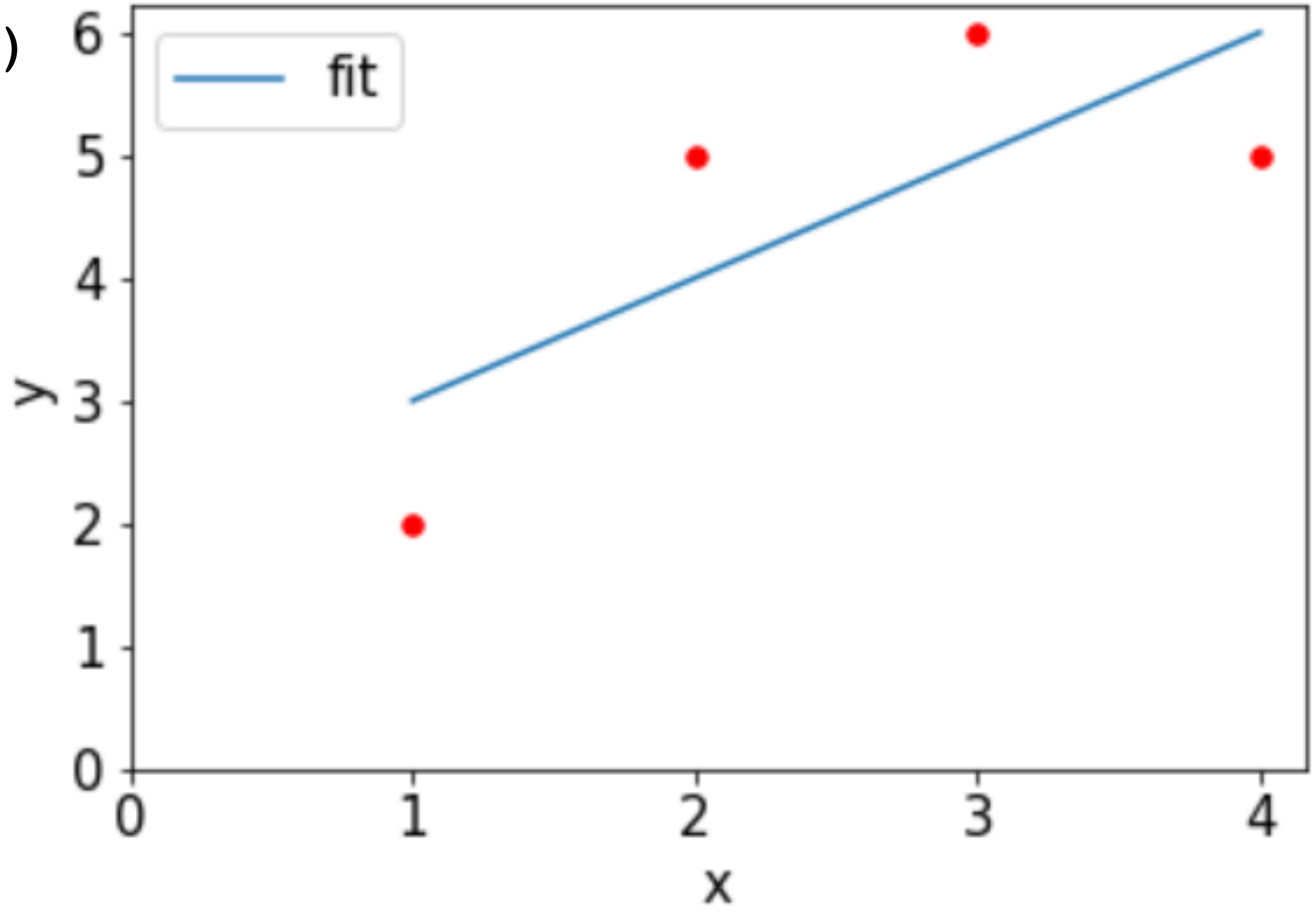
this is a complete example. You can copy/paste and just change the input data

```
res = np.linalg.lstsq(df[["x", "1"]], df["y"], rcond=None)

# res is a tuple: (COEFFICIENTS, VALUE, VALUE, VALUE)
coefficients = res[0] # coefficients is (m,n)
m = coefficients[0] # slope is 1
n = coefficients[1] # intercept is 2
ax = df.plot.scatter(x='x', y='y', c='red', s=30, xlim=0, ylim=0)

df["fit"] = df["x"] * m + n
df.plot.line(x='x', y='fit', ax=ax)
```

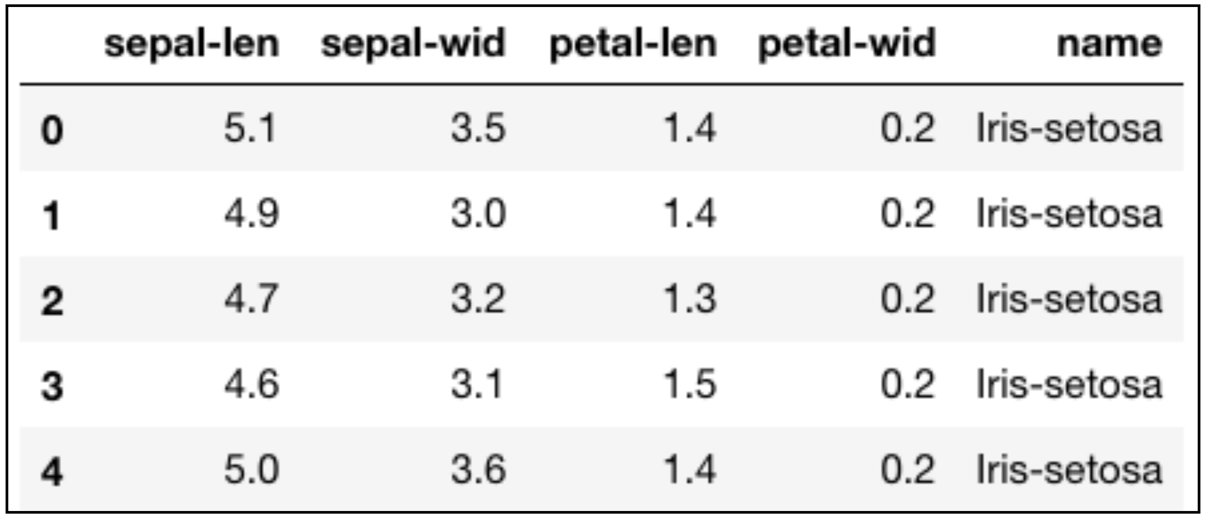
df:



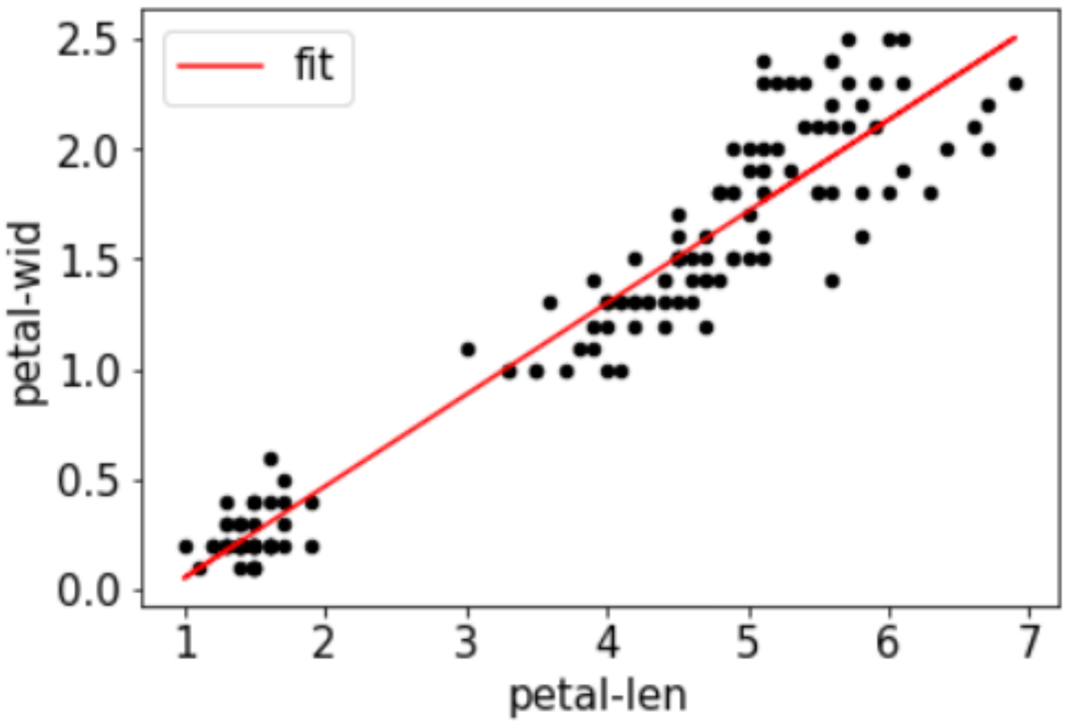
	x	1	y	fit
0	1	1.0	2	3.0
1	2	1.0	5	4.0
2	3	1.0	6	5.0
3	4	1.0	5	6.0



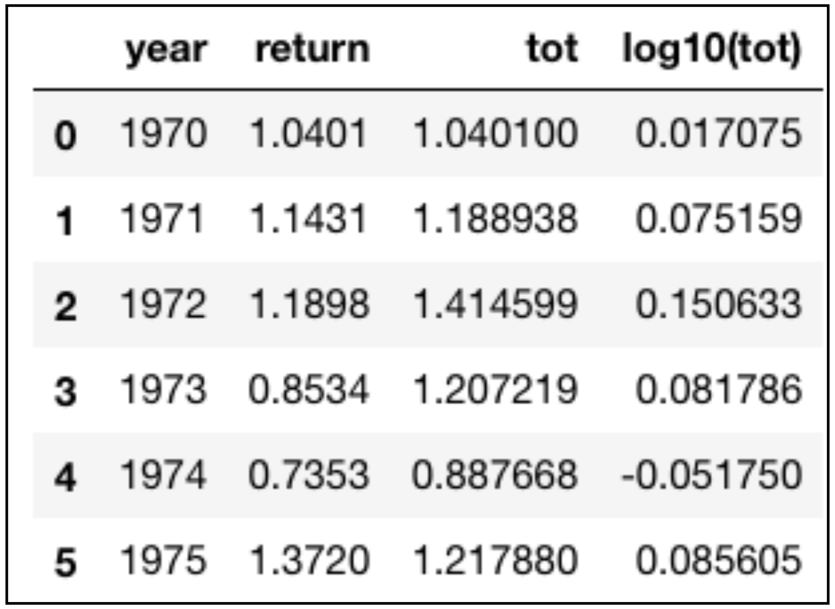
Demo 2: draw real fit line on Iris data



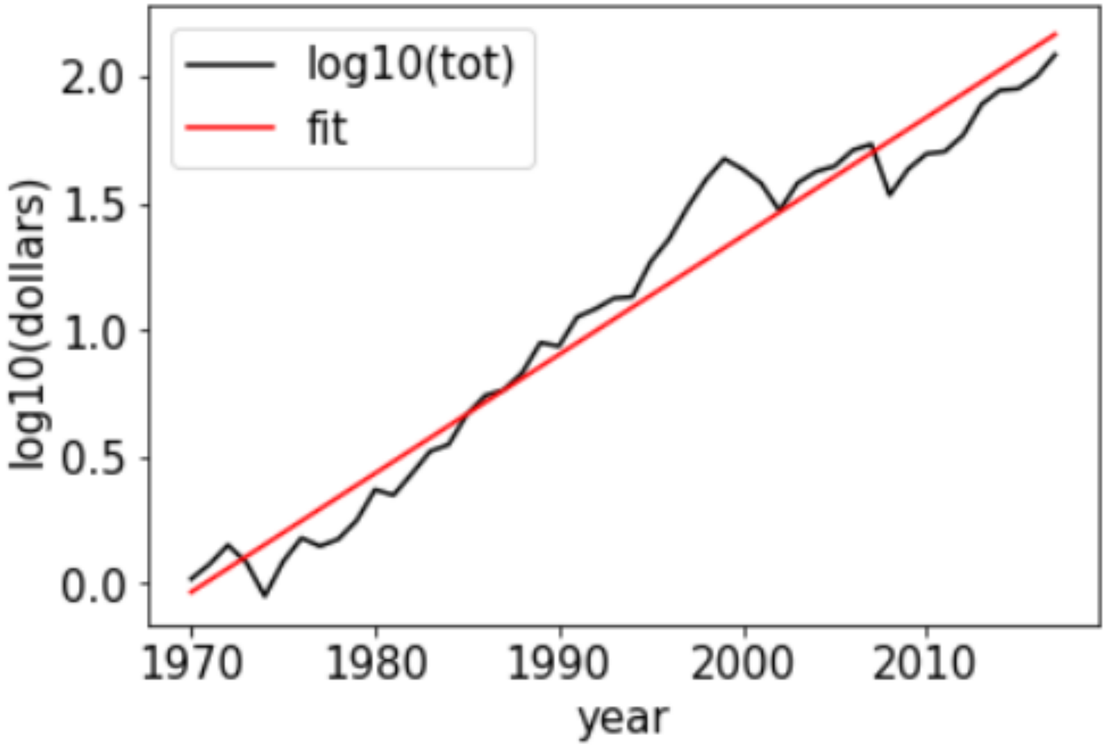
	sepal-len	sepal-wid	petal-len	petal-wid	name
0	5.1	3.5	1.4	0.2	Iris-setosa
1	4.9	3.0	1.4	0.2	Iris-setosa
2	4.7	3.2	1.3	0.2	Iris-setosa
3	4.6	3.1	1.5	0.2	Iris-setosa
4	5.0	3.6	1.4	0.2	Iris-setosa



Demo 3: fit line to S&P 500 returns



	year	return	tot	log10(tot)
0	1970	1.0401	1.040100	0.017075
1	1971	1.1431	1.188938	0.075159
2	1972	1.1898	1.414599	0.150633
3	1973	0.8534	1.207219	0.081786
4	1974	0.7353	0.887668	-0.051750
5	1975	1.3720	1.217880	0.085605


log10(tot)
fit
log10(dollars)
year