CS 301: Recursion

The Art of Self Reference

Tyler Caraza-Harter

Goal: use self-reference is a meaningful way

Hofstadter's Law*: "It always takes longer than you expect, even when you take into account* ***Hofstadter's Law****."*

(From Gödel, Escher, Bach)

good advice for CS 301 assignments!

https://en.wikipedia.org/wiki/Circular_definition

Goal: use self-reference is a meaningful way

Hofstadter's Law*: "It always takes longer than you expect, even when you take into account* ***Hofstadter's Law****."*

(From Gödel, Escher, Bach)

mountain*: "a landmass that projects conspicuously above its surroundings and is higher than a* ***hill****"*

hill*: "a usually rounded natural elevation of land lower than a* ***mountain****"*

(Example of **unhelpful** self reference from Merriam-Webster dictionary)

https://en.wikipedia.org/wiki/Circular_definition

Overview: Learning Objectives

Recursive information

- What is a **recursive definition/structure**?
- Arbitrarily vs. infinitely

Recursive code

- What is **recursive code**?
- Why write recursive code?
- Where do computers keep local variables for recursive calls?
- What happens to programs with **infinite recursion**?

Read *Think Python*

- ✦ Ch 5: “Recursion” through “Infinite Recursion”
- ✦ Ch 6: “More Recursion” through end

Overview: Learning Objectives

Recursive information

- What is a **recursive definition/structure**?
- Arbitrarily vs. infinitely

Recursive code

- What is **recursive code**?
- Why write recursive code?
- Where do computers keep local variables for recursive calls?
- What happens to programs with **infinite recursion**?

Read *Think Python*

- Ch 5: “Recursion” through “Infinite Recursion”
- Ch 6: “More Recursion” through end

What is Recursion?

Recursive definitions contain the term in the body

- Dictionaries, mathematical definitions, etc

A number **x** is a positive even number if:

- **x** is 2

OR

- **x** equals another positive even number plus two

What is Recursion?

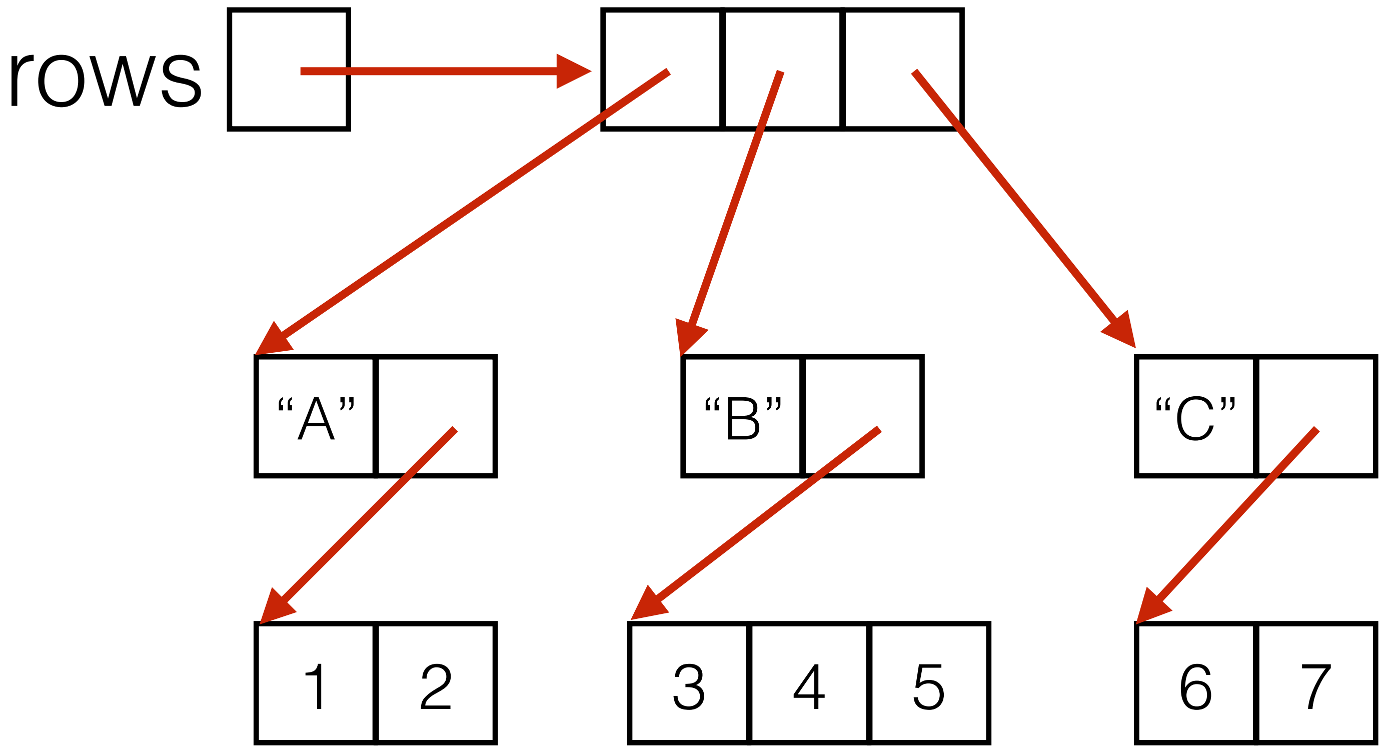
Recursive definitions contain the term in the body

- Dictionaries, mathematical definitions, etc

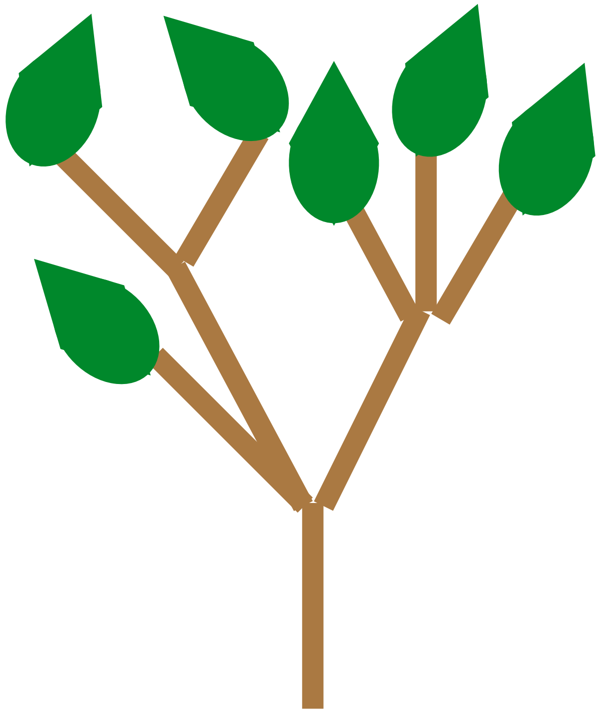
Recursive structures may refer to structures of the same type

- data structures or real-world structures

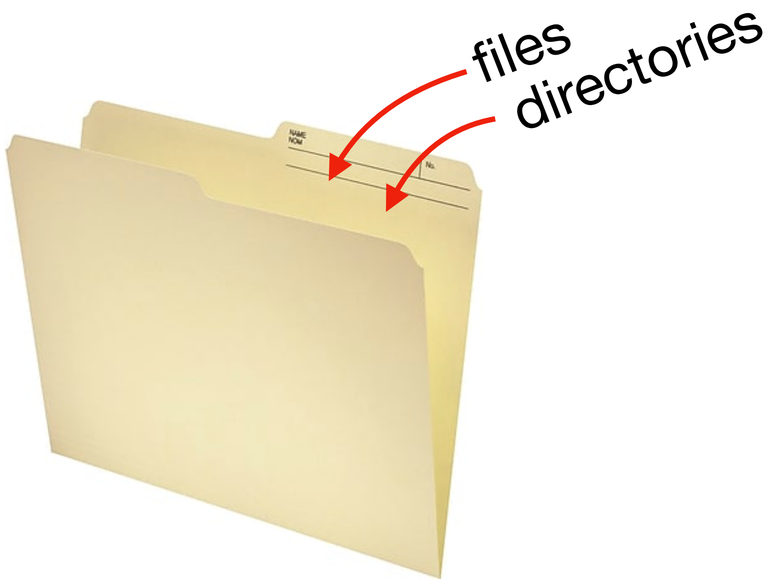
```
rows = [
   [“A”,[1,2]],
   [“B”,[3,4,5]],
   [“C”,[6,7]]
]
```



Recursive structures are EVERYWHERE!



nature



files

```
{
  “name”: “alice”,
  “grade”: “A”,
  “score”: 96,
  “exams”: {
    “midterm”: {“points”:94,
                “total”:100},
    “final”: {“points”: 98,
              “total”: 100}
  }
}
```

formats

Example: Trees (Finite Recursion)

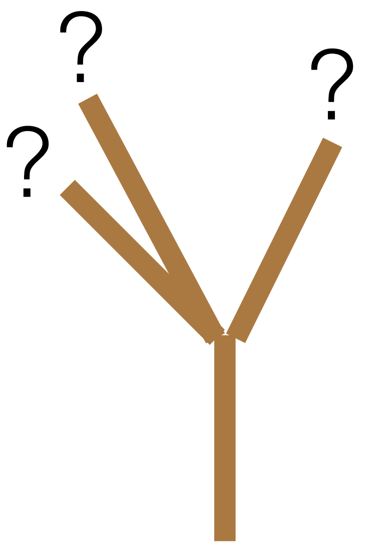
Term: branch

Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf

Example: Trees (Finite Recursion)

Term: branch

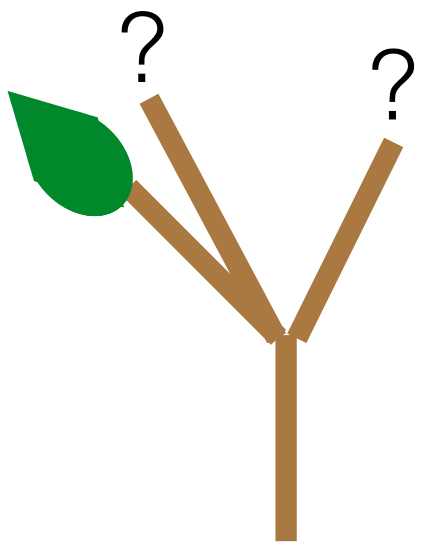
Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf



Example: Trees (Finite Recursion)

Term: branch

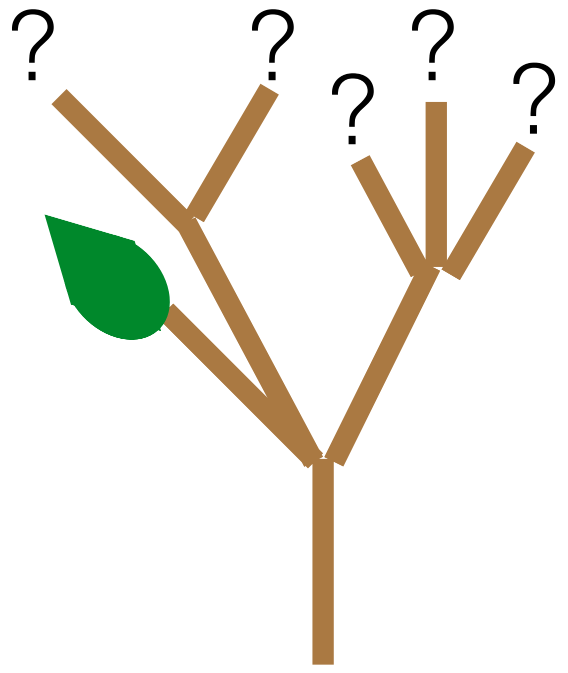
Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf



Example: Trees (Finite Recursion)

Term: branch

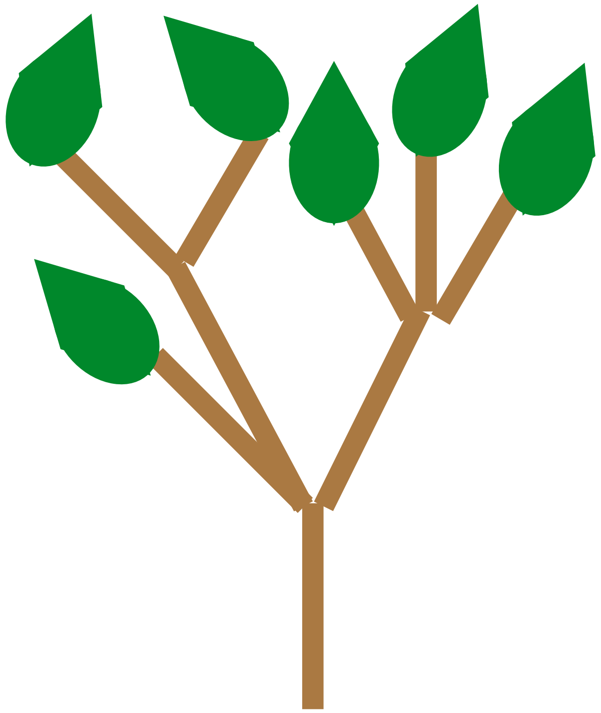
Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf



Example: Trees (Finite Recursion)

Term: branch

Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf

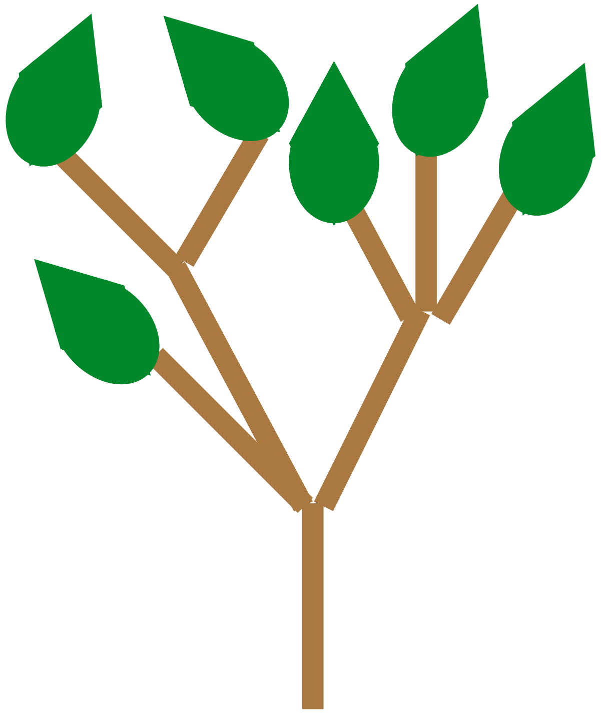


Example: Trees (Finite Recursion)

Term: branch

Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf

trees are arbitrarily large: **recursive case** allows indefinite growth



arbitrarily != infinitely

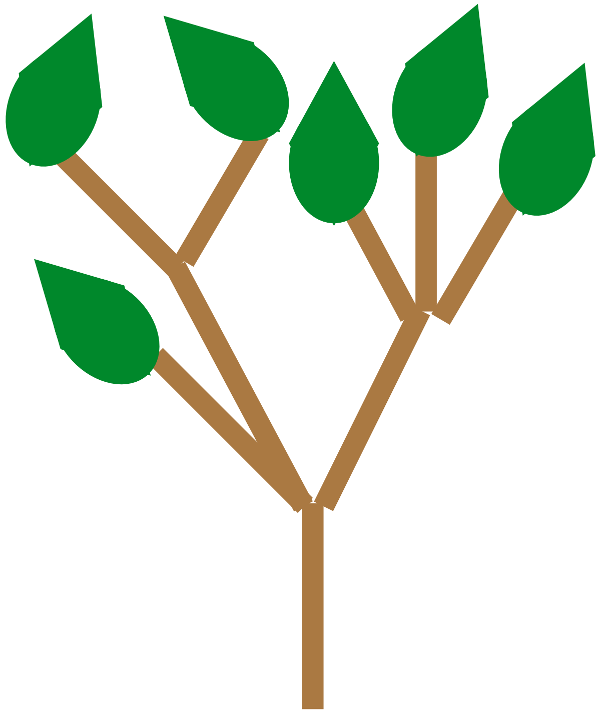
Example: Trees (Finite Recursion)

Term: branch

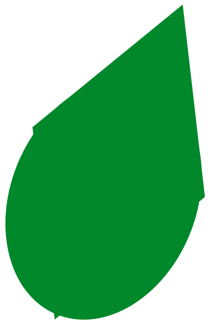
Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf

trees are finite: eventual **base case** allows completion

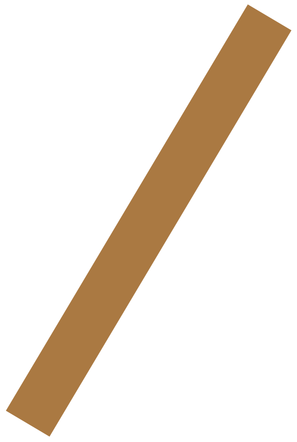
trees are arbitrarily large: **recursive case** allows indefinite growth



arbitrarily != infinitely



base case (leaf)



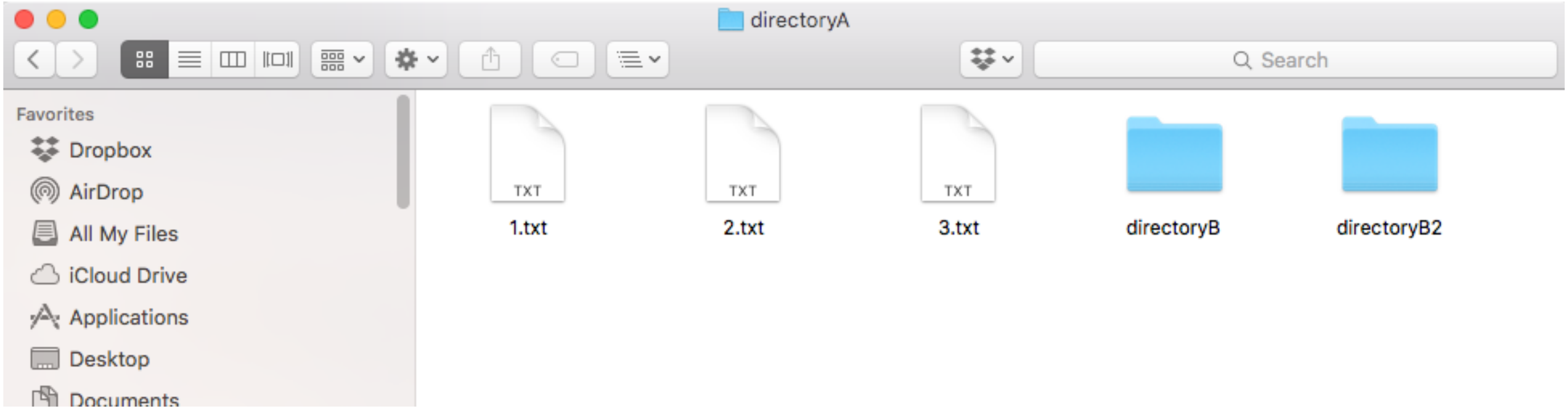
recursive case (branch)

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories

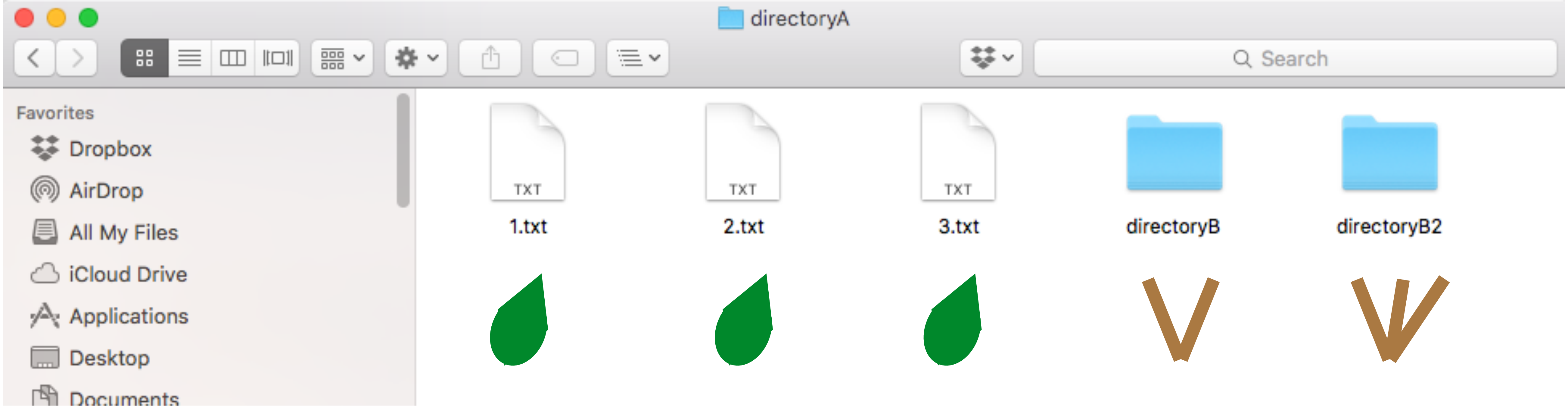


Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



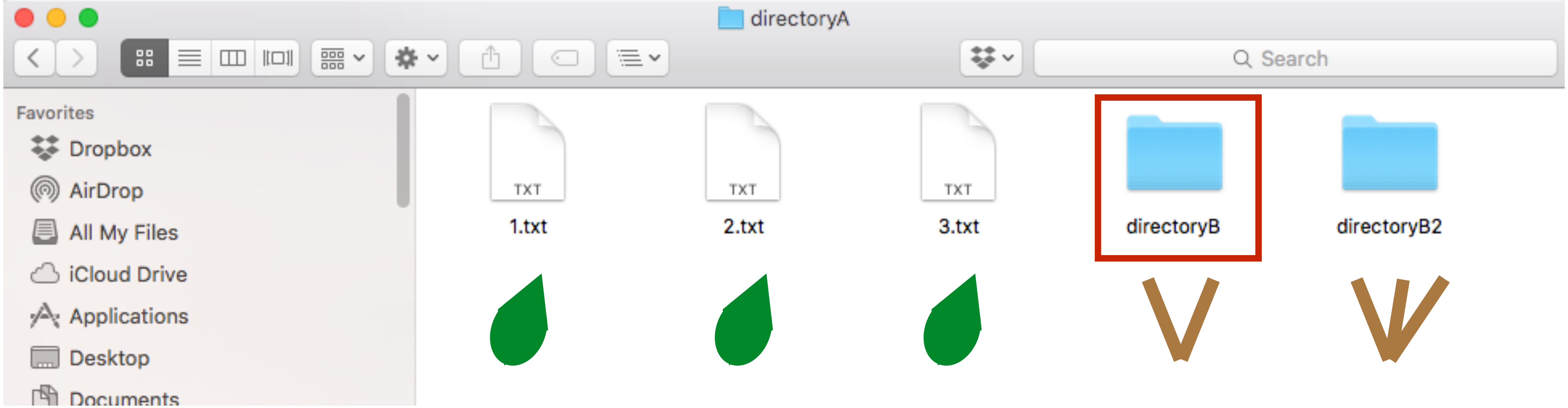
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



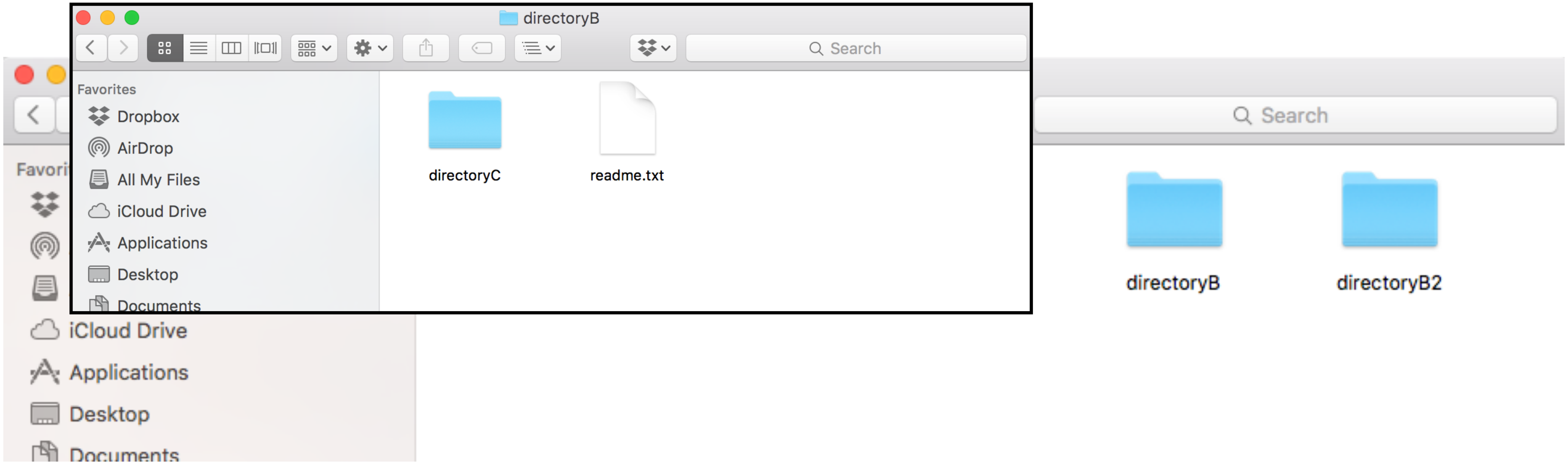
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



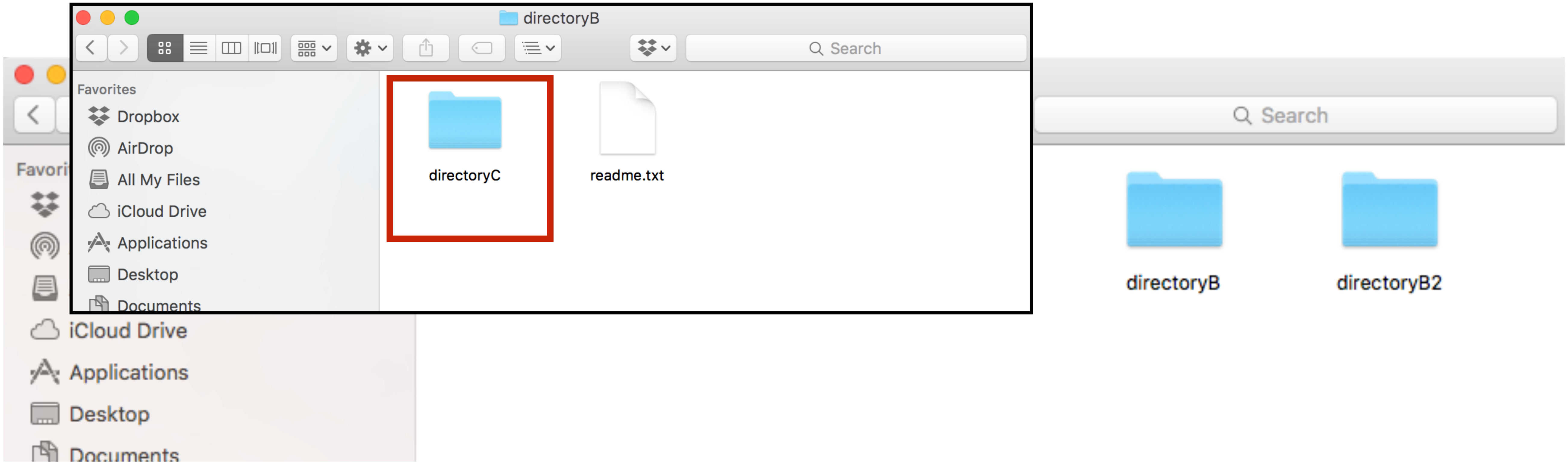
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



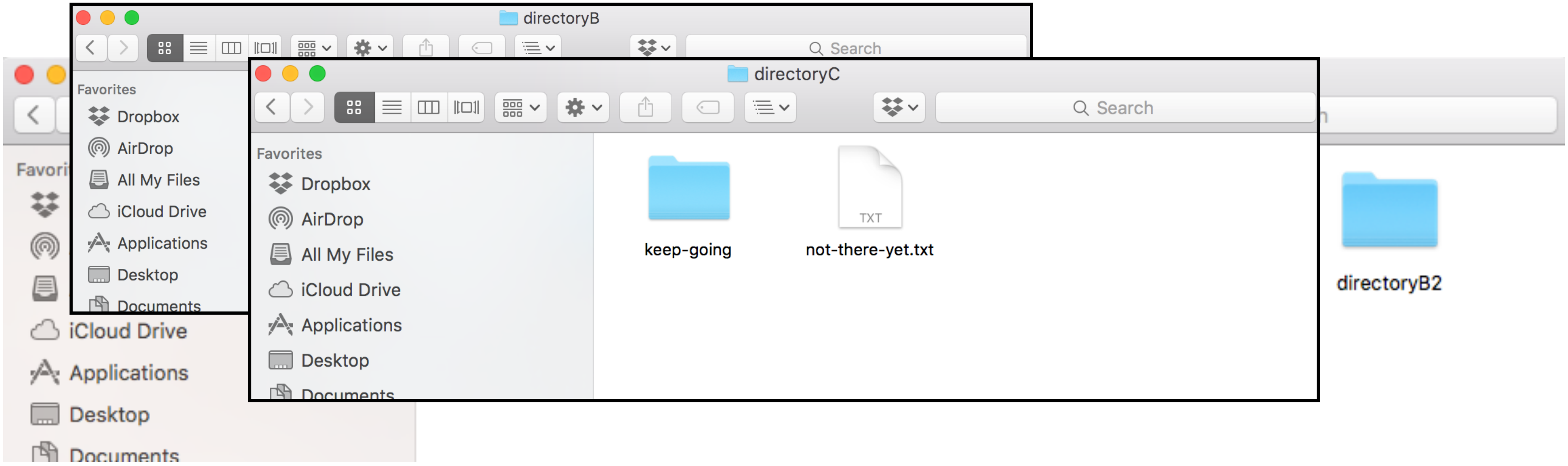
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



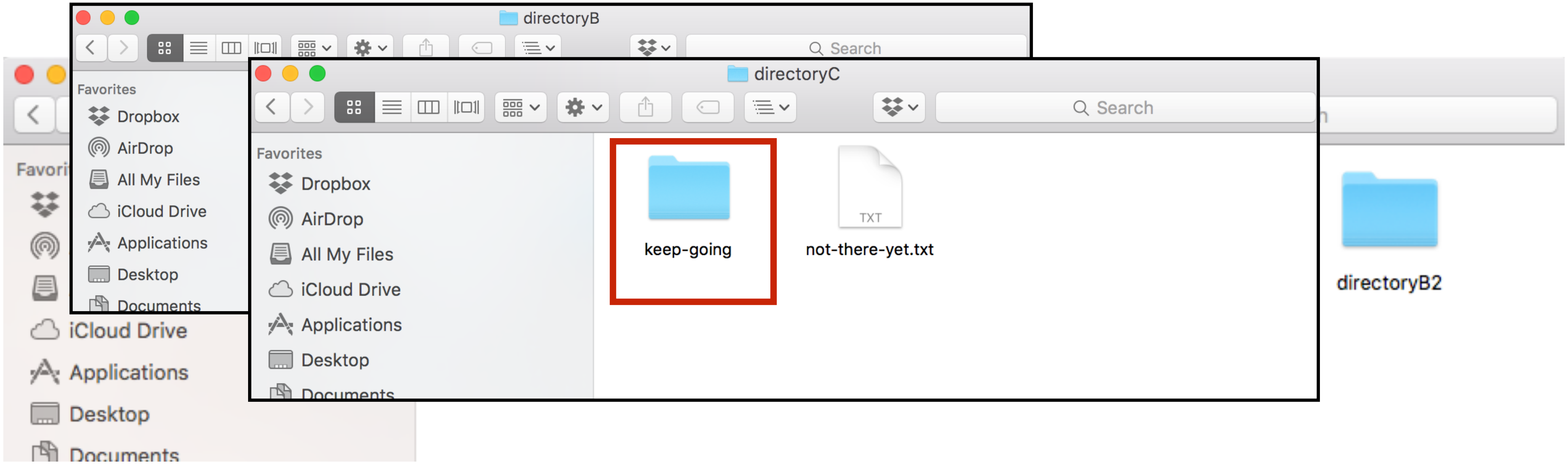
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



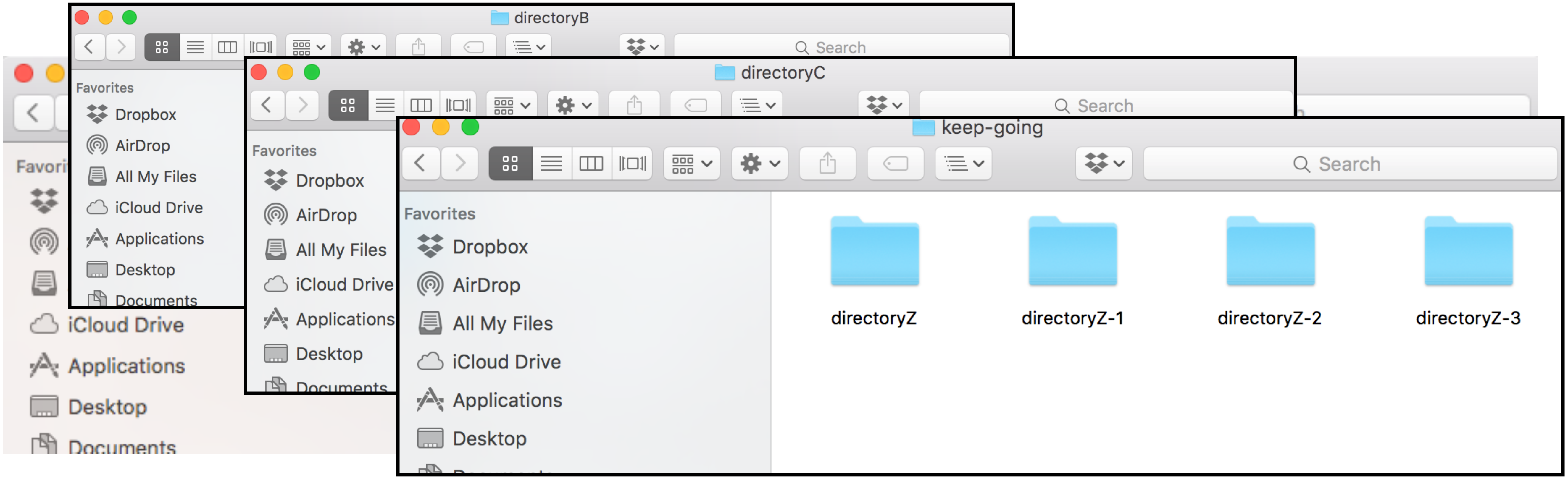
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



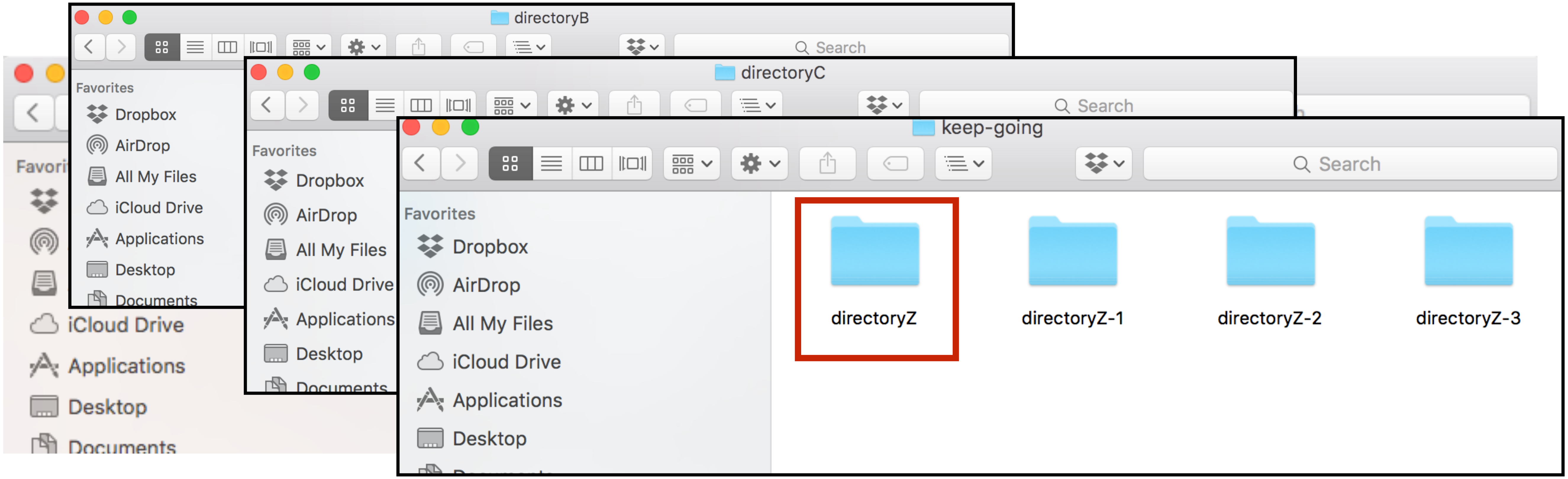
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



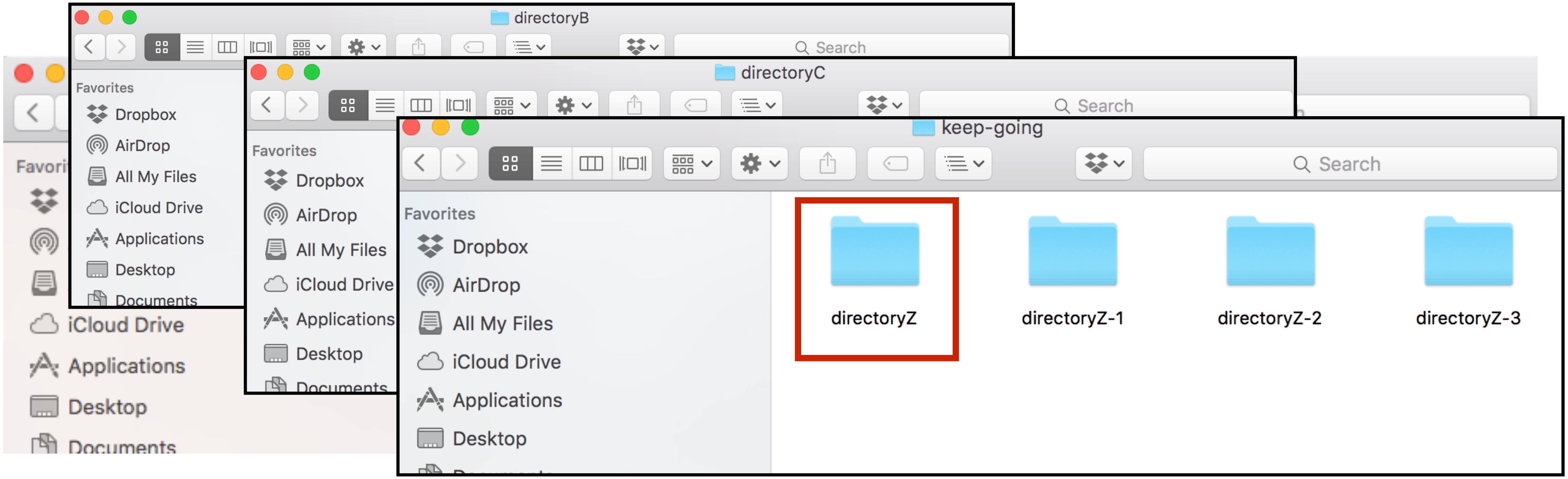
file system tree

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



file system tree

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96
}
```

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96
}
```

Term: *json-dict*
Def: a set of *json-mapping*'s

Example: (simplified) JSON Format

Example JSON Dictionary:

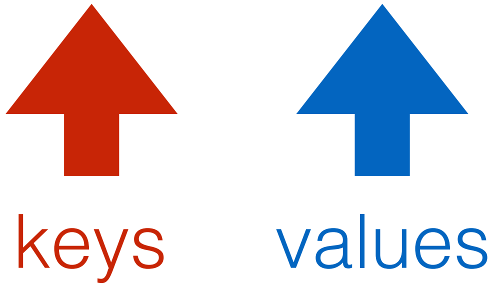
```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96
}
```

Term: *json-dict*
Def: a set of *json-mapping*'s

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96
}
```


keys

values

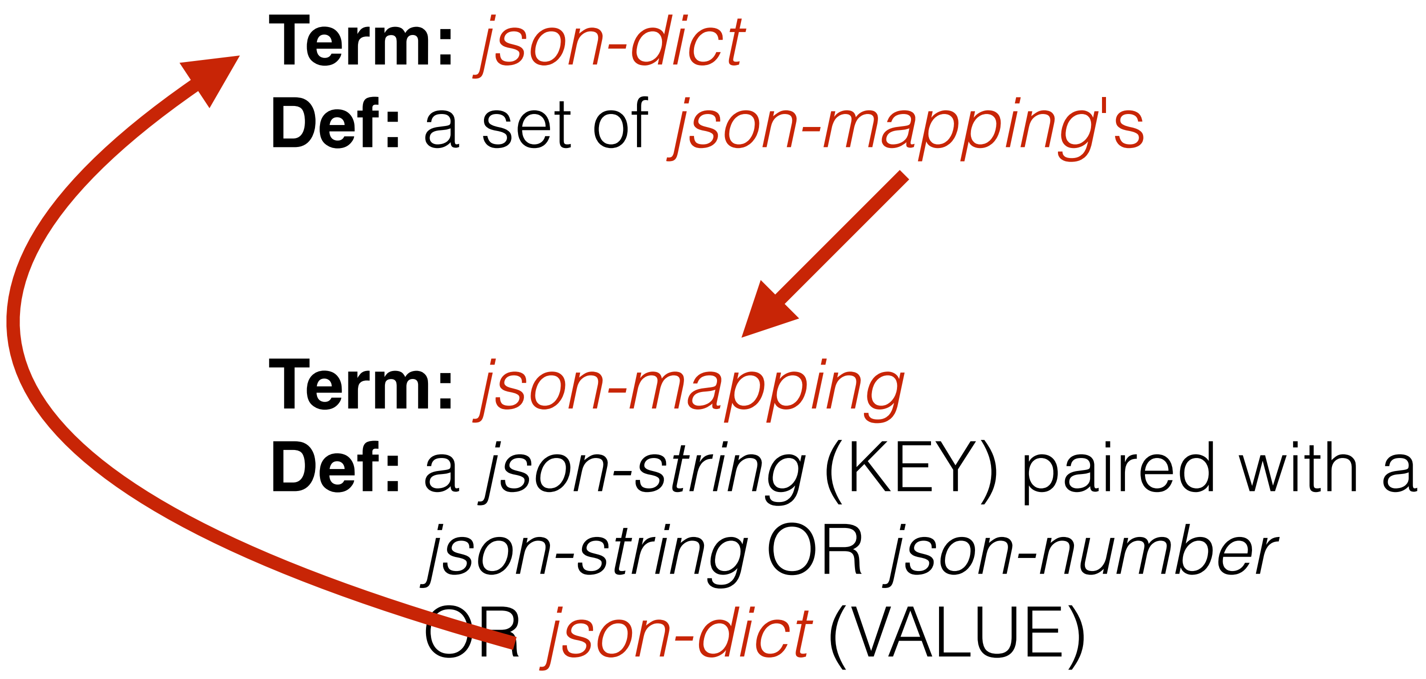
Term: *json-dict*
Def: a set of *json-mapping*'s

Term: *json-mapping*
Def: a *json-string* (KEY) paired with a *json-string* OR *json-number* OR *json-dict* (VALUE)

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96
}
```



Term: *json-dict*
Def: a set of *json-mapping*'s

Term: *json-mapping*
Def: a *json-string* (KEY) paired with a
json-string OR *json-number*
OR *json-dict* (VALUE)

recursive self reference isn’t always direct!

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96,
 “exams”: {
   “midterm”: 94,
   “final”: 98
 }
}
```

Term: *json-dict*
Def: a set of *json-mapping*'s

Term: *json-mapping*
Def: a *json-string* (KEY) paired with a
json-string OR *json-number*
OR *json-dict* (VALUE)

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
 “name”: “alice”,
 “grade”: “A”,
 “score”: 96,
 “exams”: {
   “midterm”: {“points”:94,
               “total”:100},
   “final”: {“points”: 98,
             “total”: 100}
 }
}
```

Term: *json-dict*
Def: a set of *json-mapping*'s

Term: *json-mapping*
Def: a *json-string* (KEY) paired with a *json-string* OR *json-number* OR *json-dict* (VALUE)

Example: (simplified) JSON Format

Example JSON Dictionary:

```
{
  “name”: “alice”,
  “grade”: “A”,
  “score”: 96,
  “exams”: {
    “midterm”: {“points”:94,
                “total”:100},
    “final”: {“points”: 98,
              “total”: 100}
  }
}
```

Term: *json-dict*
Def: a set of *json-mapping*'s

Term: *json-mapping*
Def: a *json-string* (KEY) paired with a *json-string* OR *json-number* OR *json-dict* (VALUE)

Overview: Learning Objectives

Recursive information

- What is a **recursive definition/structure**?
- Arbitrarily vs. infinitely

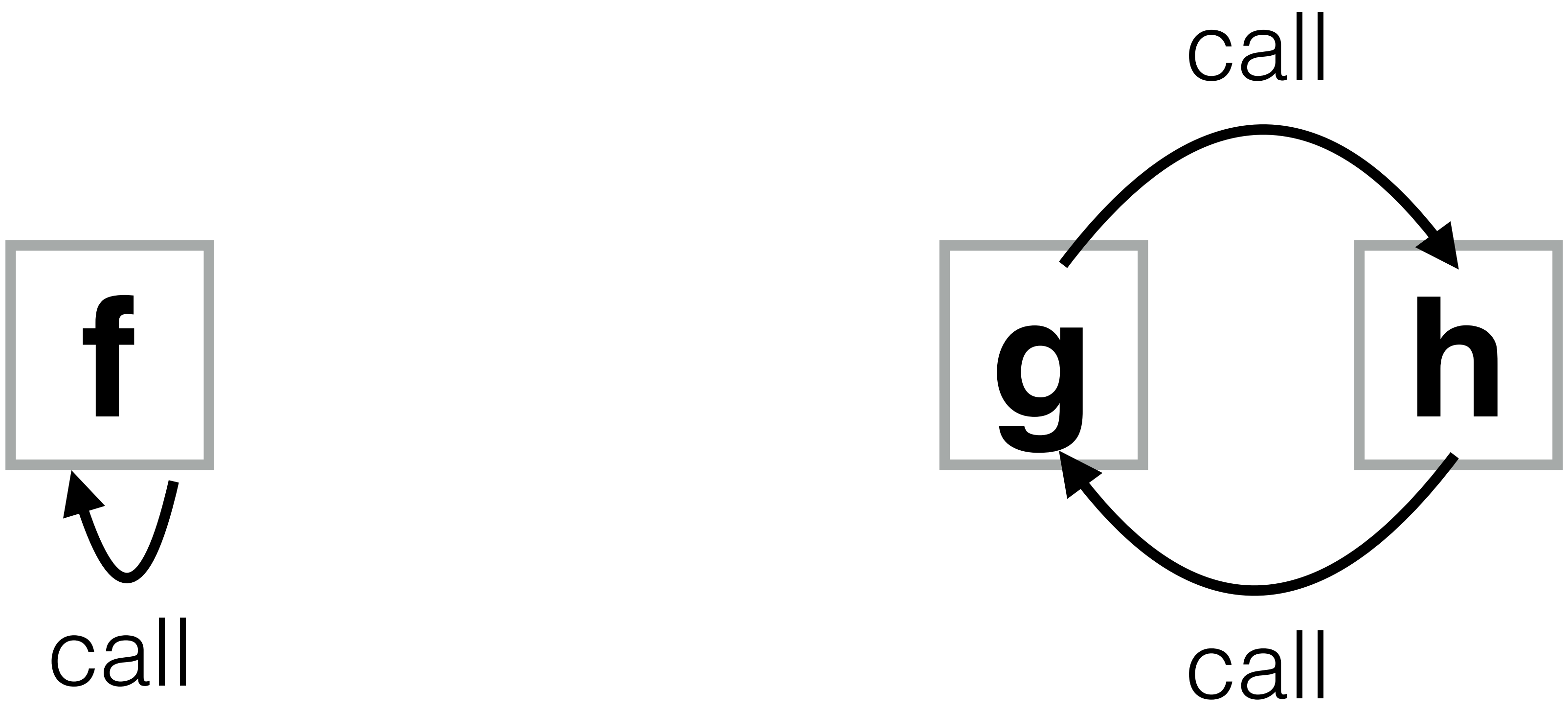
Recursive code

- What is **recursive code**?
- Why write recursive code?
- Where do computers keep local variables for recursive calls?
- What happens to programs with **infinite recursion**?

Recursive Code

What is it?

- A function that calls itself (possible indirectly)

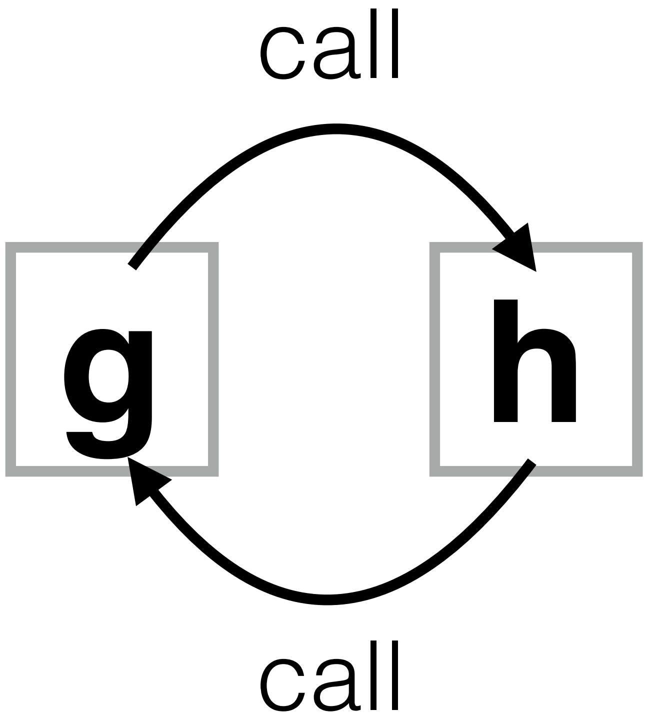


Recursive Code

What is it?

- A function that calls itself (possible indirectly)

```
def f():
    # other code
    f()
    # other code
```



call

g

h

call

Recursive Code

What is it?

- A function that calls itself (possible indirectly)

```
def f():
    # other code
    f()
    # other code
```

```
def g():
    # other code
    h()
    # other code

def h():
    # other code
    g()
    # other code
```

Recursive Code

What is it?

- A function that calls itself (possible indirectly)

Motivation: don’t know how big the data is before execution

- Need either **iteration** or **recursion**
- In theory, these techniques are equally powerful

Recursive Code

What is it?

- A function that calls itself (possible indirectly)

Motivation: don't know how big the data is before execution

- Need either **iteration** or **recursion**
- In theory, these techniques are equally powerful

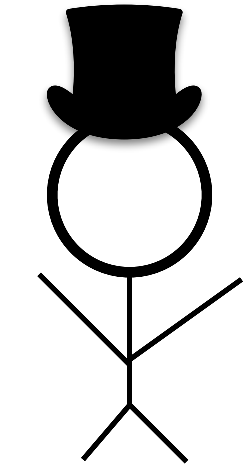
Why recurse? (instead of always iterating)

- in practice, often easier
- recursive code corresponds to recursive data
- reduce a big problem into a smaller problem

https://texastreesurgeons.com/services/tree-removal/

Recursive Students

eager CS 301 students
in the front row



wise and benevolent
teacher wearing a top hat

Example from https://courses.cs.washington.edu/courses/cse143/17au/

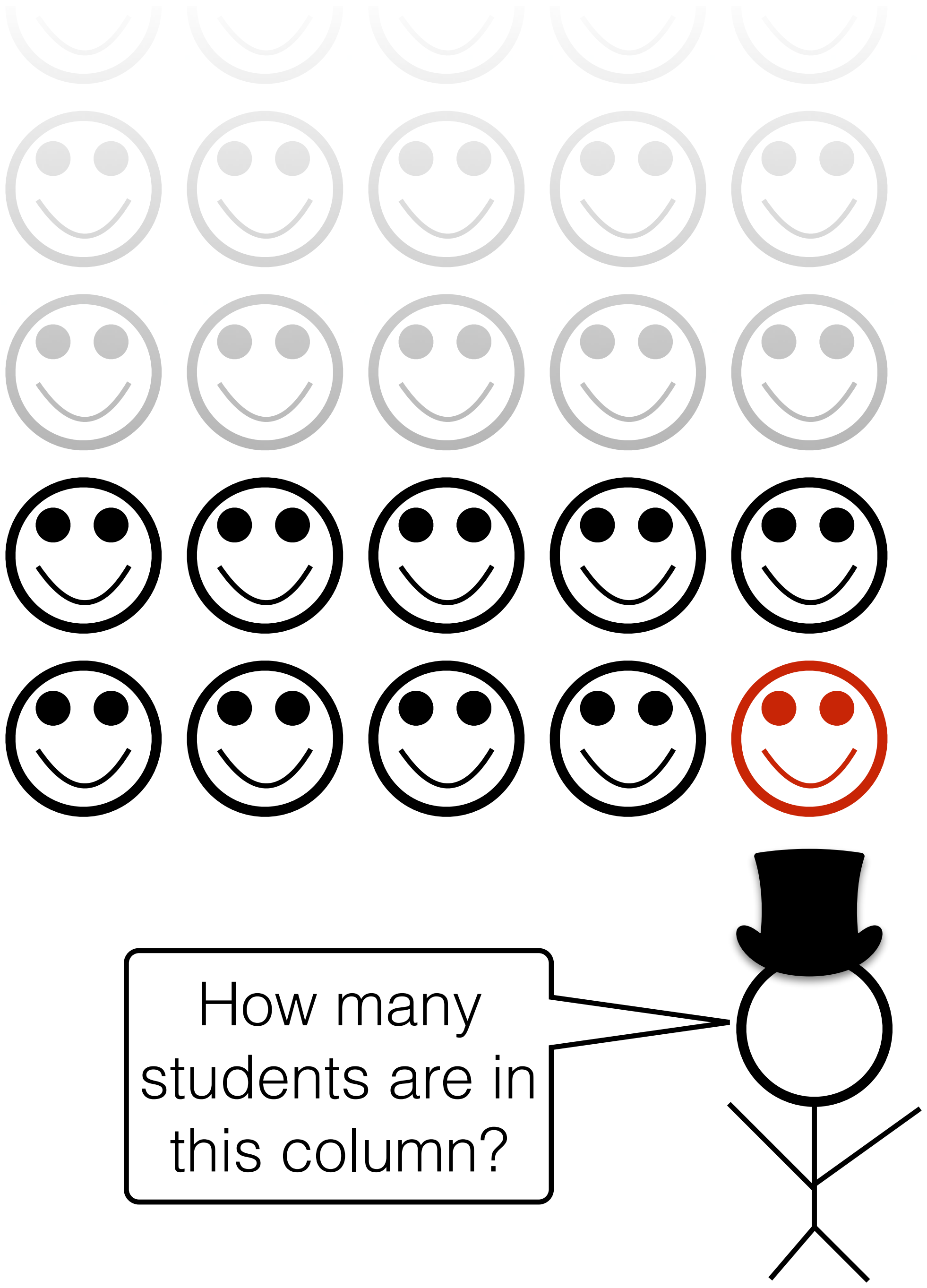
Recursive Students

Imagine:

A teacher wants to know how many students are in a column. **What should each student ask the person behind them?**

Constraints:

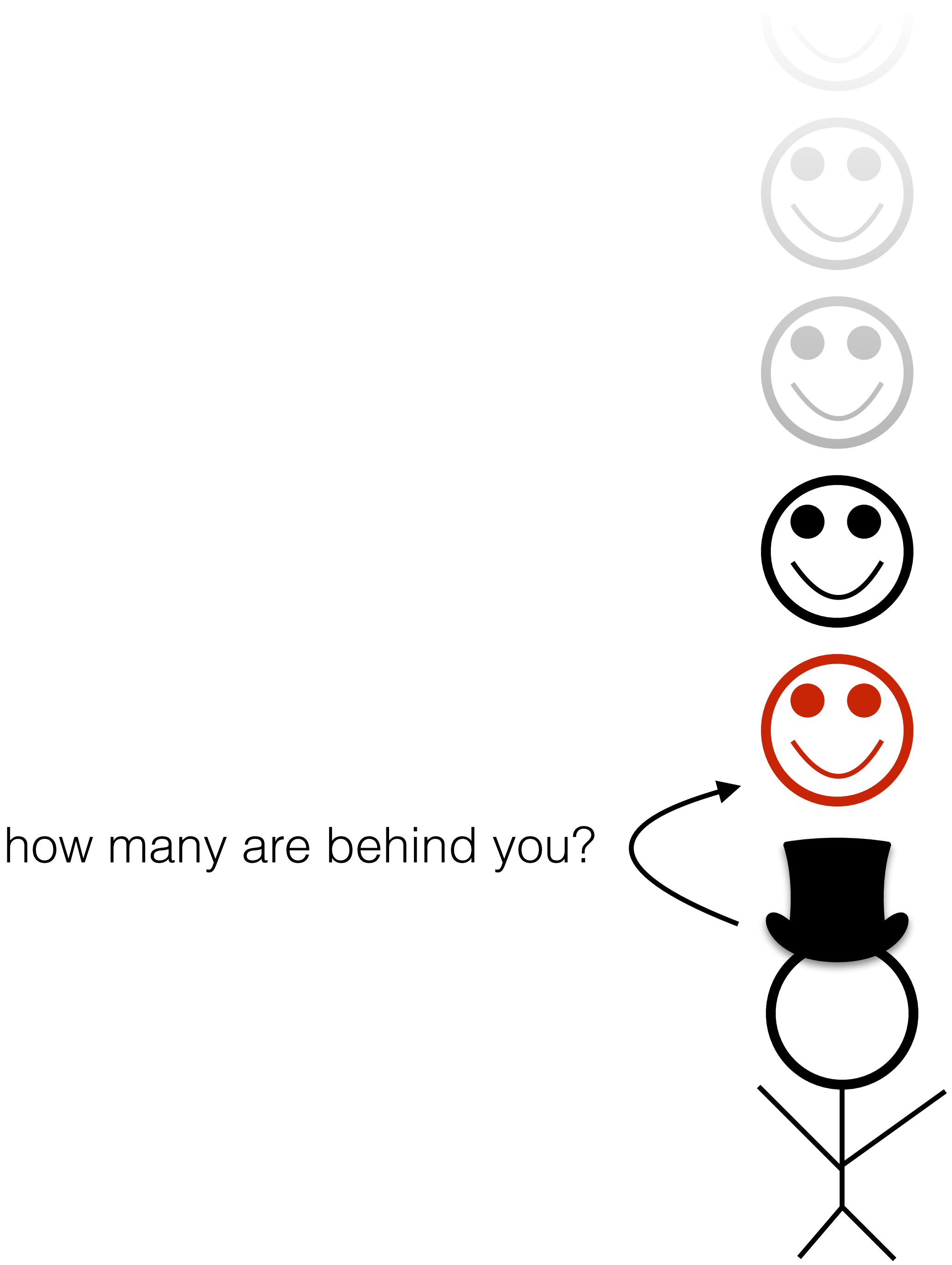
- It is dark, you **can't** see the back
- You **can't** get up to count
- You **may** talk to adjacent students
- Mic is broken (students in back can't hear from front)



Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *“how many students are behind you?”*

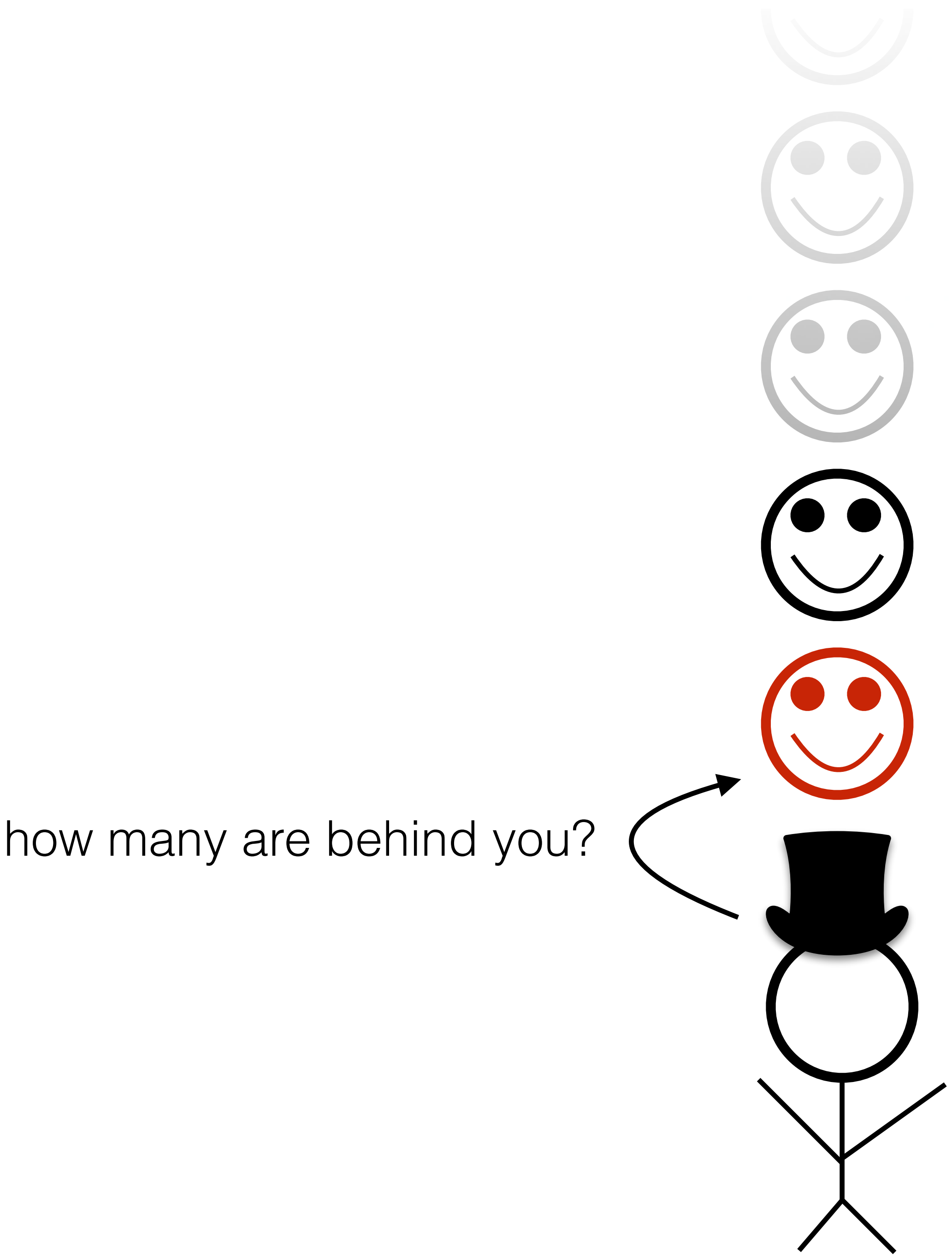


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Reframing is the hardest part

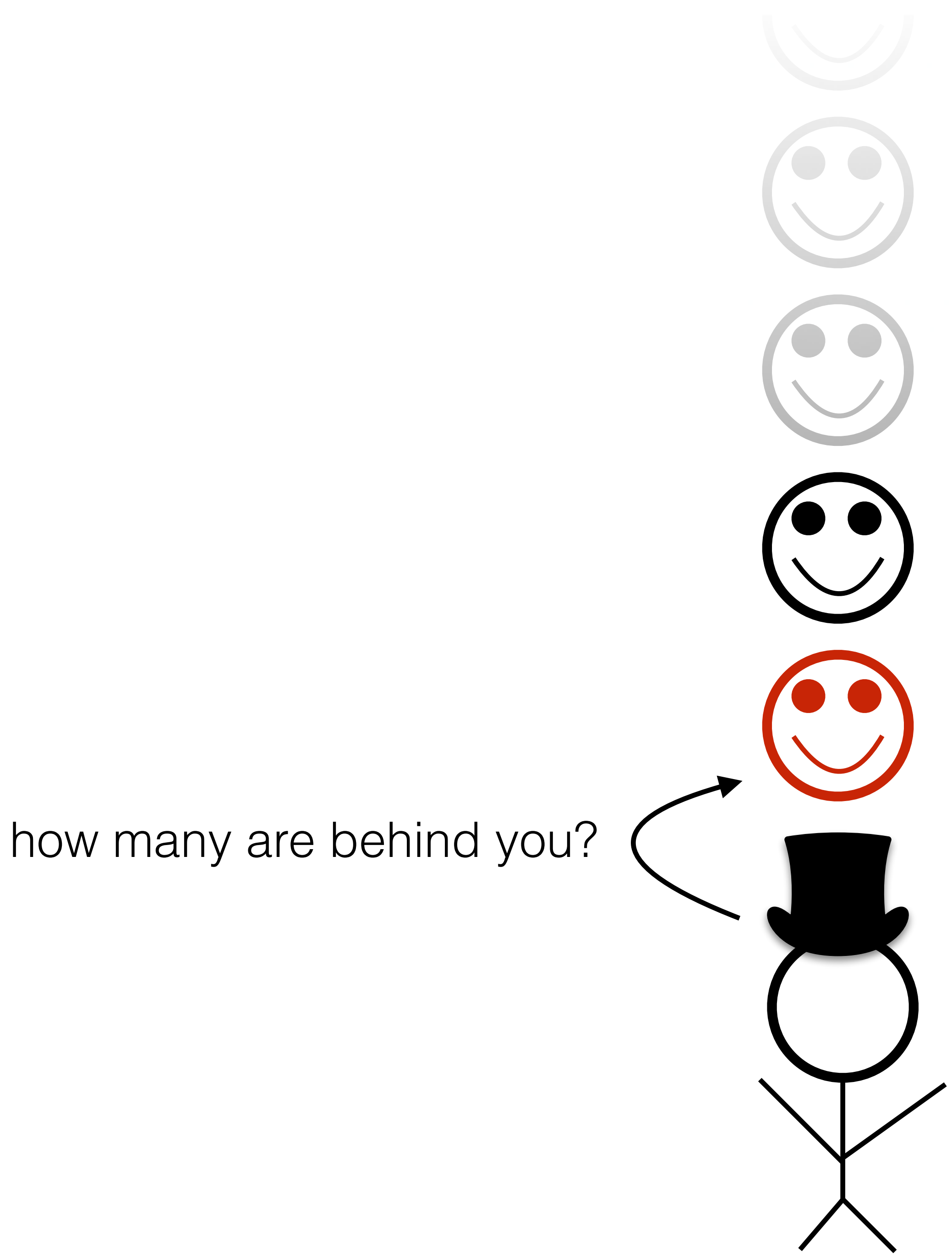


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *“how many students are behind you?”*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1

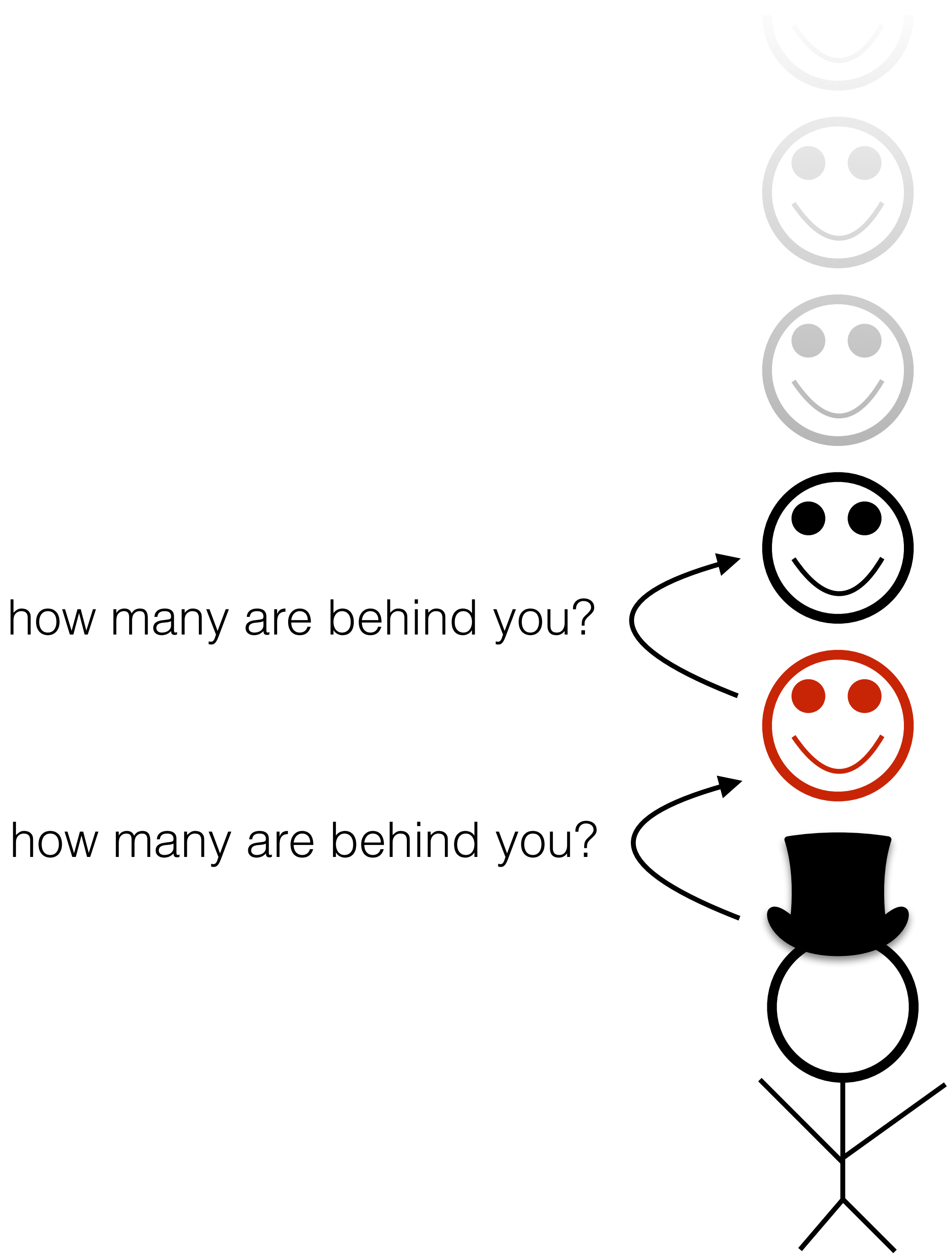


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1



how many are behind you?

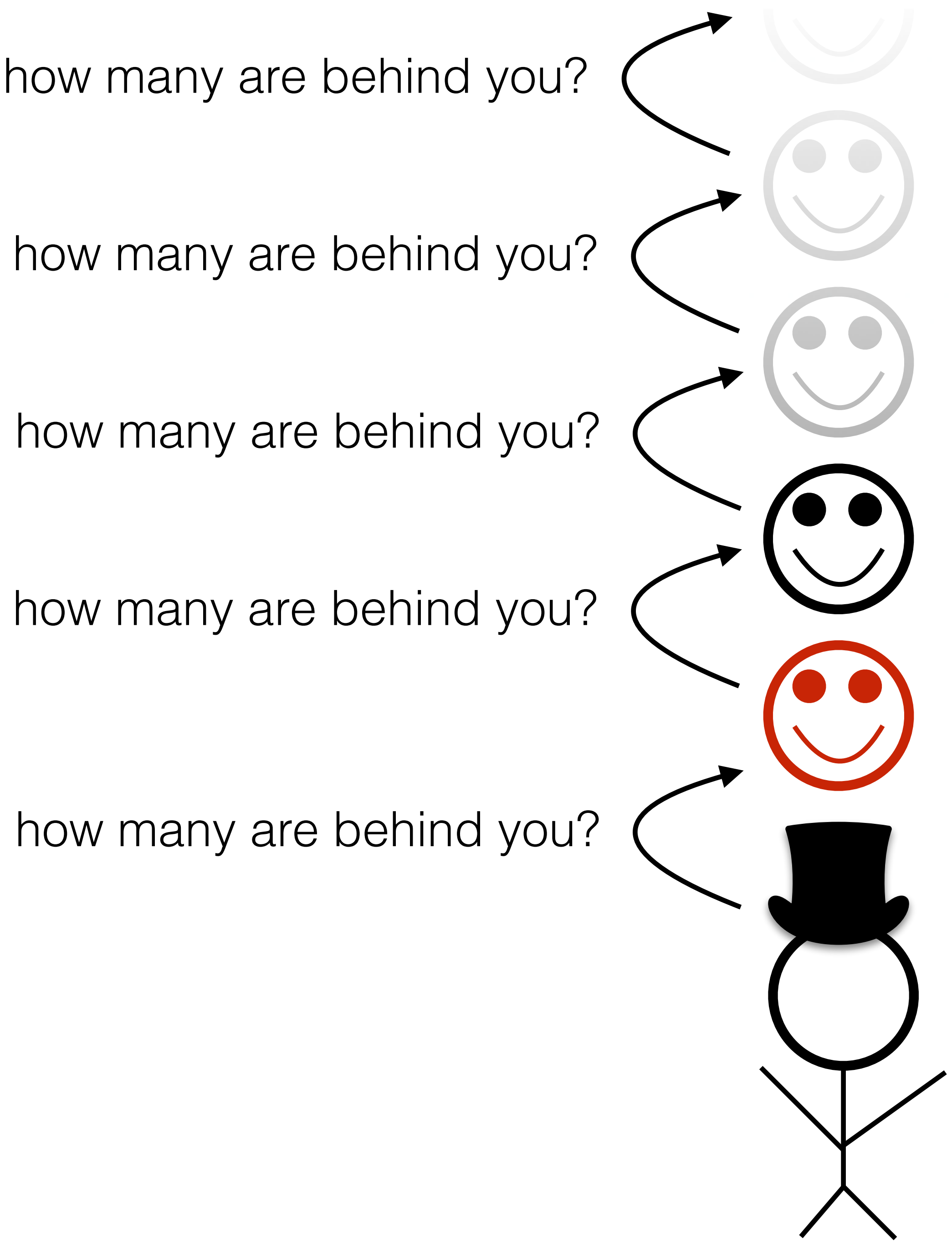
how many are behind you?

Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *“how many students are behind you?”*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1



how many are behind you?

how many are behind you?

how many are behind you?

how many are behind you?

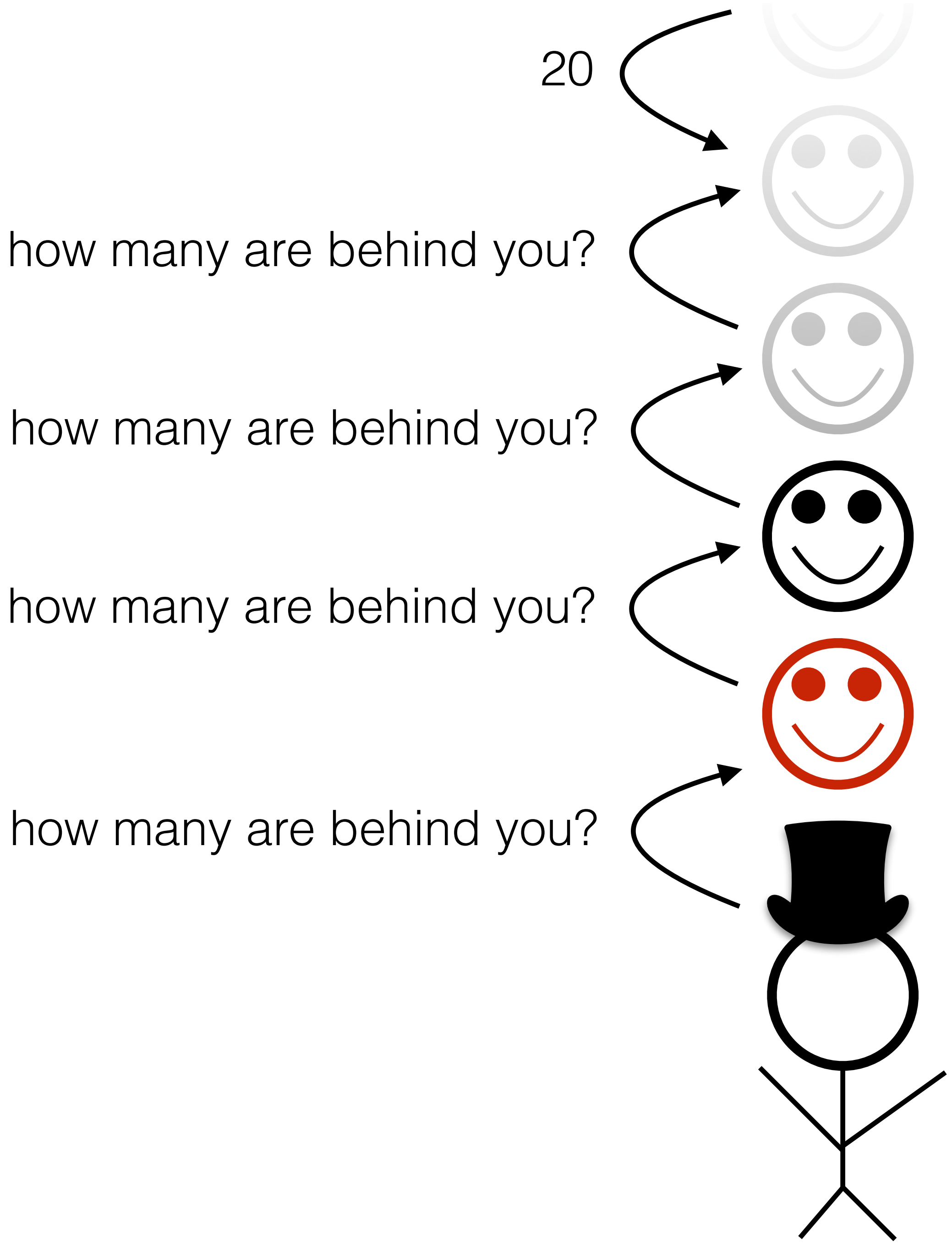
how many are behind you?

Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1

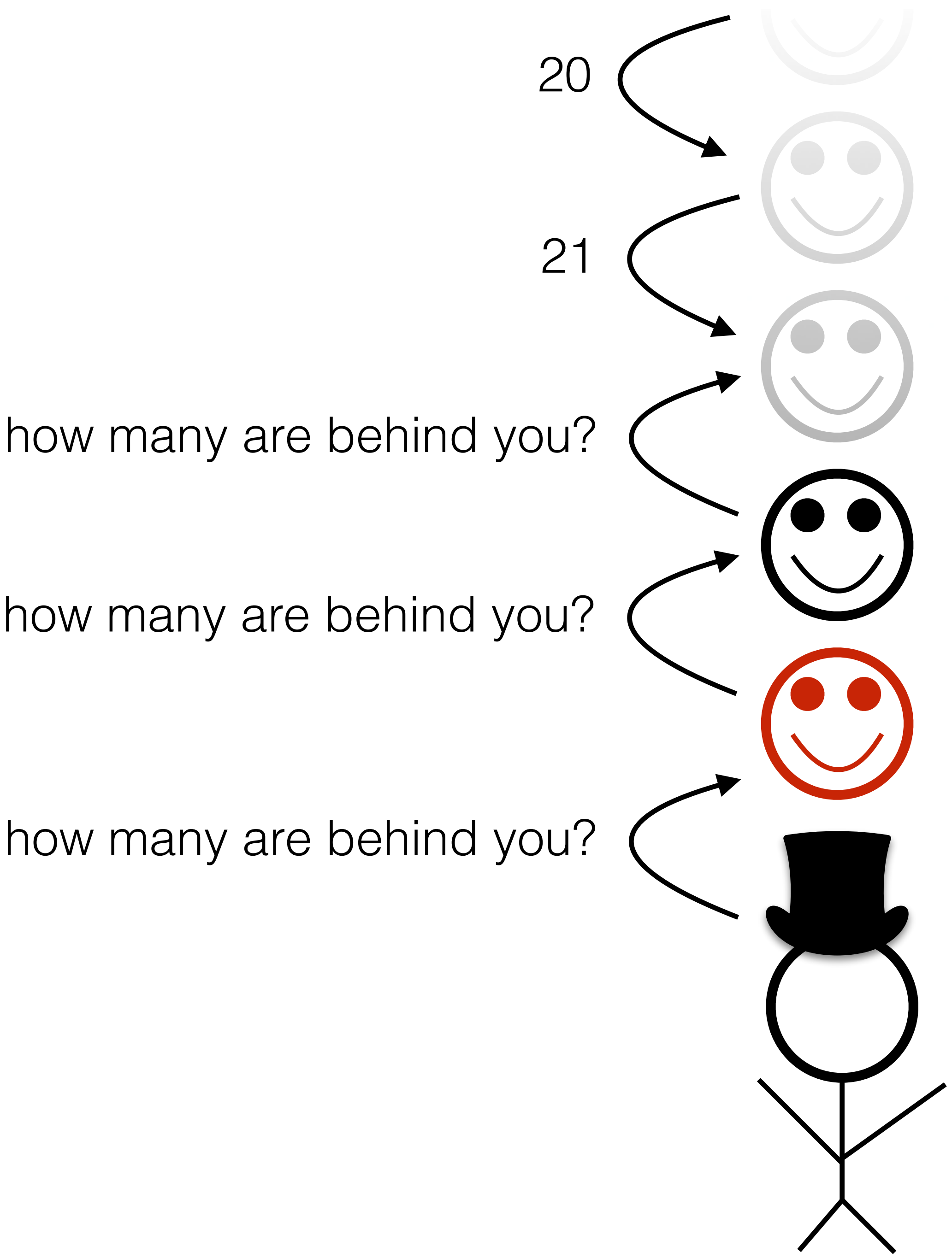


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1

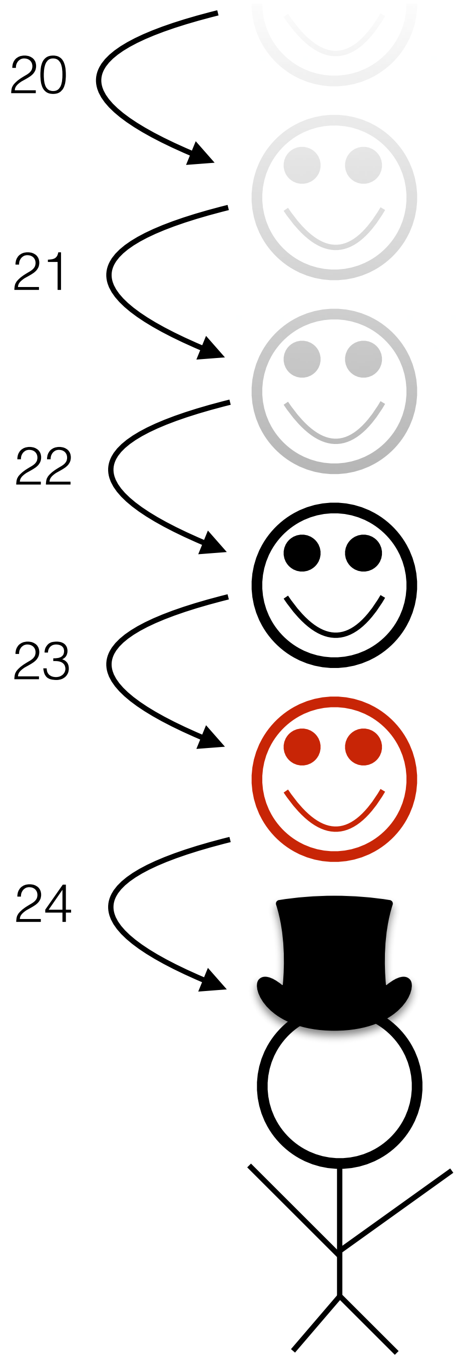


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1

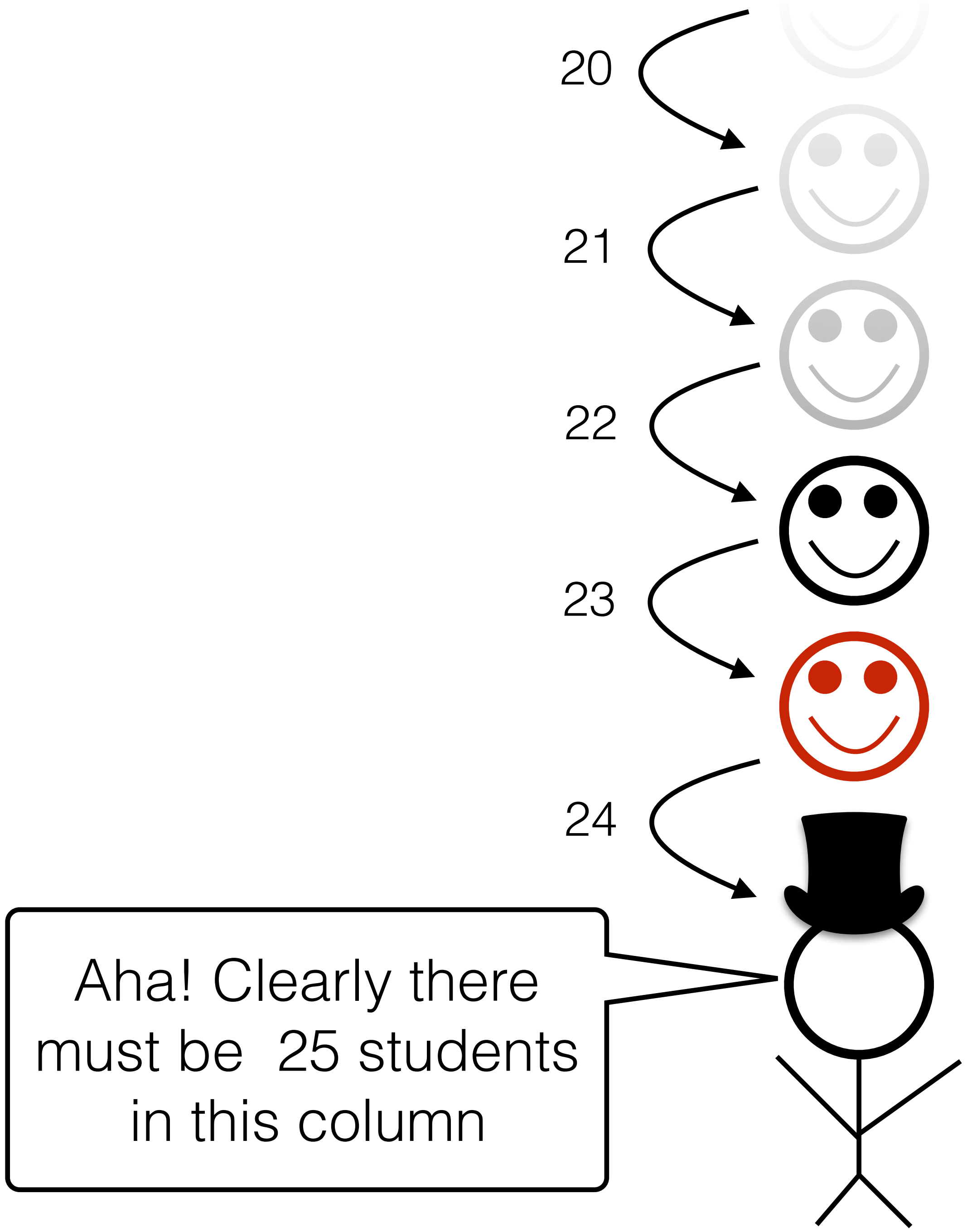


Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1



Example from https://courses.cs.washington.edu/courses/cse143/17au/

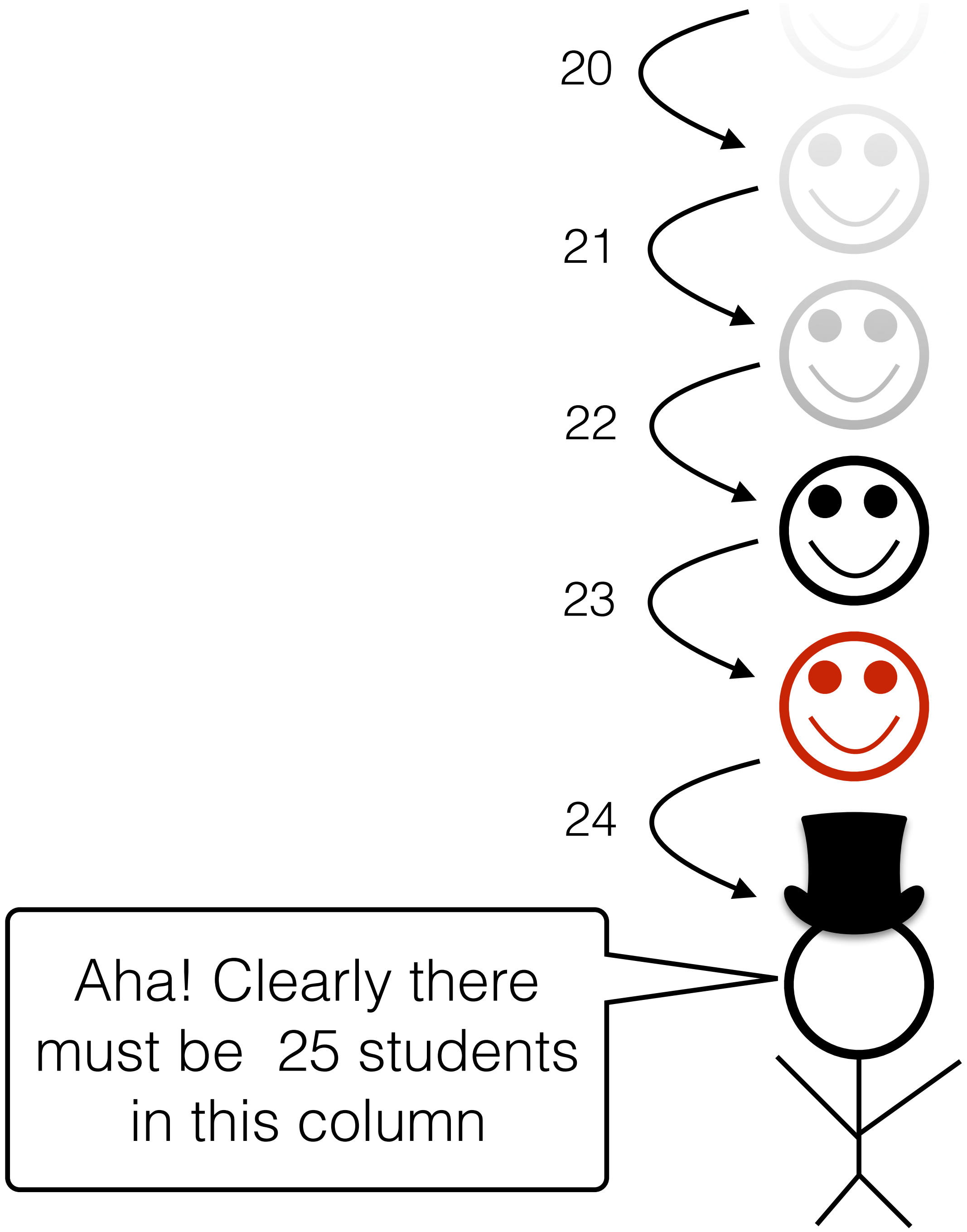
Recursive Students

Strategy: reframe question as *"how many students are behind you?"*

Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1

Observations:

- Each student runs the same "code"
- Each student has their own "state"



Example from https://courses.cs.washington.edu/courses/cse143/17au/

Practice: Reframing Factorials

$$N! = 1 \times 2 \times 3 \times \ldots \times (N-2) \times (N-1) \times N$$

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

3. Recursive Definition:

4. Python Code:

```python
def fact(n):
    pass # TODO
```

Goal: work from examples to get to recursive code

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

look for patterns that allow rewrites with self reference

3. Recursive Definition:

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

look for patterns that allow rewrites with self reference

3. Recursive Definition:

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! =
2! =
3! =
4! =
5! = 4! * 5
```

3. Recursive Definition:

4. Python Code:

```python
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! =
2! =
3! =
4! =
5! = 4! * 5
```

3. Recursive Definition:

4. Python Code:

```python
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! =
2! =
3! =
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! =
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

4. Python Code:

```python
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

don't need a pattern at the start

3. Recursive Definition:

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

convert self-referring examples to a recursive definition

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

4. Python Code:

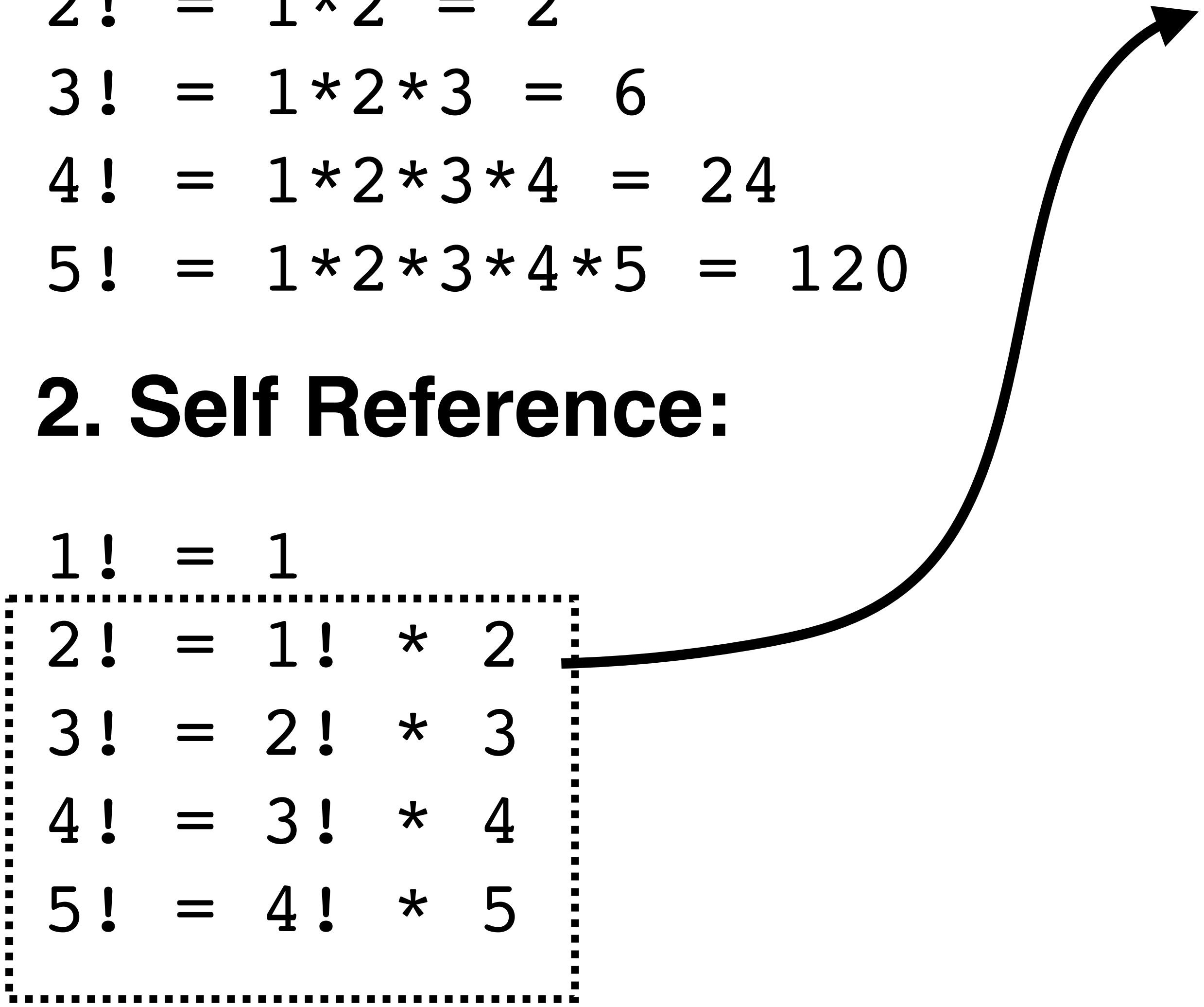
```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:



```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

`N!` is `????` for N>1

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

`N!` is `(N-1)! * N` for N>1

4. Python Code:

```python
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

`N!` is `(N-1)! * N` for N>1

4. Python Code:

```python
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

`N!` is `(N-1)! * N` for N>1

4. Python Code:

```
def fact(n):
    if n == 1:
        return 1
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

`N!` is `(N-1)! * N` for N>1

4. Python Code:

```python
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

```
1! = 1
2! = 1! * 2
3! = 2! * 3
4! = 3! * 4
5! = 4! * 5
```

3. Recursive Definition:

`1!` is `1`

`N!` is `(N-1)! * N` for N>1

4. Python Code:

```
def fact(n):
    if n == 1:
        return 1
    p =fact(n-1)
    return n * p
```

Let’s “run” it!

Tracing Factorial

somebody
called `fact(4)`

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

fact(n=4)

Note, this is **not** a stack frame!
We're tracing code line-by-line.
Boxes represent which invocation.

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

fact(n=4)

if n == 1:

Tracing Factorial

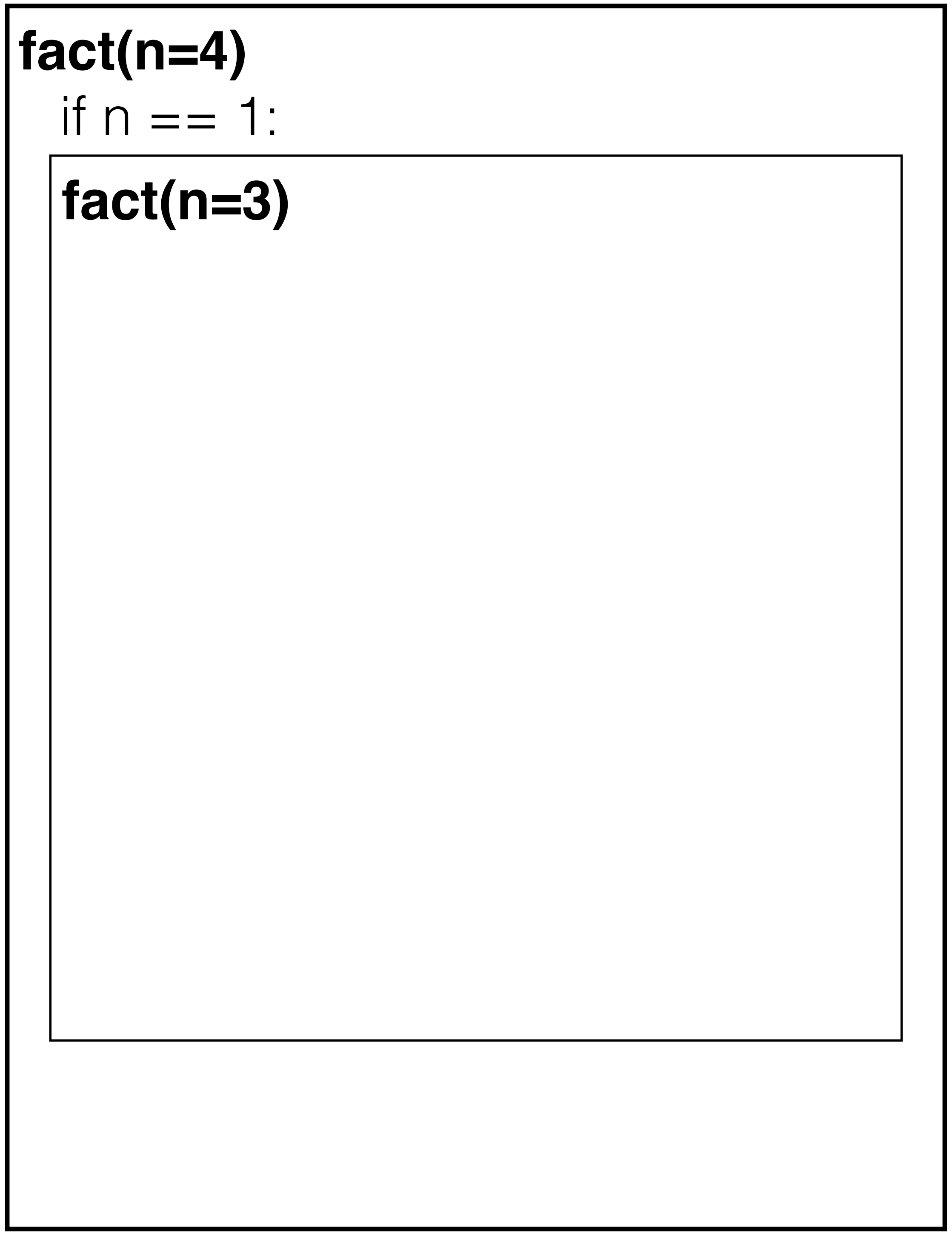
```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

fact(n=4)

```
  if n == 1:
```

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



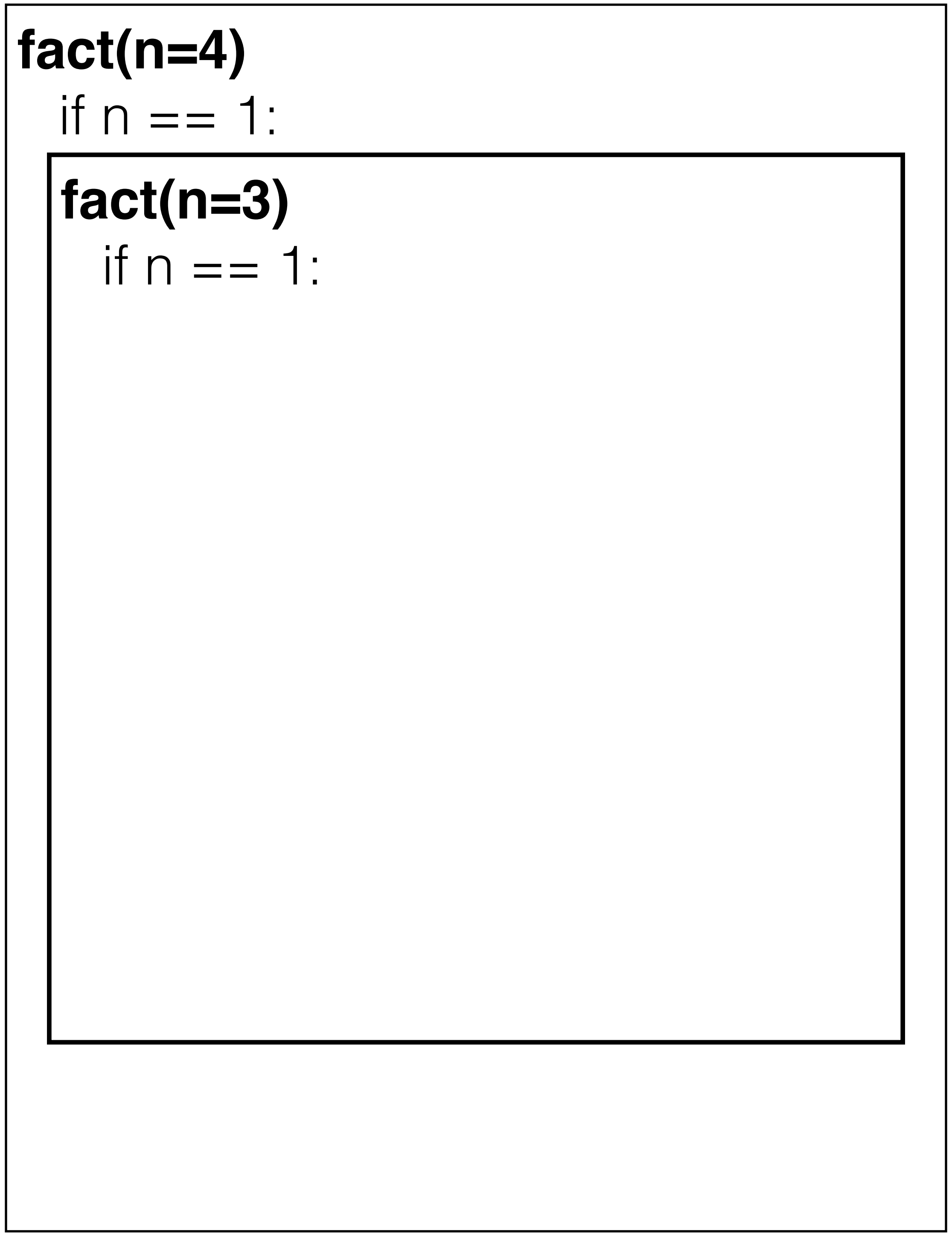
fact(n=4)

if n == 1:

fact(n=3)

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)

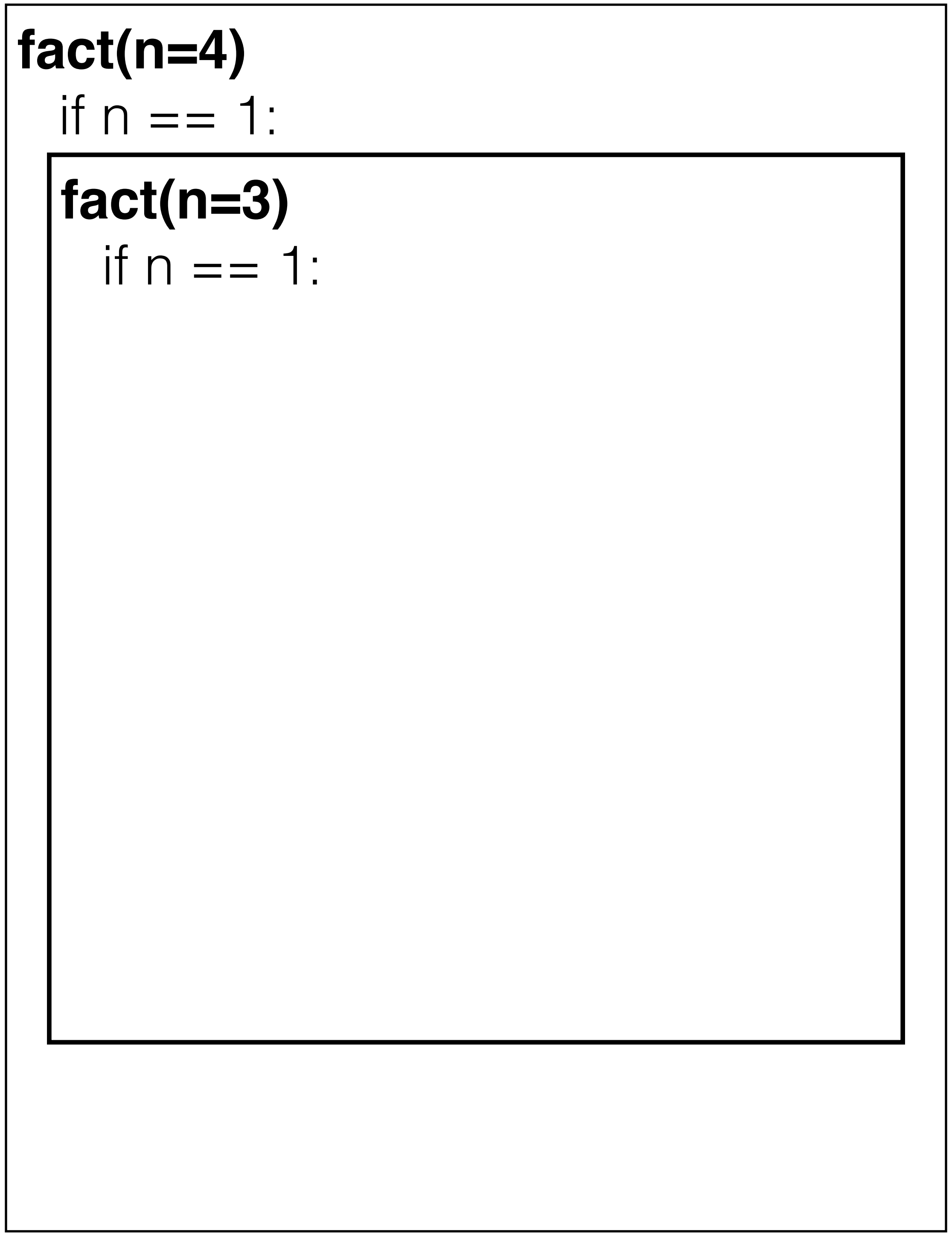
if n == 1:

fact(n=3)

if n == 1:

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)

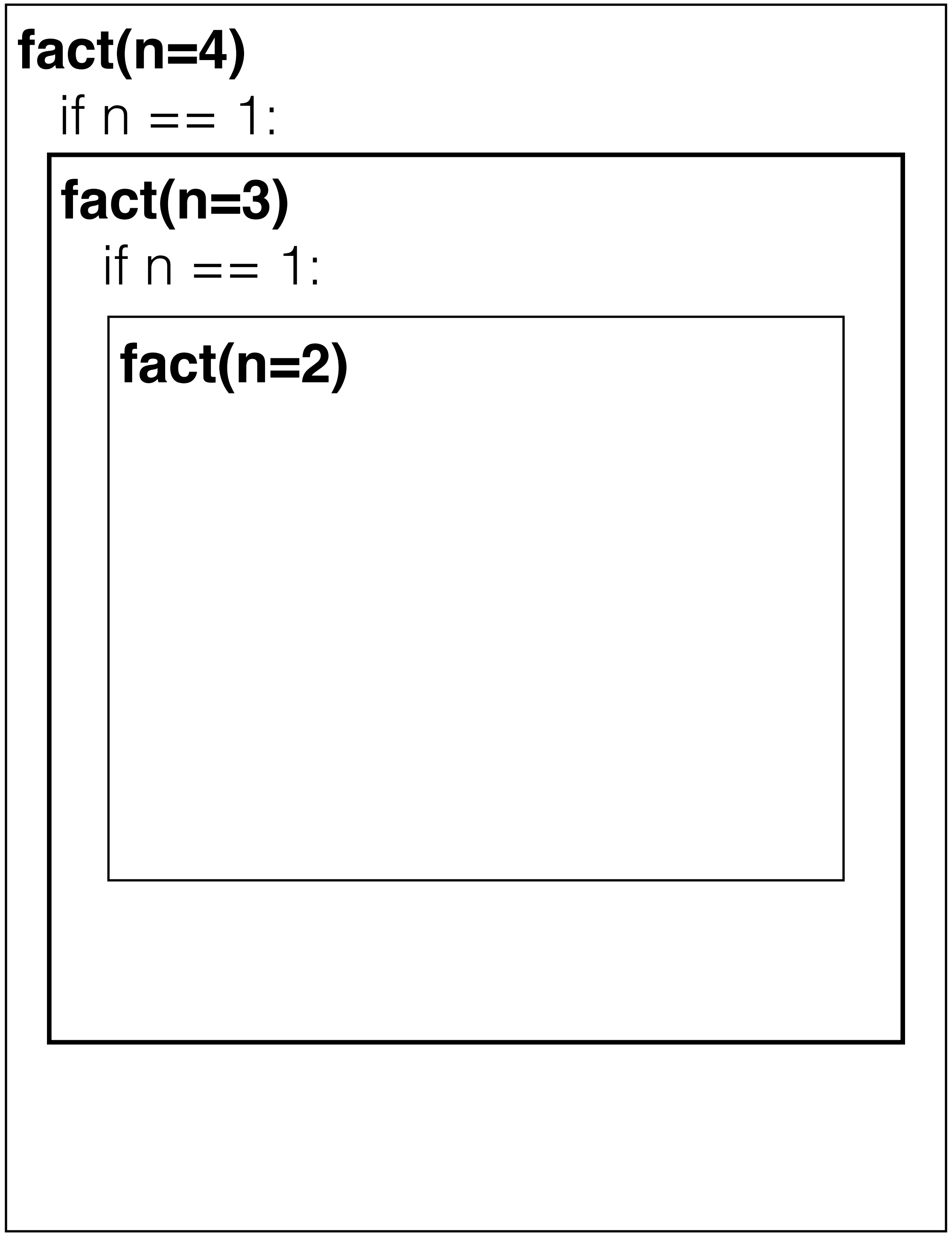
if n == 1:

fact(n=3)

if n == 1:

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)

if n == 1:

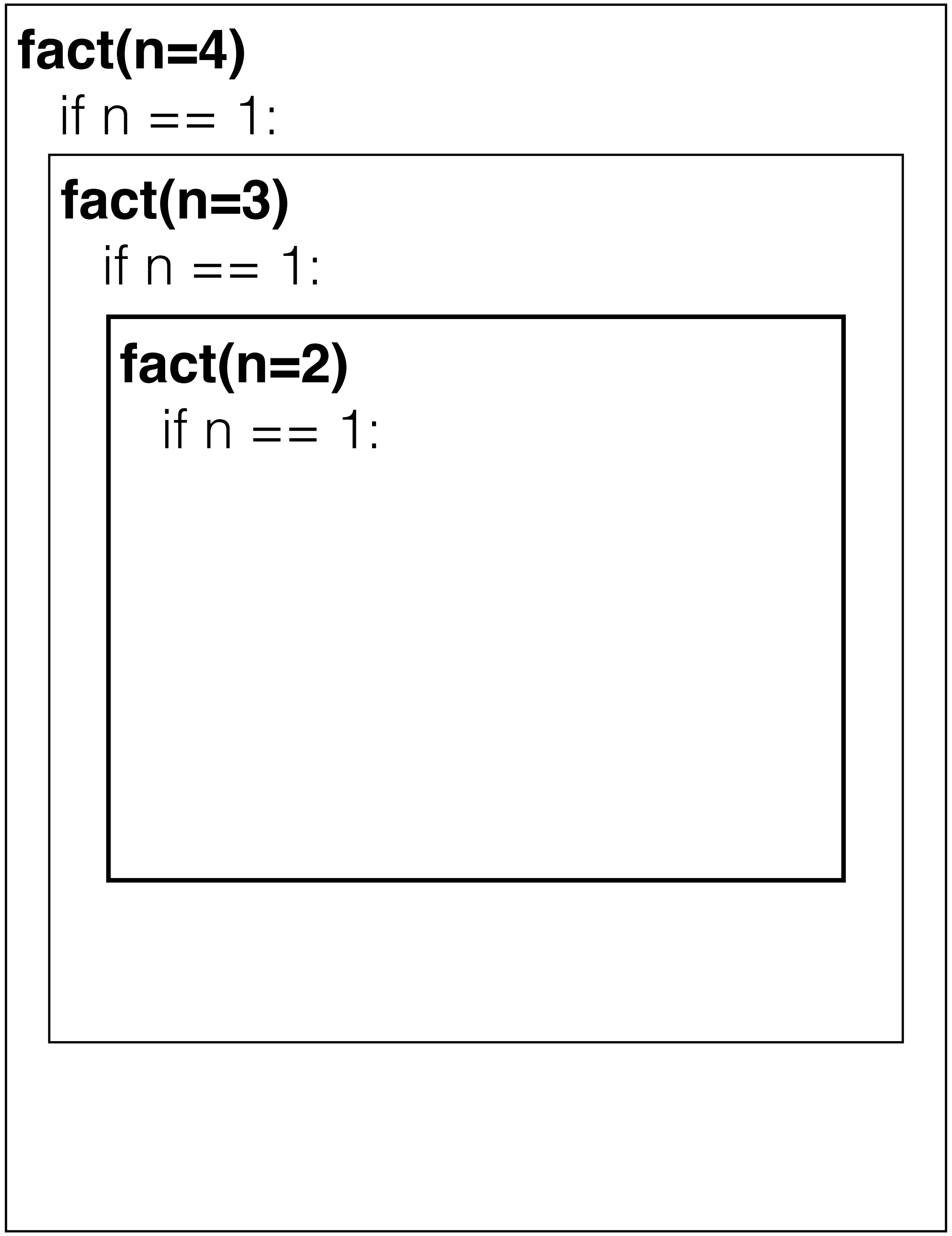
fact(n=3)

if n == 1:

fact(n=2)

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)

if n == 1:

fact(n=3)

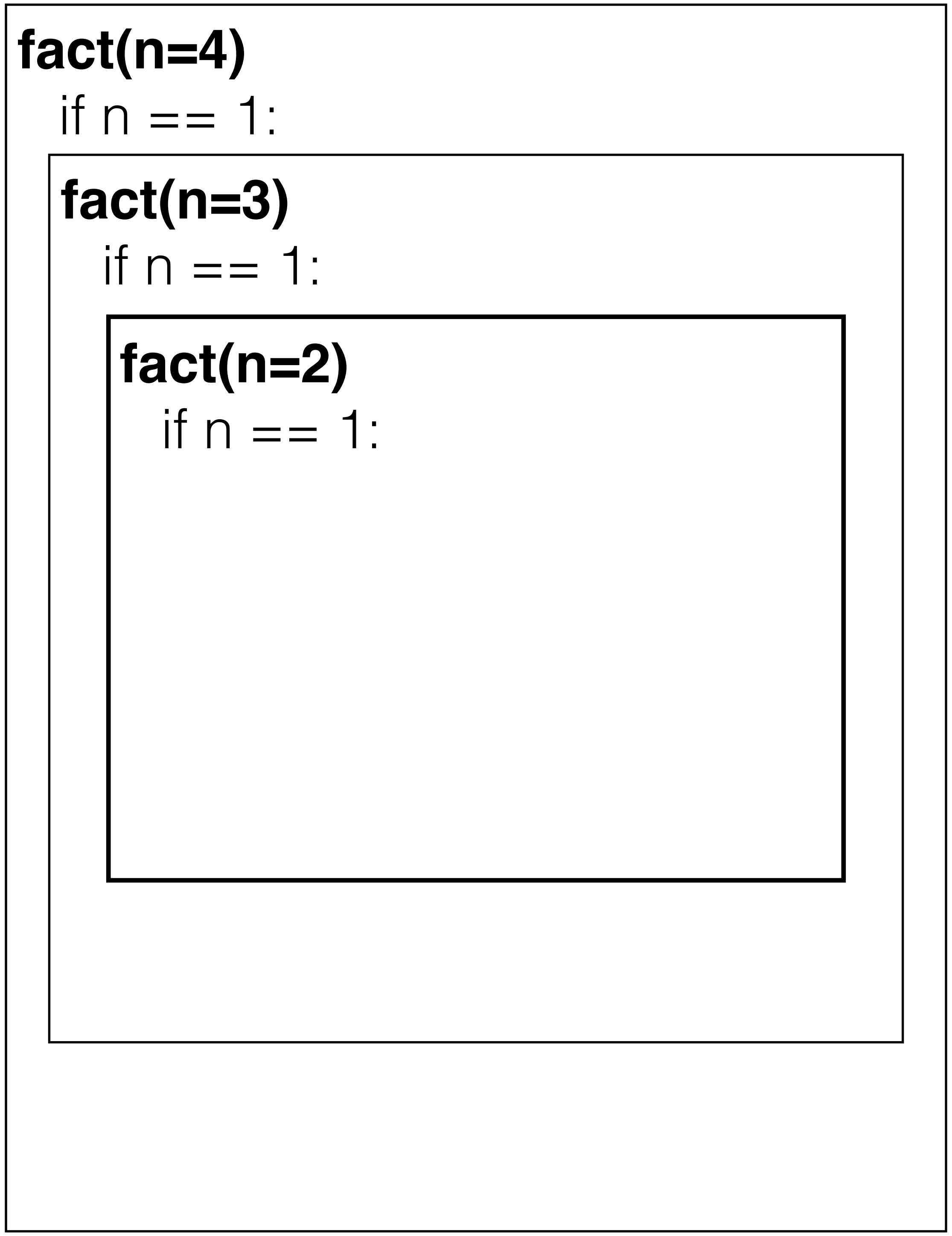
if n == 1:

fact(n=2)

if n == 1:

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



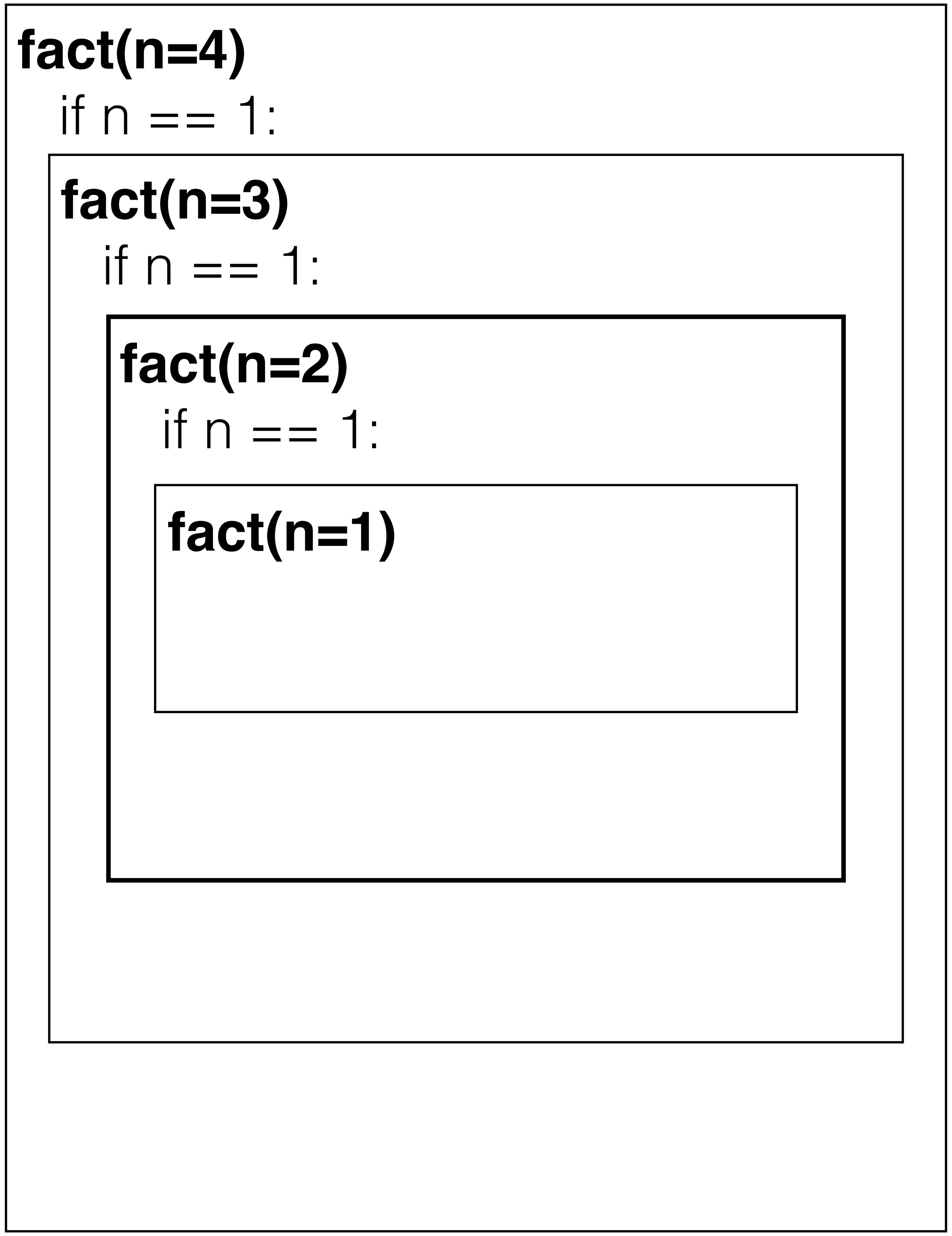
fact(n=4)
  if n == 1:

fact(n=3)
  if n == 1:

fact(n=2)
  if n == 1:

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)
if n == 1:

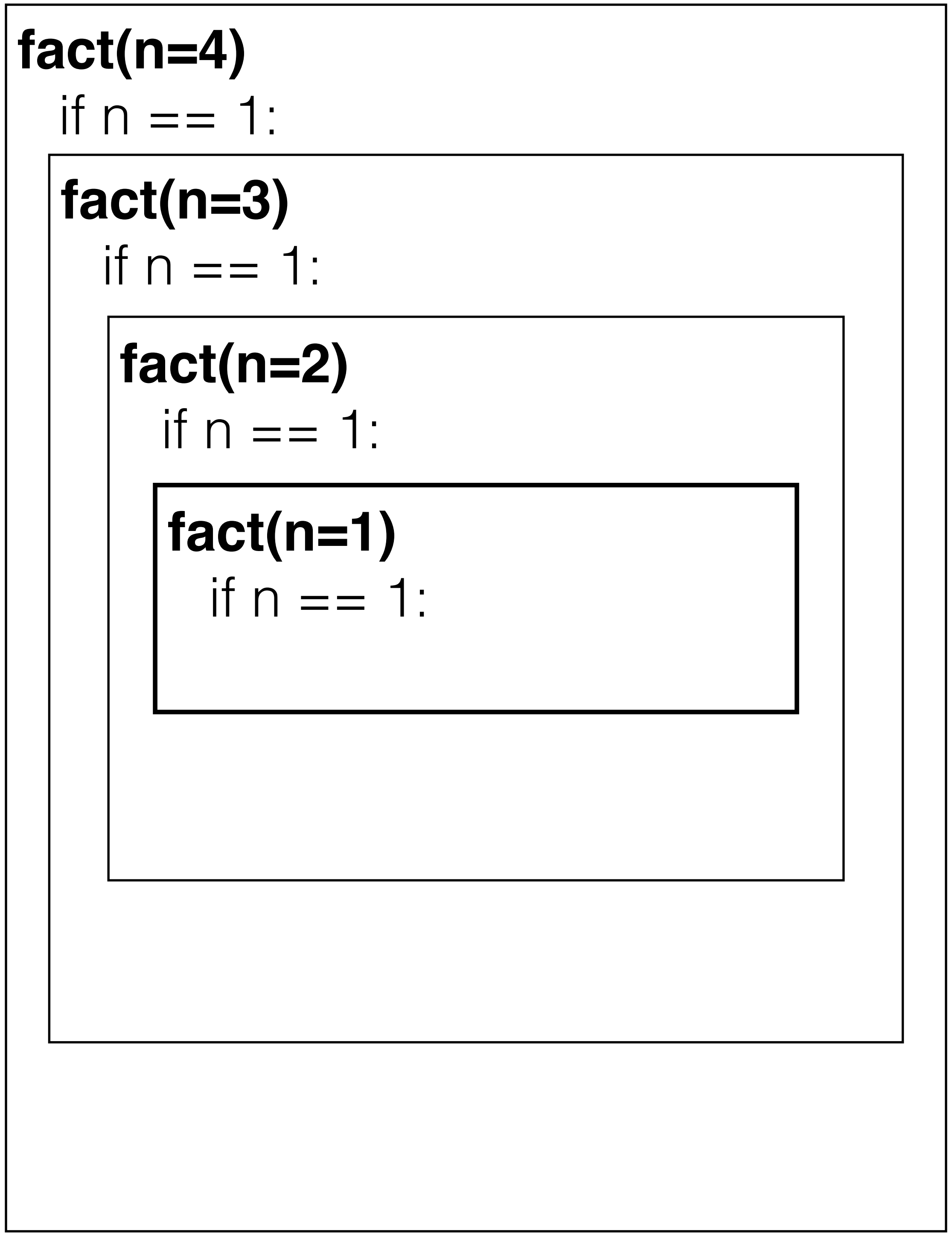
fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)
if n == 1:

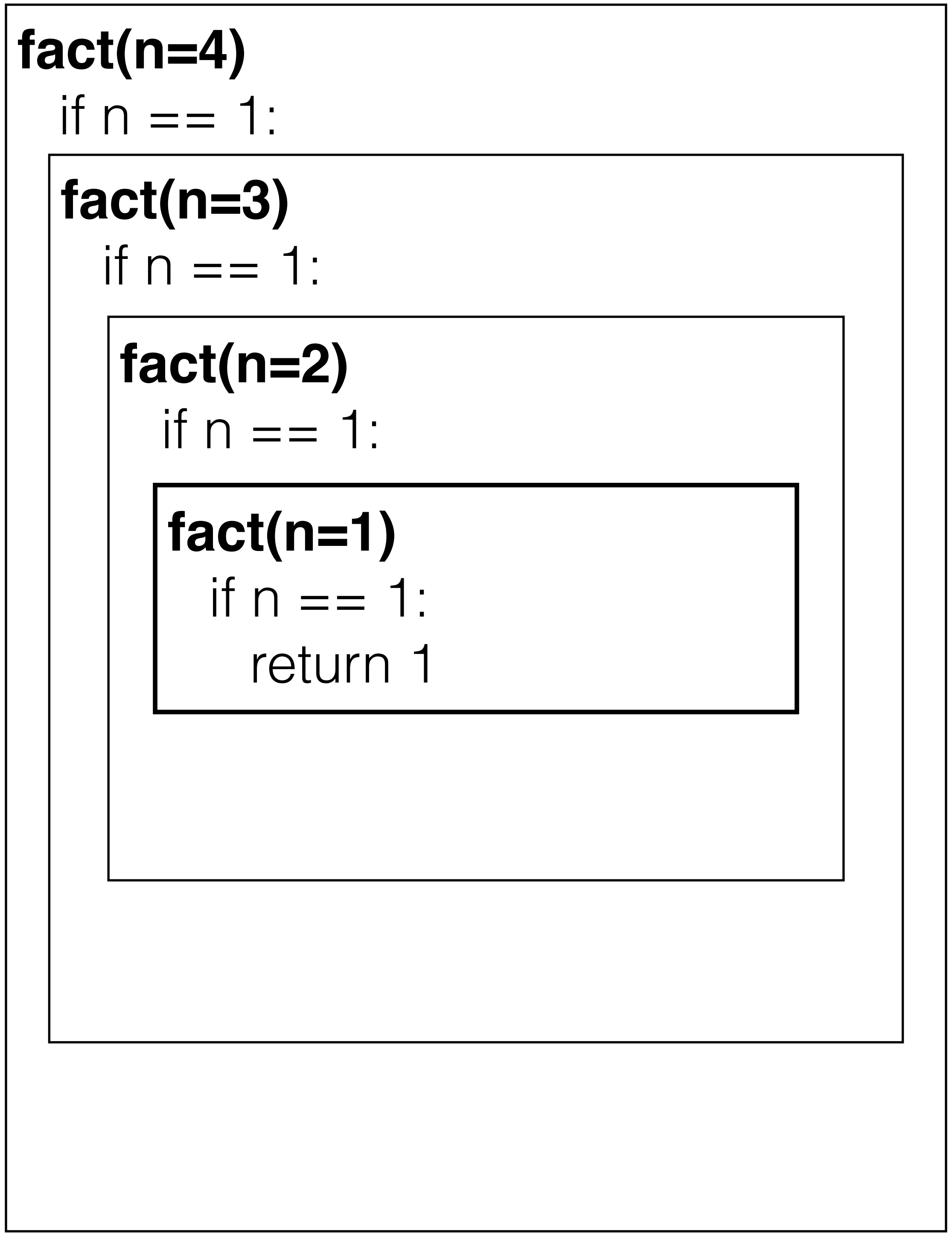
fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)
if n == 1:

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)
if n == 1:

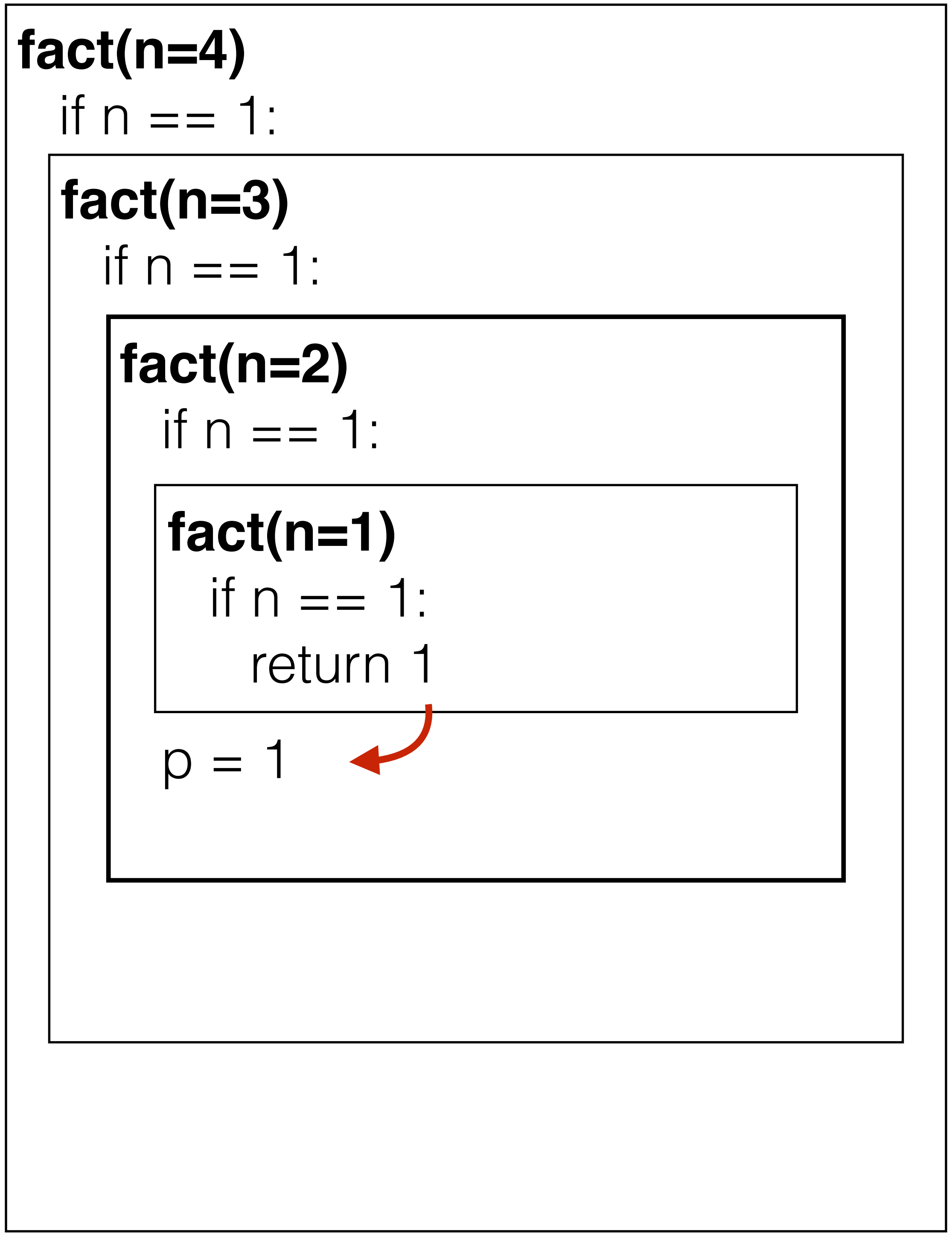
fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)
if n == 1:
return 1

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)
  if n == 1:

fact(n=3)
  if n == 1:

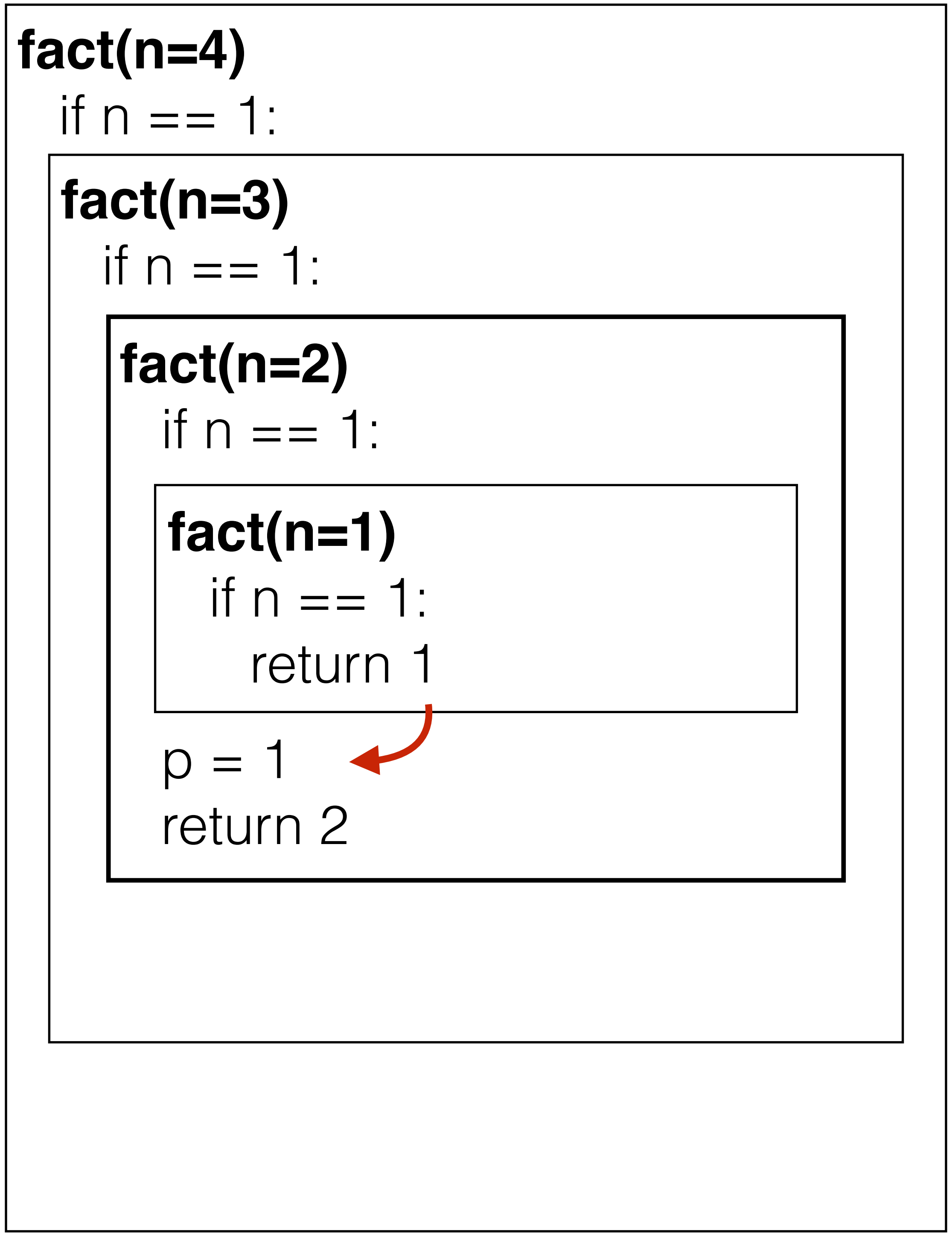
fact(n=2)
  if n == 1:

fact(n=1)
  if n == 1:
    return 1

p = 1

Tracing Factorial

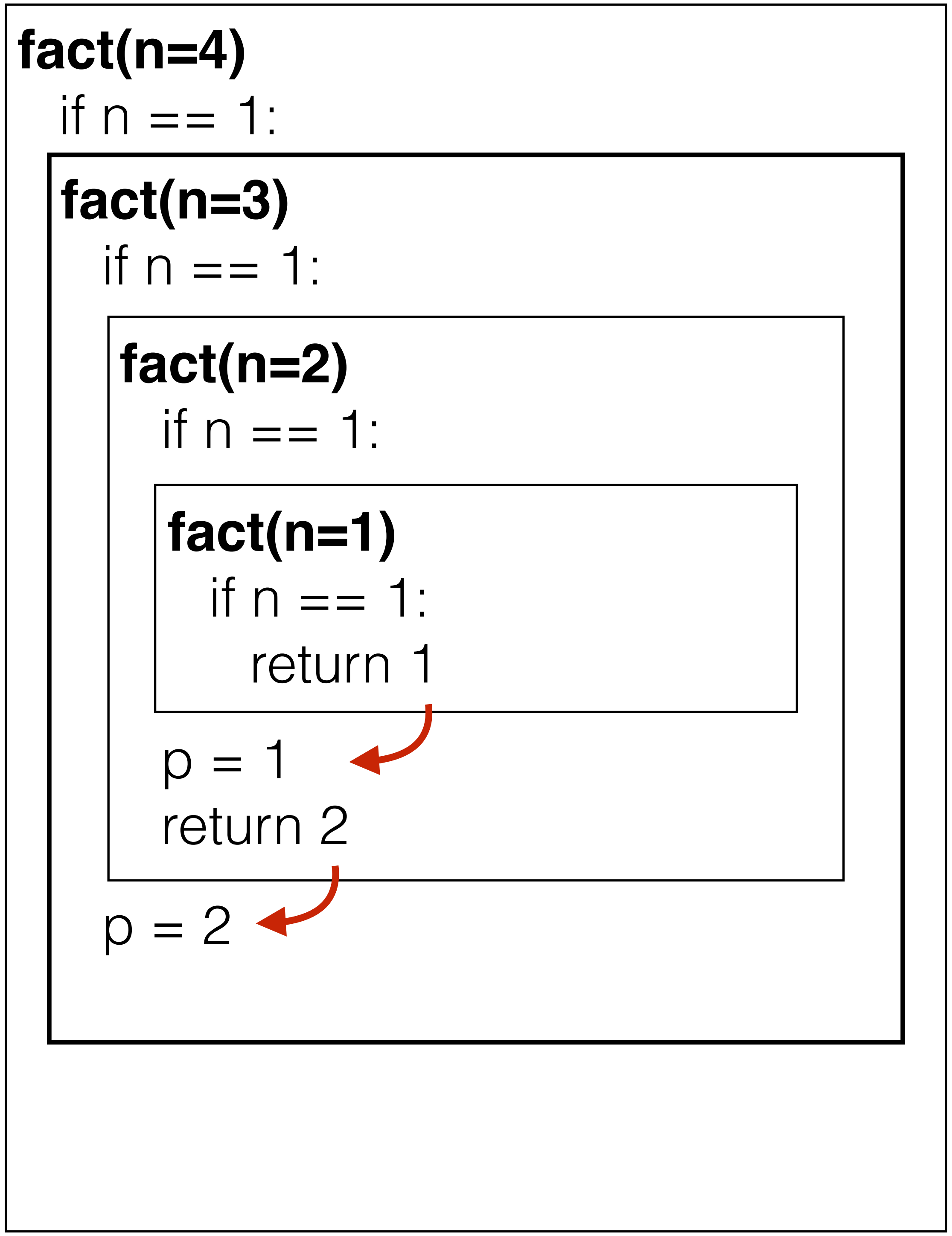
```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



```
fact(n=4)
  if n == 1:
  fact(n=3)
    if n == 1:
    fact(n=2)
      if n == 1:
      fact(n=1)
        if n == 1:
          return 1
      p = 1
      return 2
```

Tracing Factorial

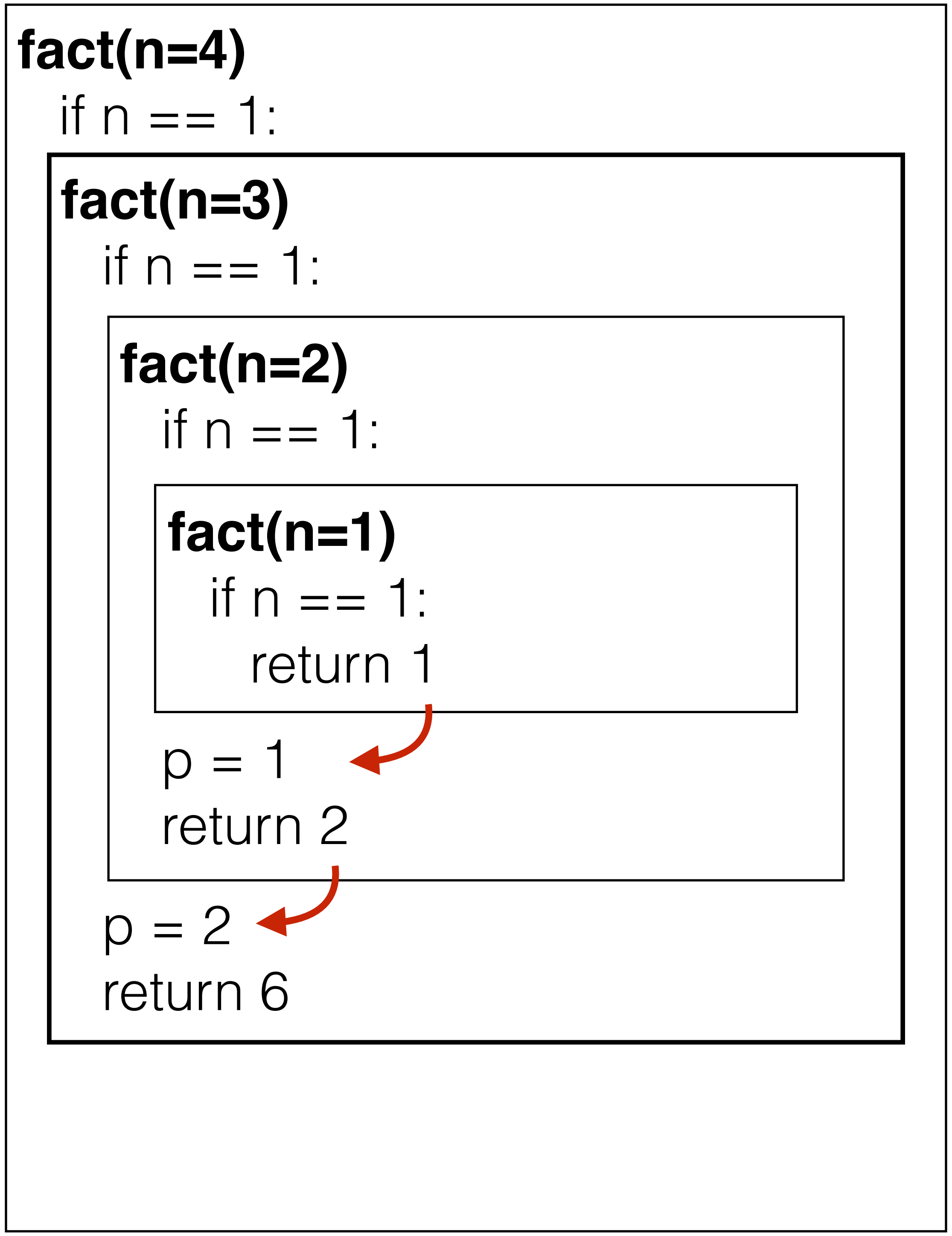
```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



```
fact(n=4)
  if n == 1:
  fact(n=3)
    if n == 1:
    fact(n=2)
      if n == 1:
      fact(n=1)
        if n == 1:
          return 1
      p = 1
      return 2
    p = 2
```

Tracing Factorial

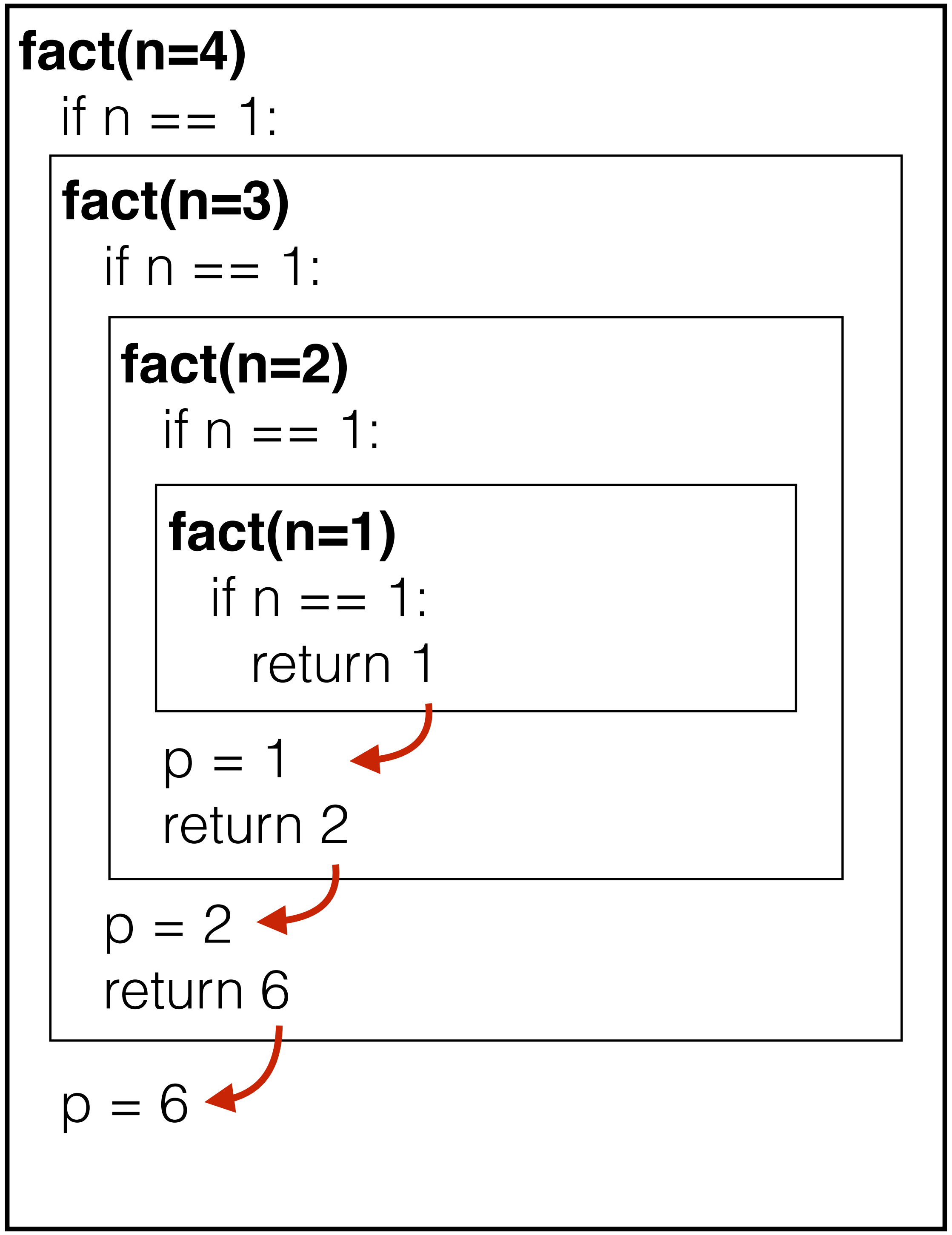
```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



```
fact(n=4)
  if n == 1:
  fact(n=3)
    if n == 1:
    fact(n=2)
      if n == 1:
      fact(n=1)
        if n == 1:
          return 1
      p = 1
      return 2
    p = 2
    return 6
```

Tracing Factorial

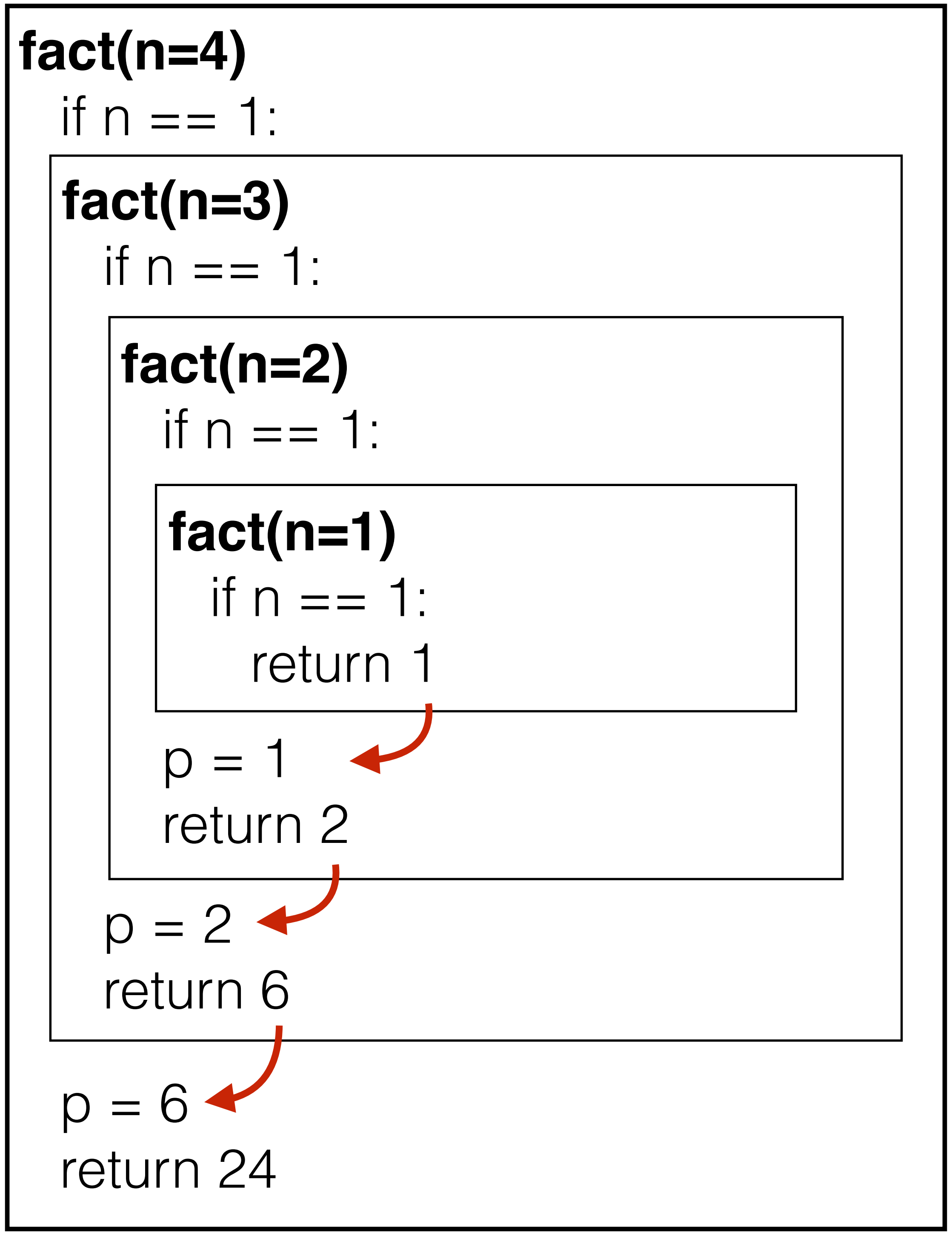
```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)
if n == 1:
fact(n=3)
if n == 1:
fact(n=2)
if n == 1:
fact(n=1)
if n == 1:
return 1
p = 1
return 2
p = 2
return 6
p = 6

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact(n=4)
if n == 1:

fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)
if n == 1:
return 1

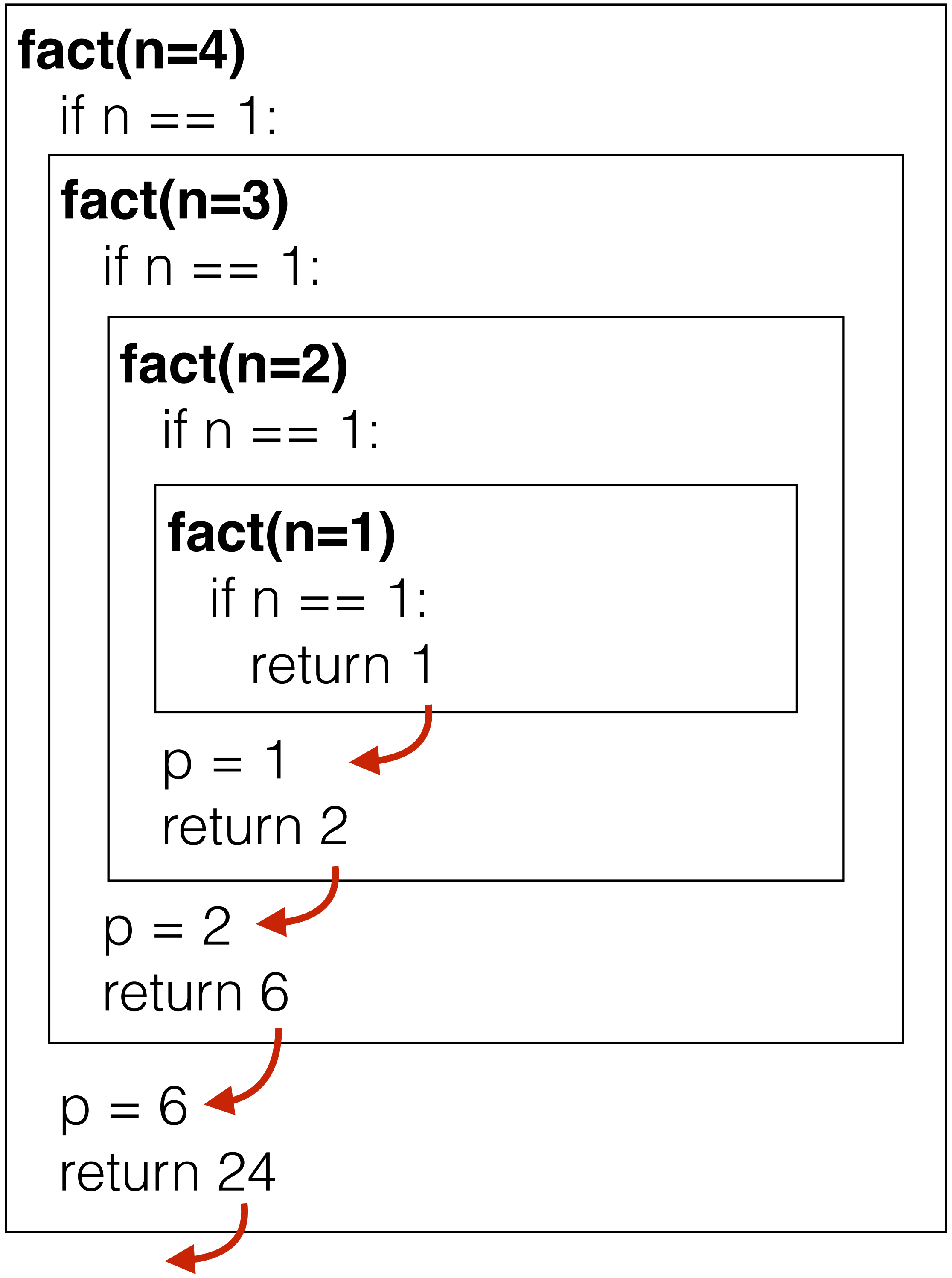
p = 1
return 2

p = 2
return 6

p = 6
return 24

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

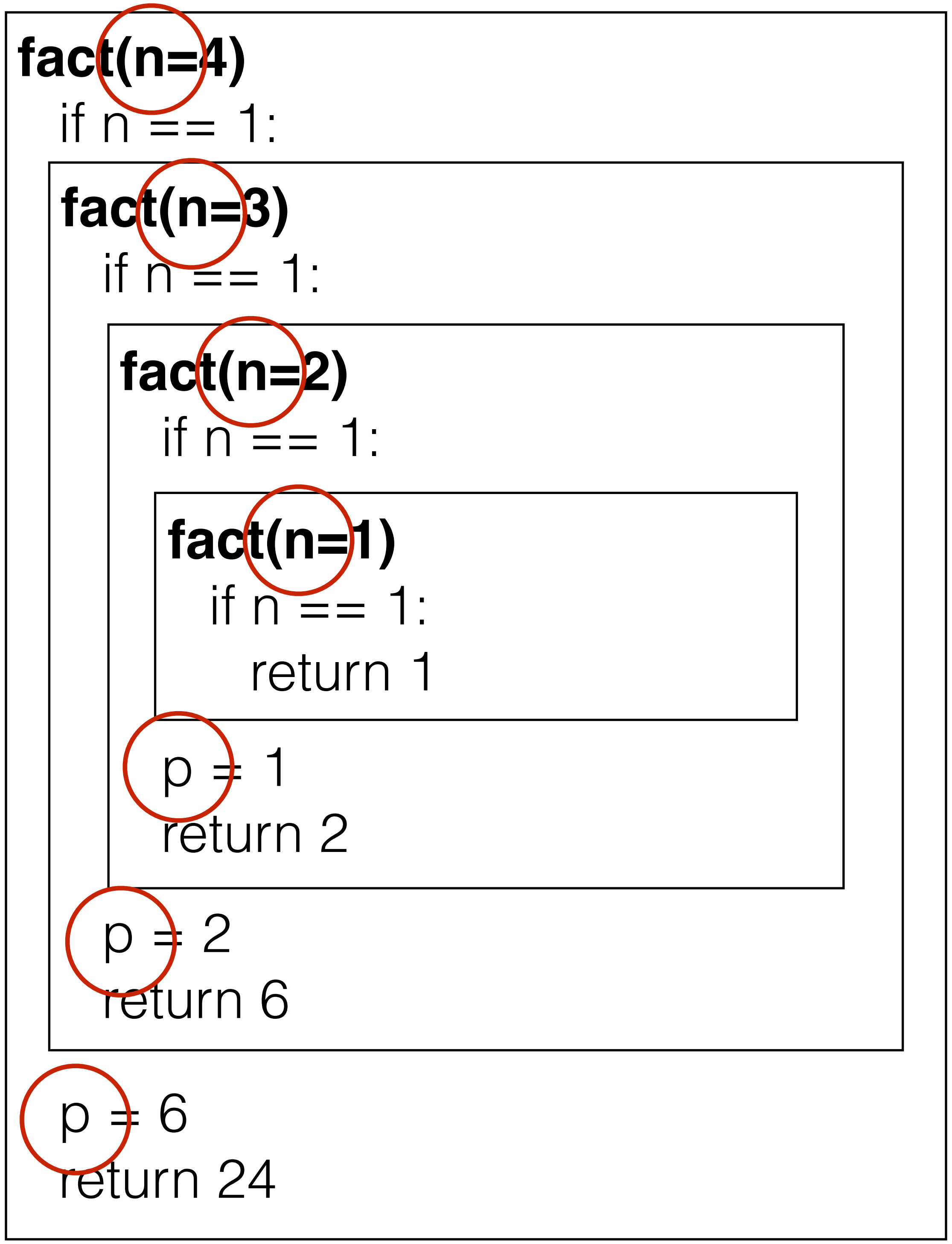


fact(n=4)
  if n == 1:
  fact(n=3)
    if n == 1:
    fact(n=2)
      if n == 1:
      fact(n=1)
        if n == 1:
          return 1
      p = 1
      return 2
    p = 2
    return 6
  p = 6
  return 24

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

How does Python keep all the variables separate?



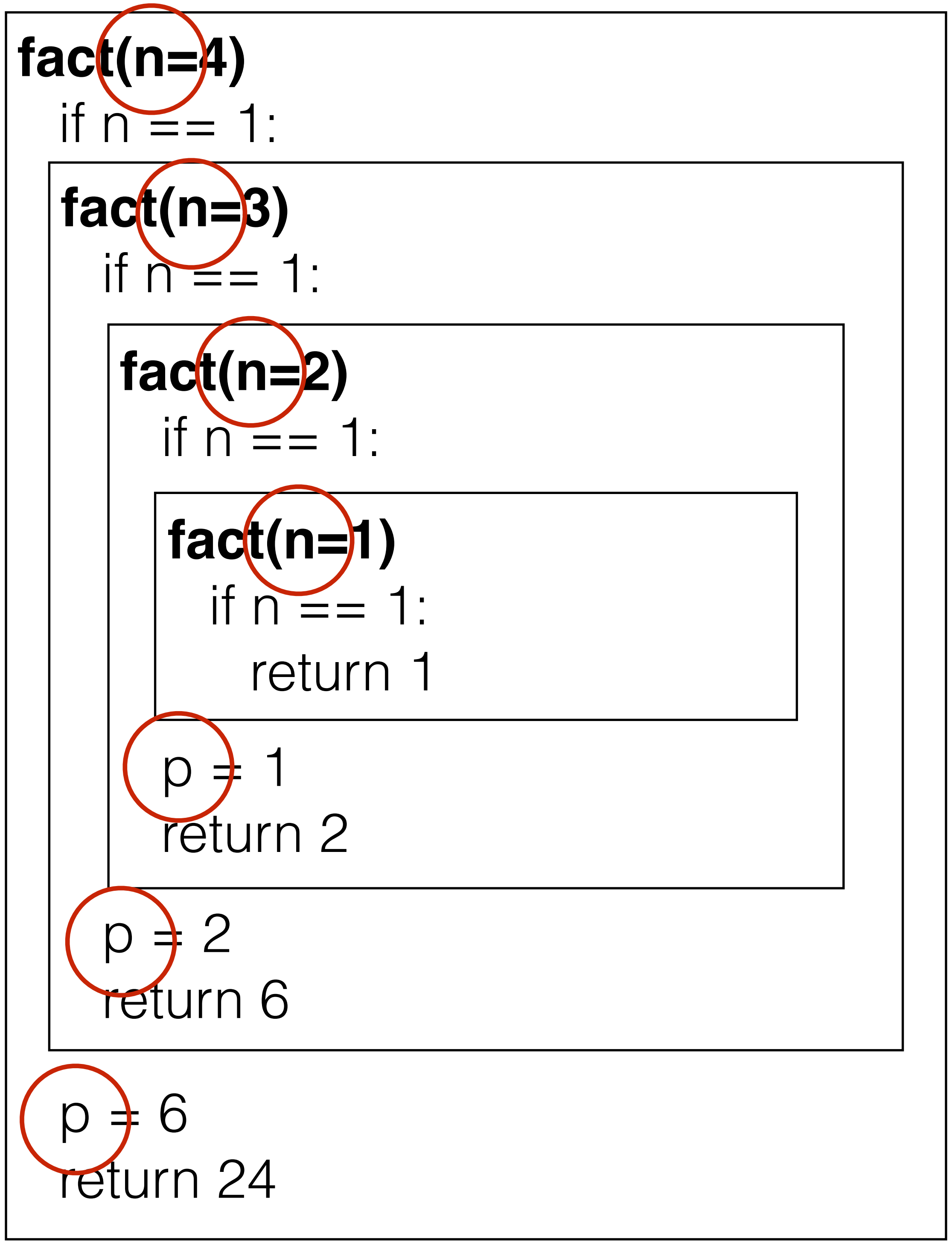
fact(n=4)
  if n == 1:
  fact(n=3)
    if n == 1:
    fact(n=2)
      if n == 1:
      fact(n=1)
        if n == 1:
          return 1
      p = 1
      return 2
    p = 2
    return 6
  p = 6
  return 24

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

How does Python keep
all the variables separate?

frames to the rescue!



```
fact(n=4)
  if n == 1:
  fact(n=3)
    if n == 1:
    fact(n=2)
      if n == 1:
      fact(n=1)
        if n == 1:
          return 1
      p = 1
      return 2
    p = 2
    return 6
  p = 6
  return 24
```

Deep Dive: Invocation State

In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Deep Dive: Invocation State

In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Variables for an invocation exist in a ***frame***

frame:

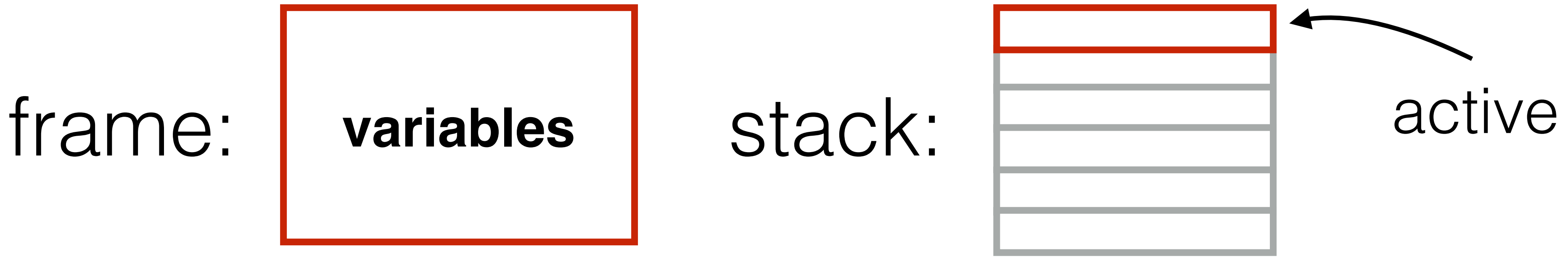
variables

Deep Dive: Invocation State

In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Variables for an invocation exist in a ***frame***

- the frames are stored in the **stack**



frame: **variables**

stack:

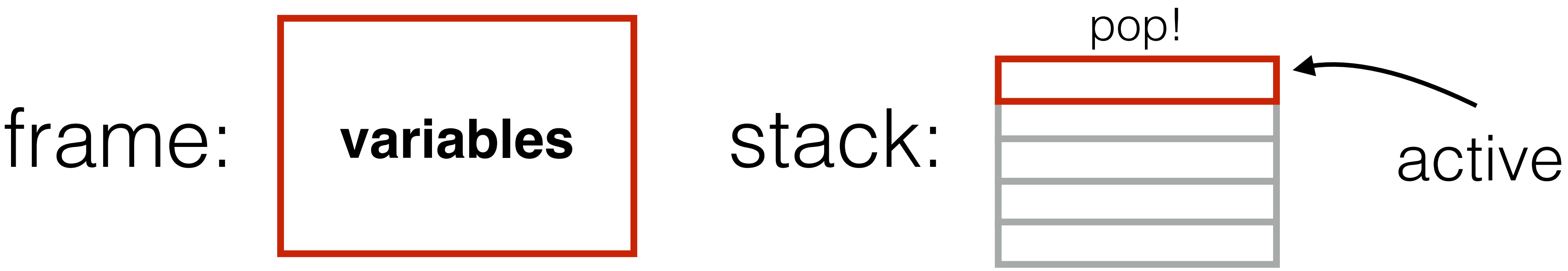
active

Deep Dive: Invocation State

In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Variables for an invocation exist in a ***frame***

- the frames are stored in the **stack**
- one invocation is active at a time: its frame is on the top of stack



Deep Dive: Invocation State

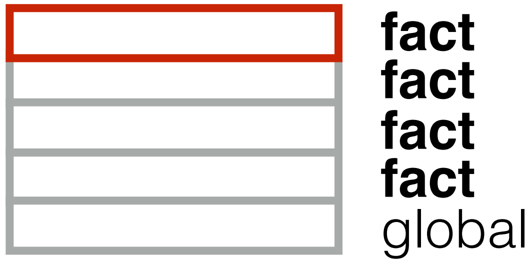
In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Variables for an invocation exist in a ***frame***

- the frames are stored in the **stack**
- one invocation is active at a time: its frame is on the top of stack
- if a function calls itself, there will be multiple frames at the same time for the multiple invocations of the same function

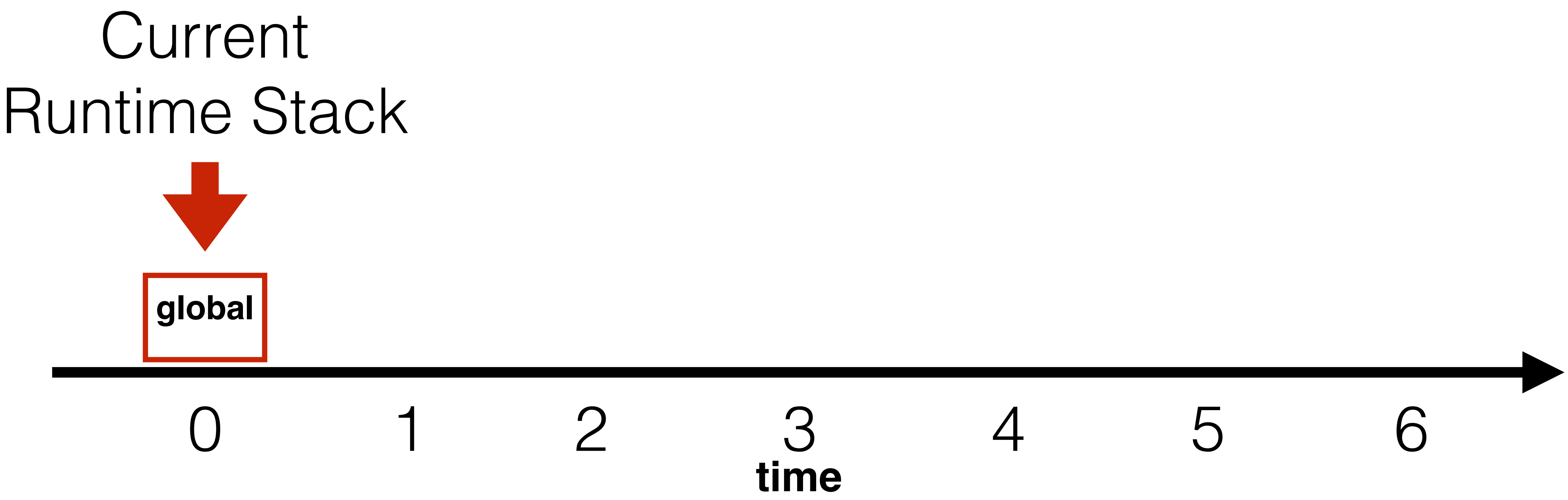
frame:

stack:



Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



Current Runtime Stack

global

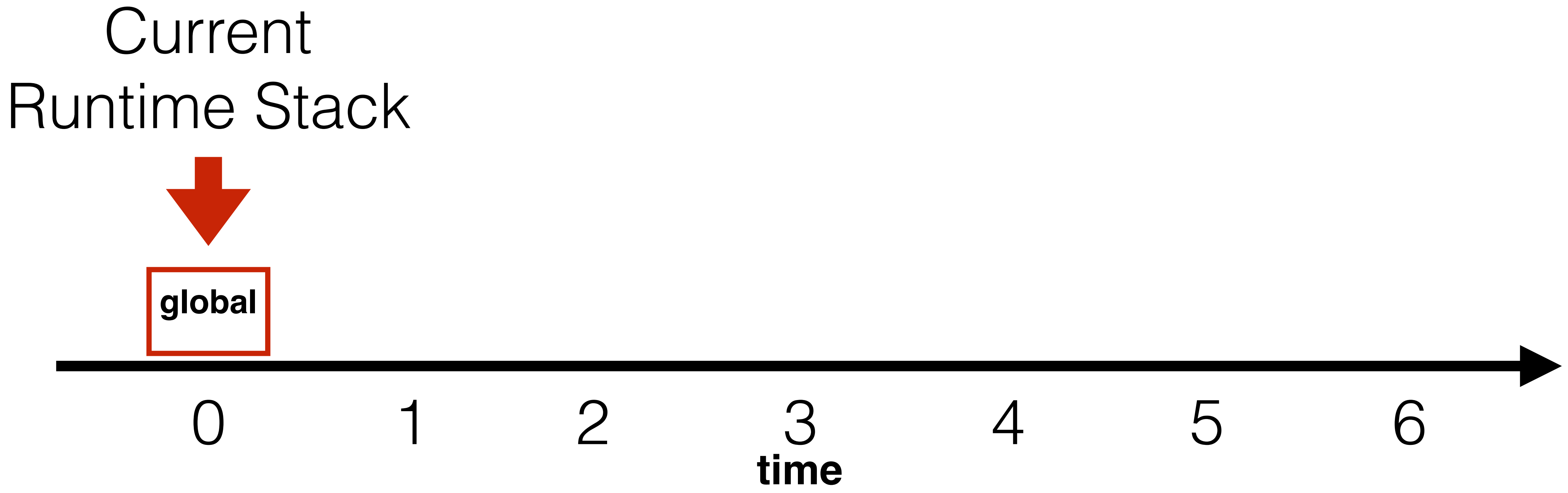
0 1 2 3 4 5 6

time

Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

call `fact(3)`



Current Runtime Stack

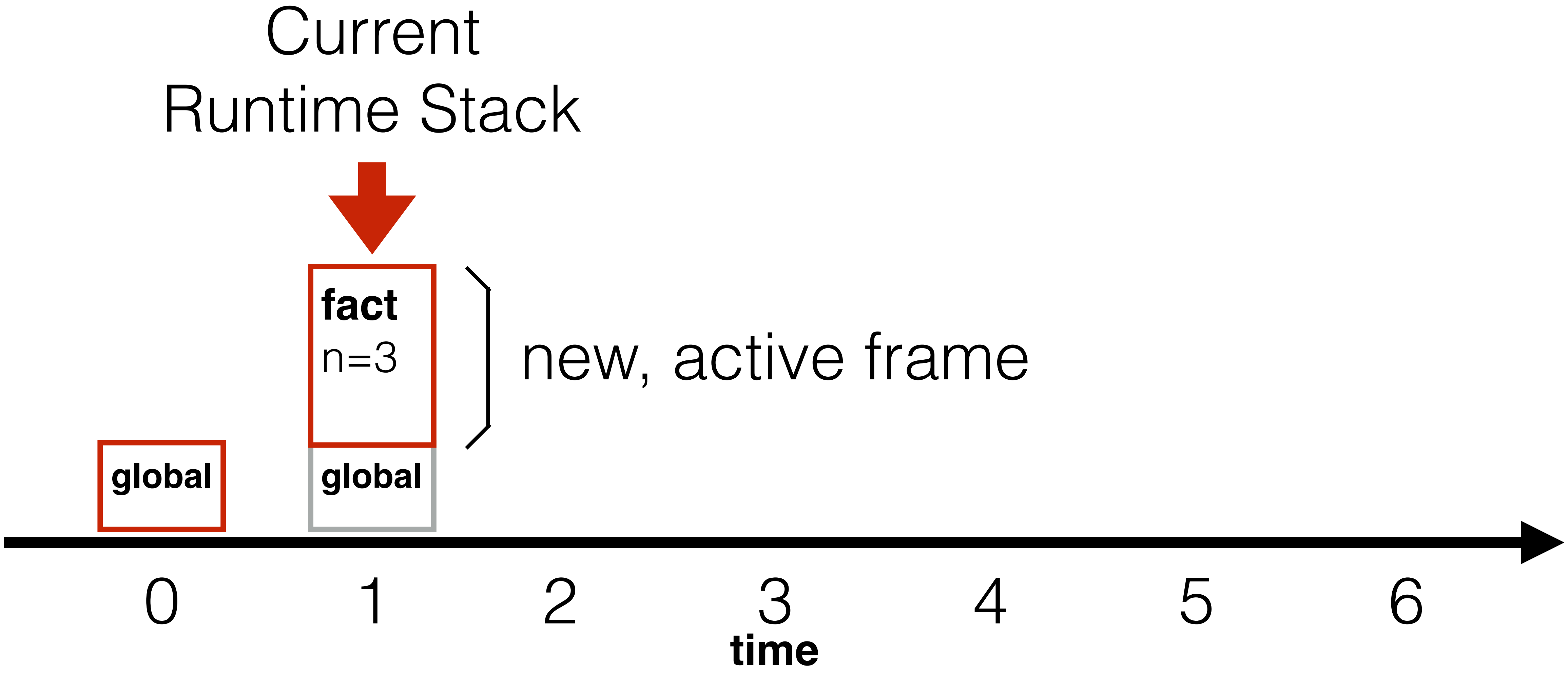
global

0 1 2 3 4 5 6

time

Deep Dive:
Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

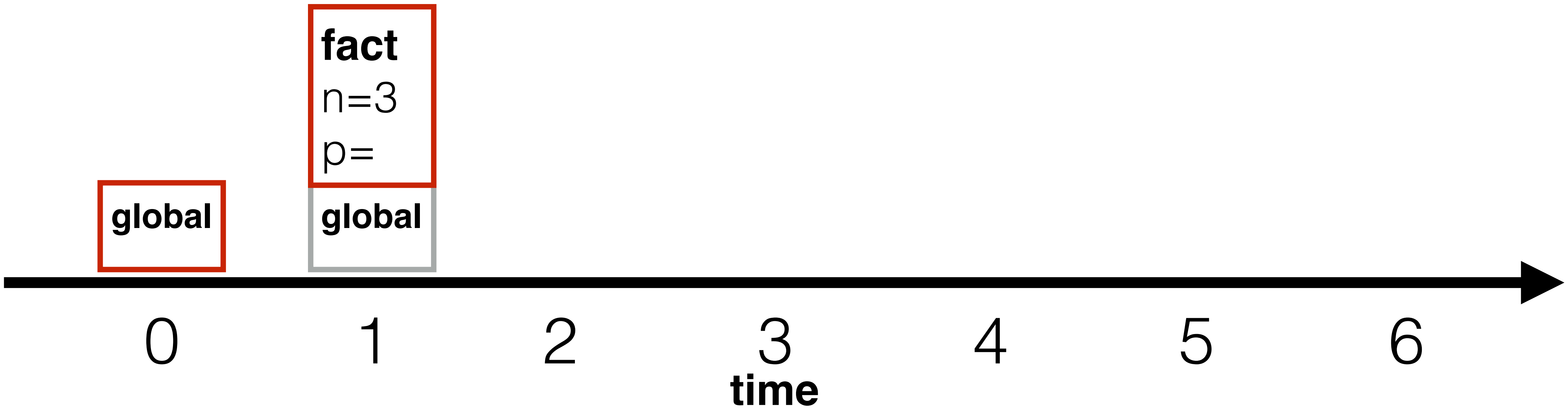


Current
Runtime Stack

new, active frame

Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact
n=3
p=

global

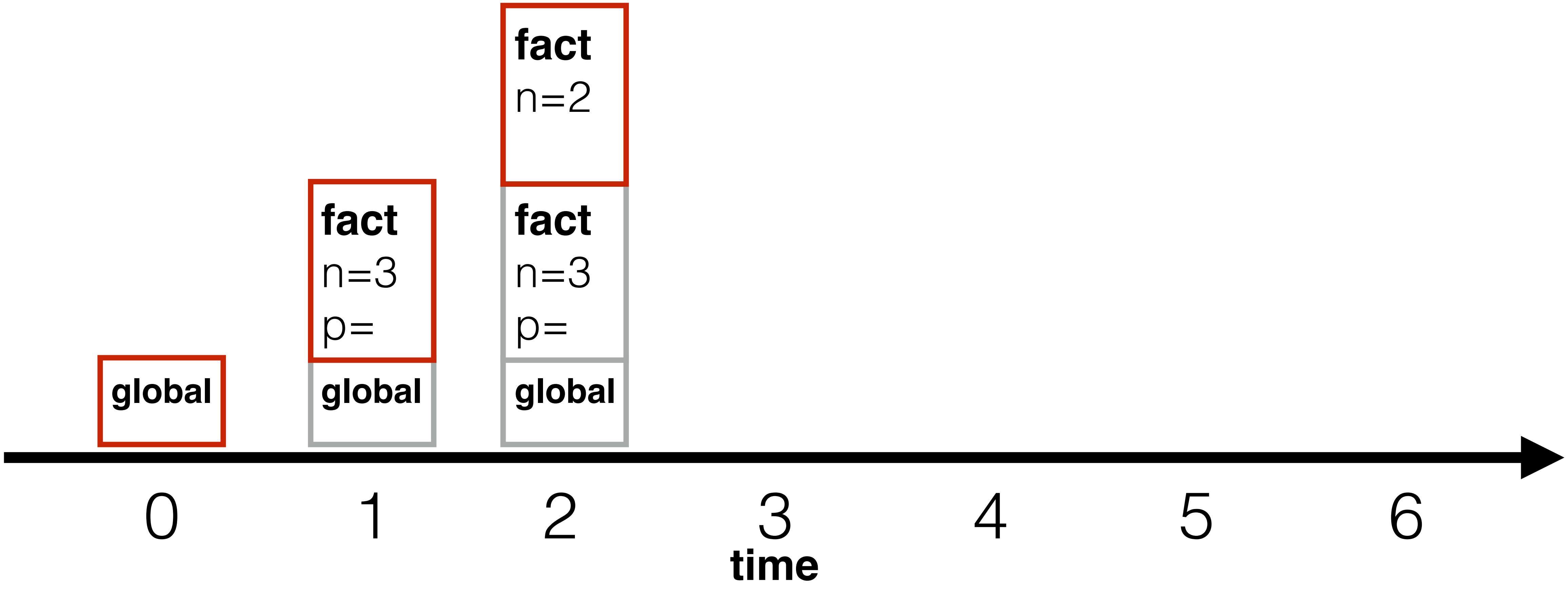
global

0 1 2 3 4 5 6

time

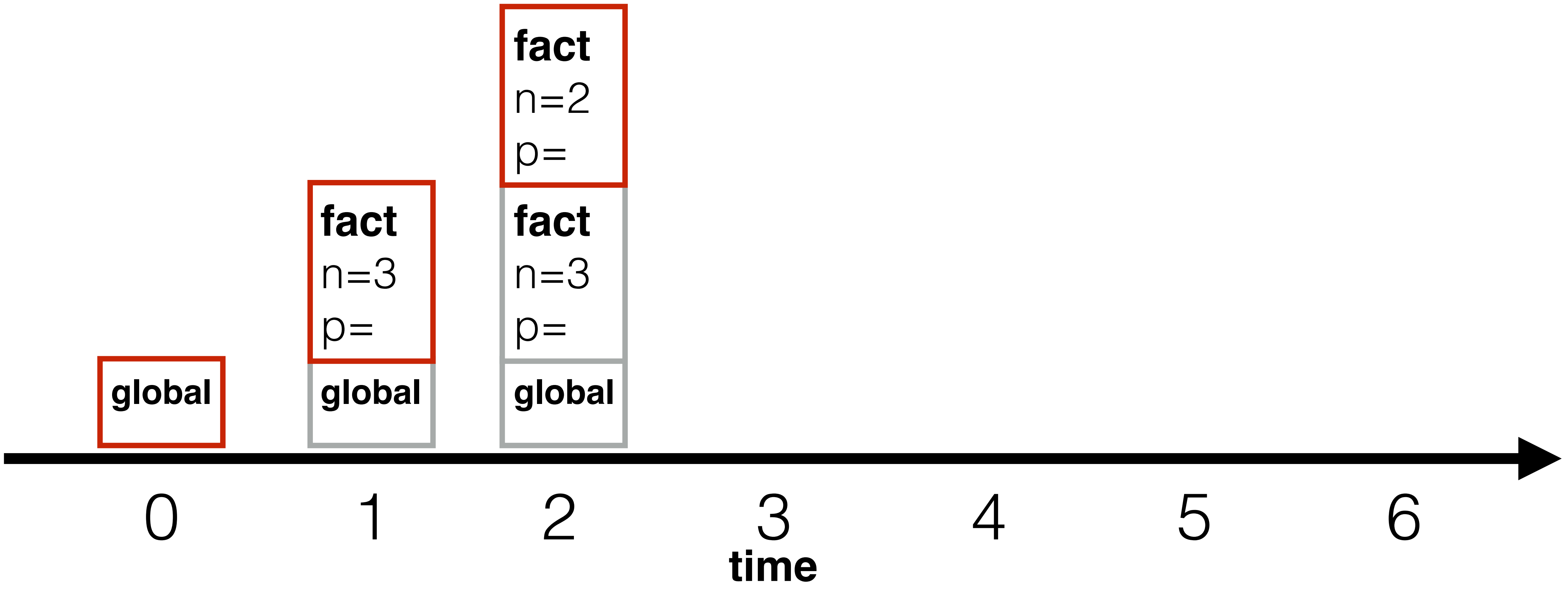
Deep Dive:
Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



Deep Dive: Runtime Stack

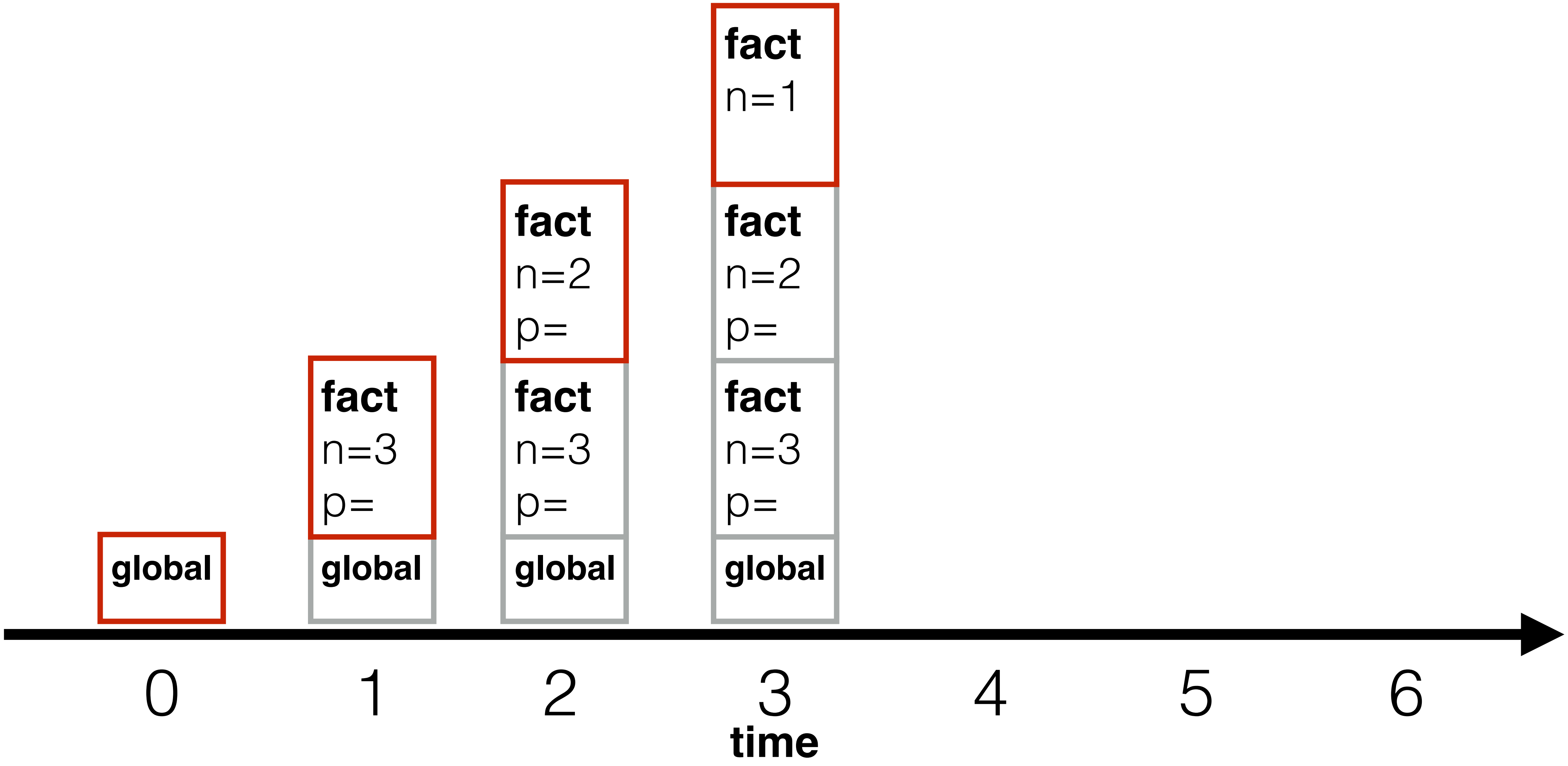
```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



fact
n=2
p=
fact
n=3
p=
fact
n=3
p=
global
global
global
0
1
2
3
4
5
6
time

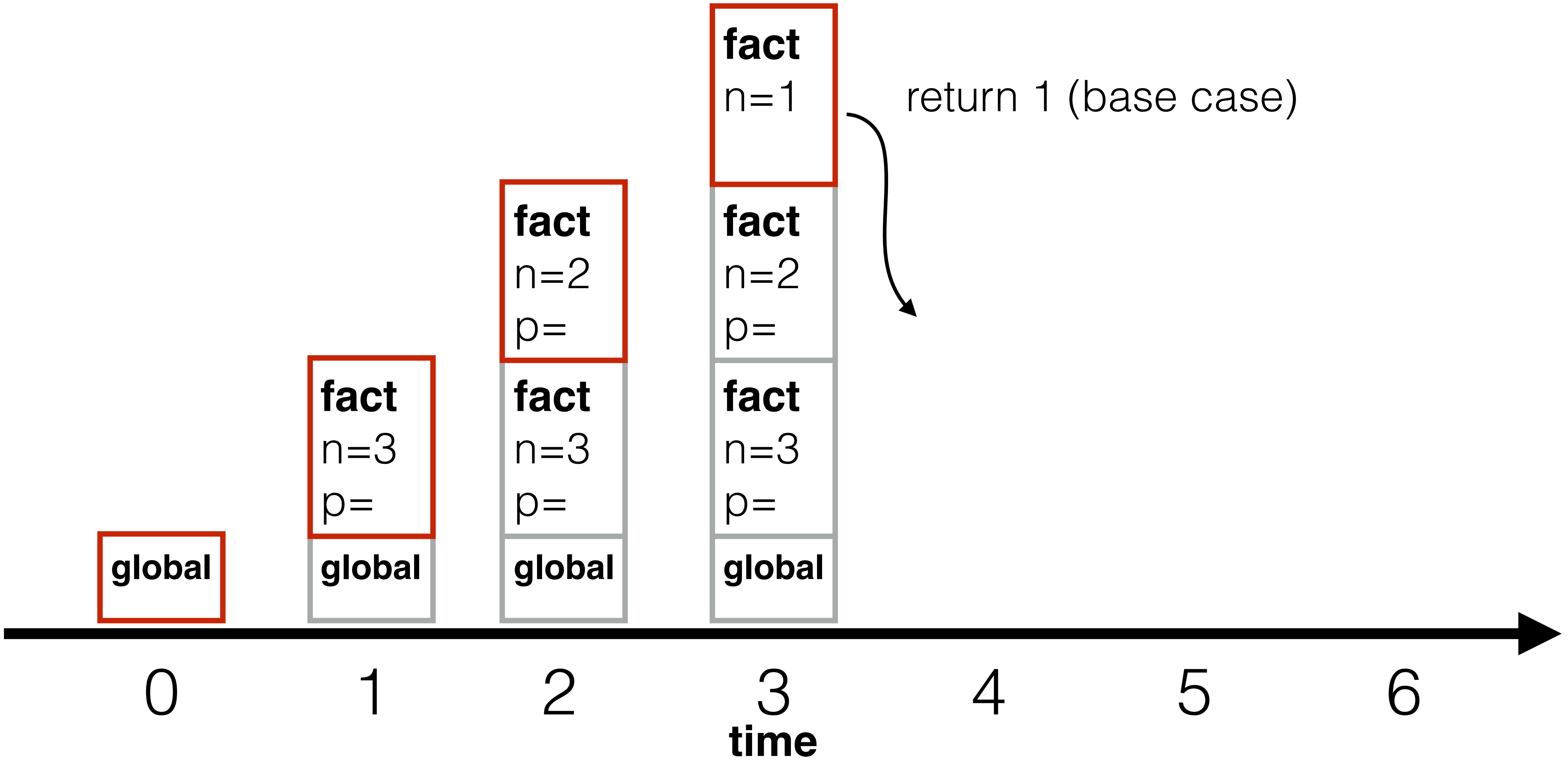
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



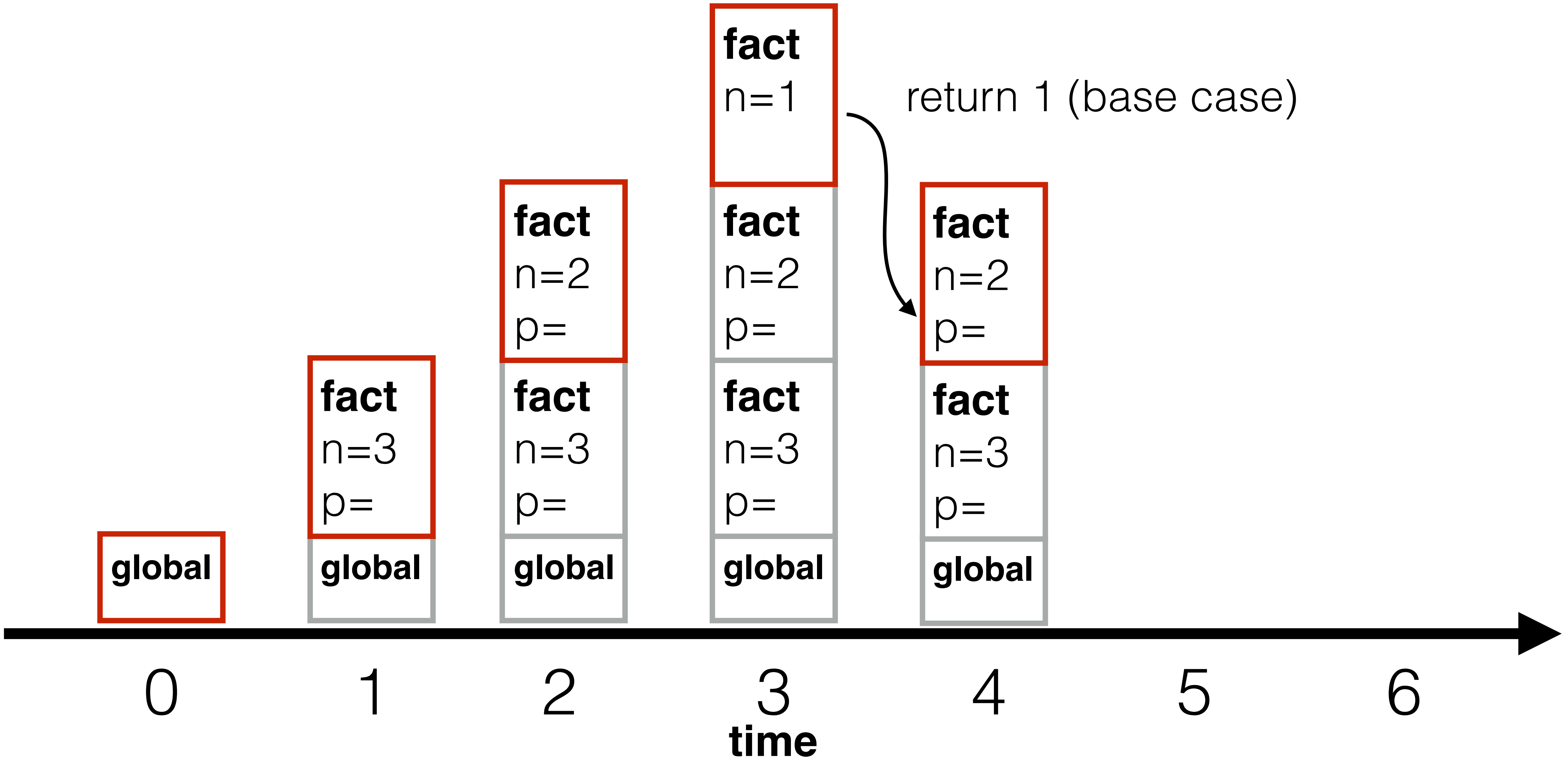
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



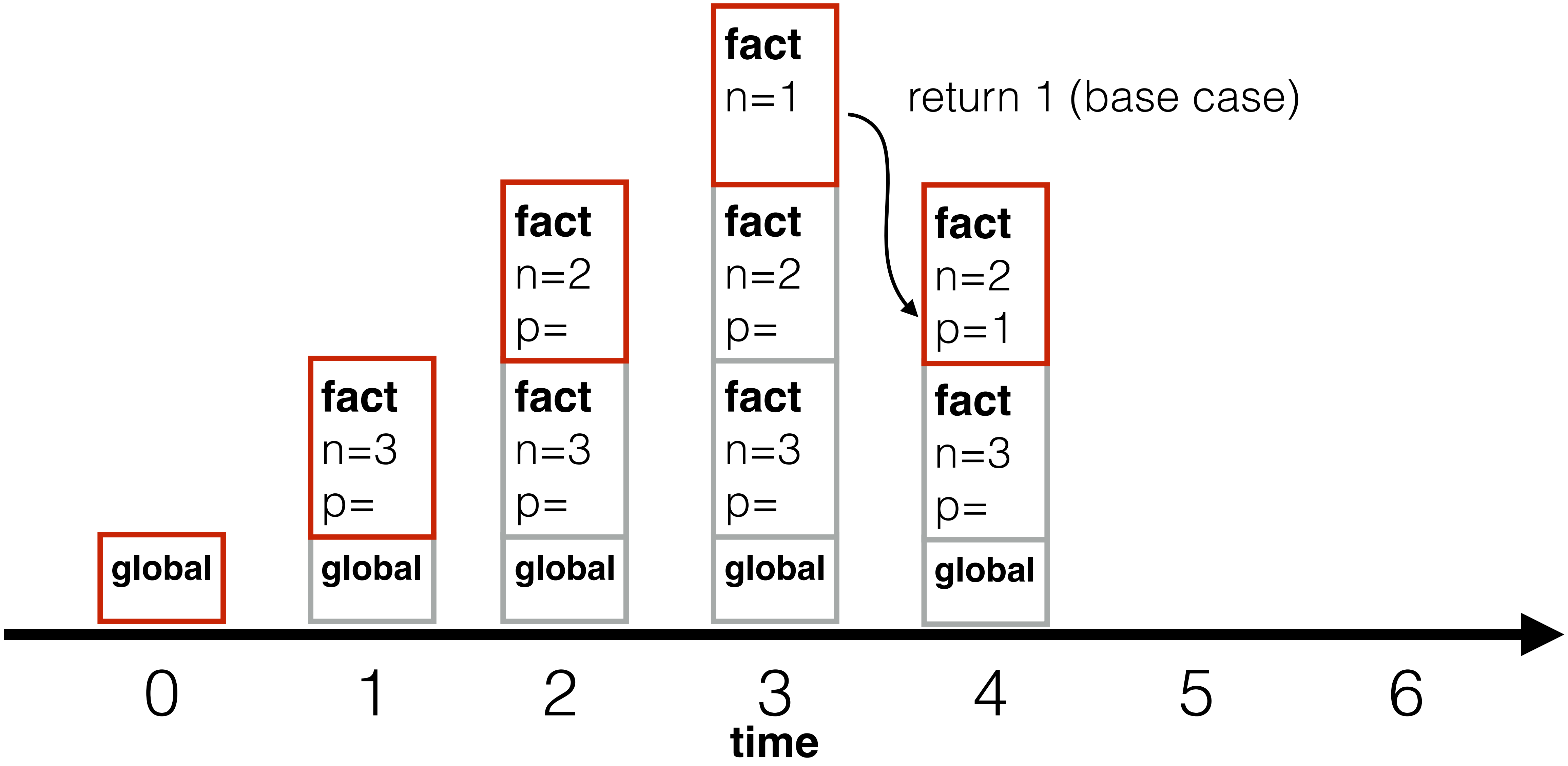
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



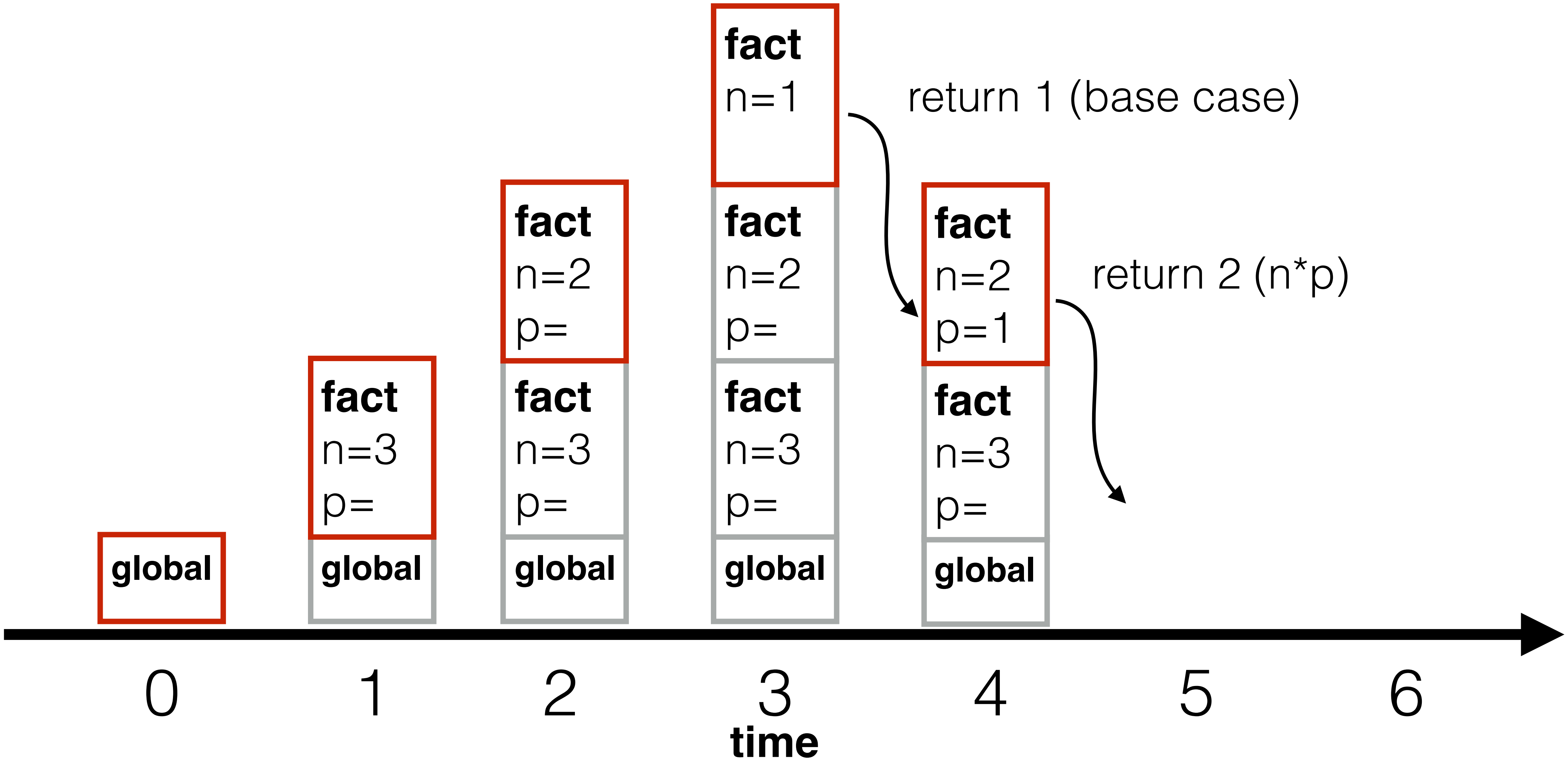
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



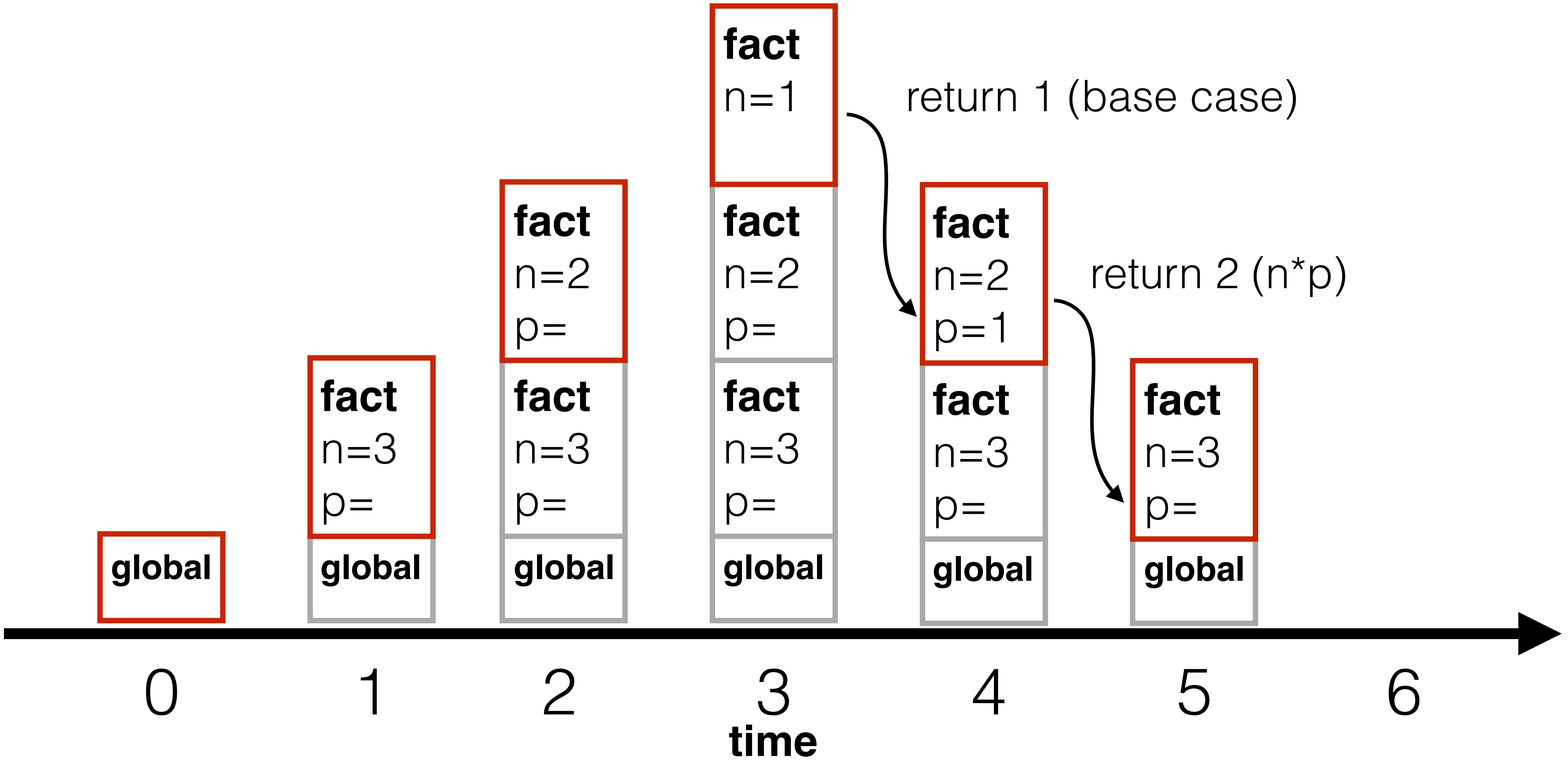
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



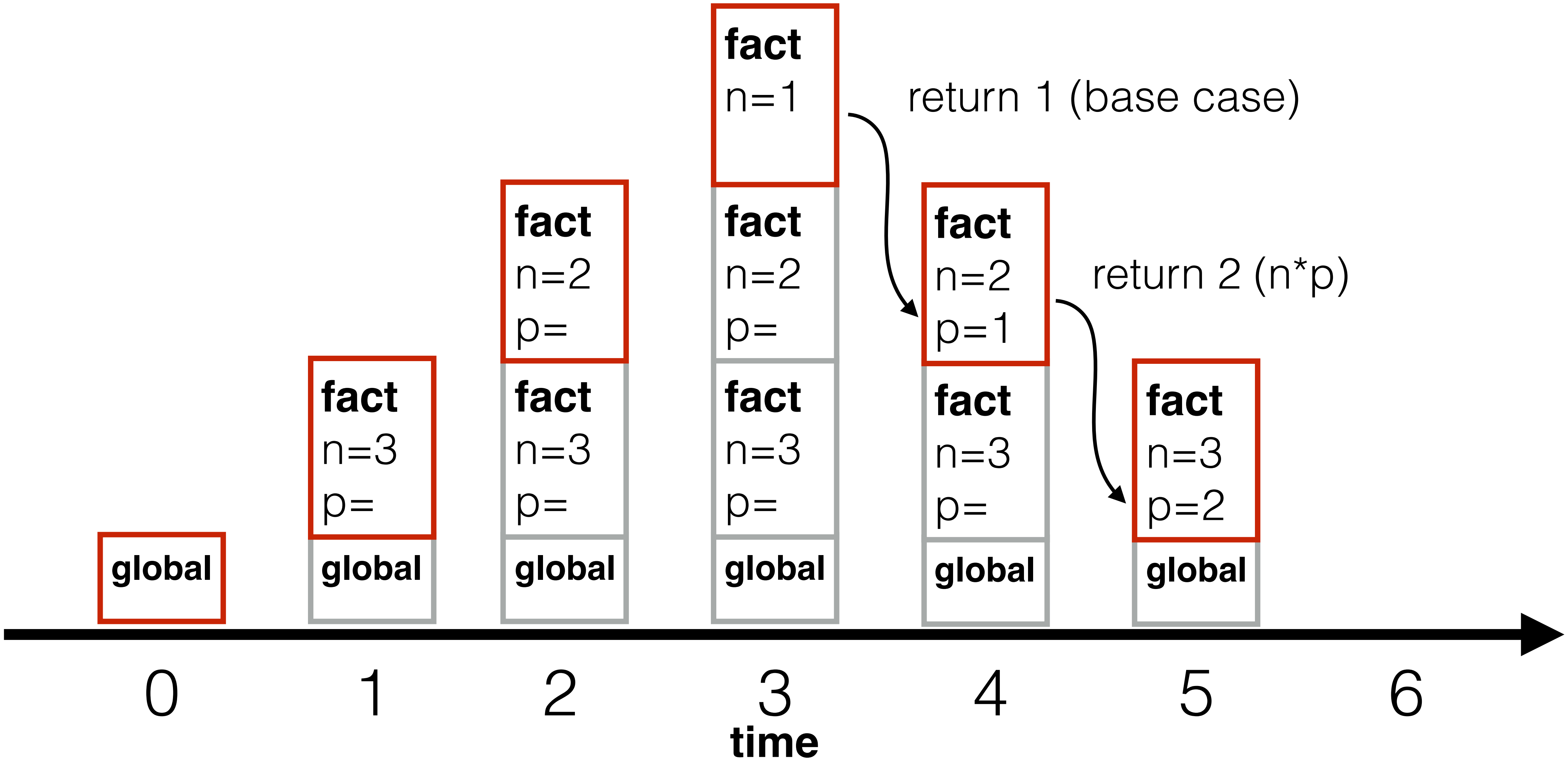
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



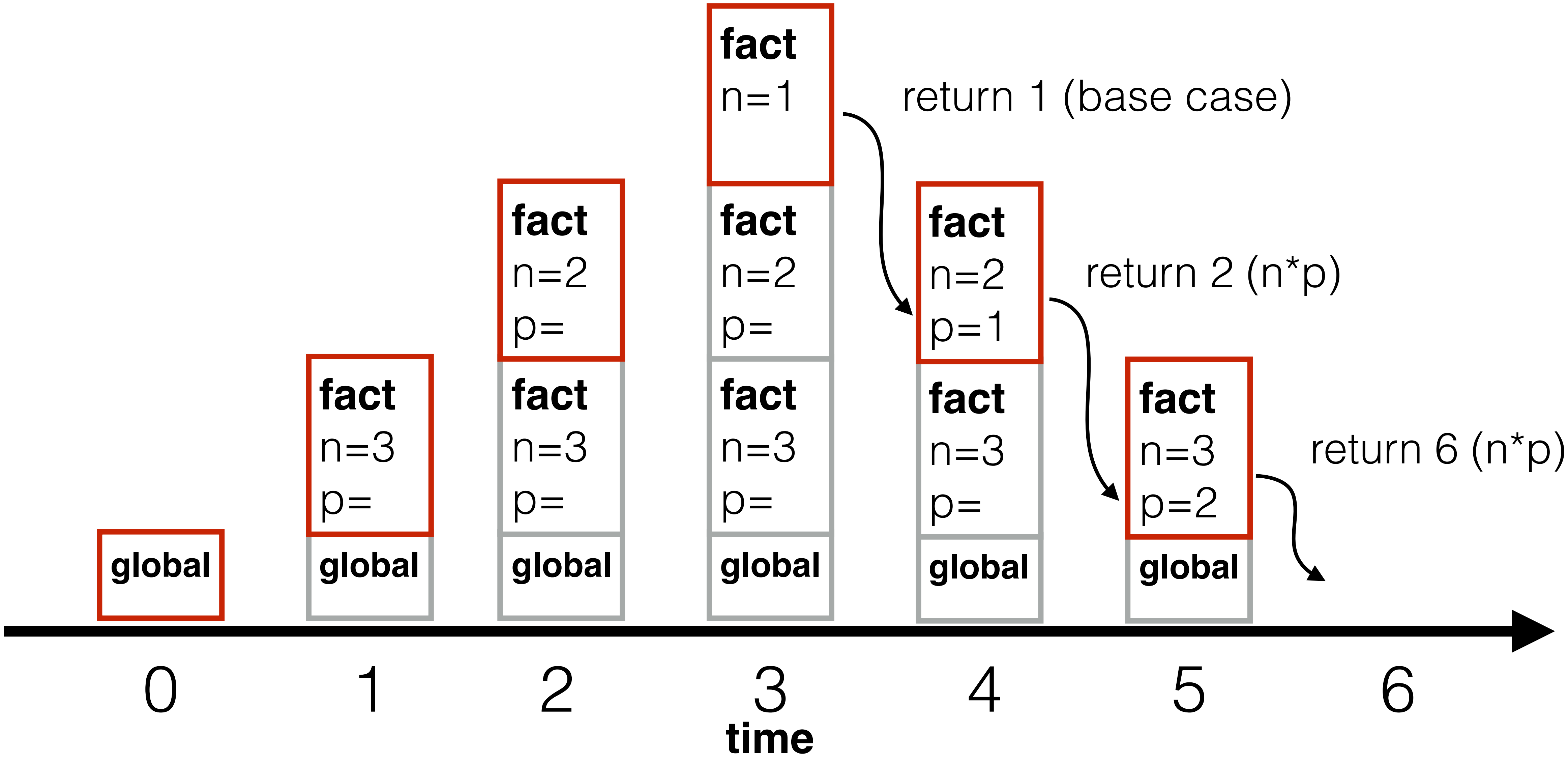
Deep Dive:
Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



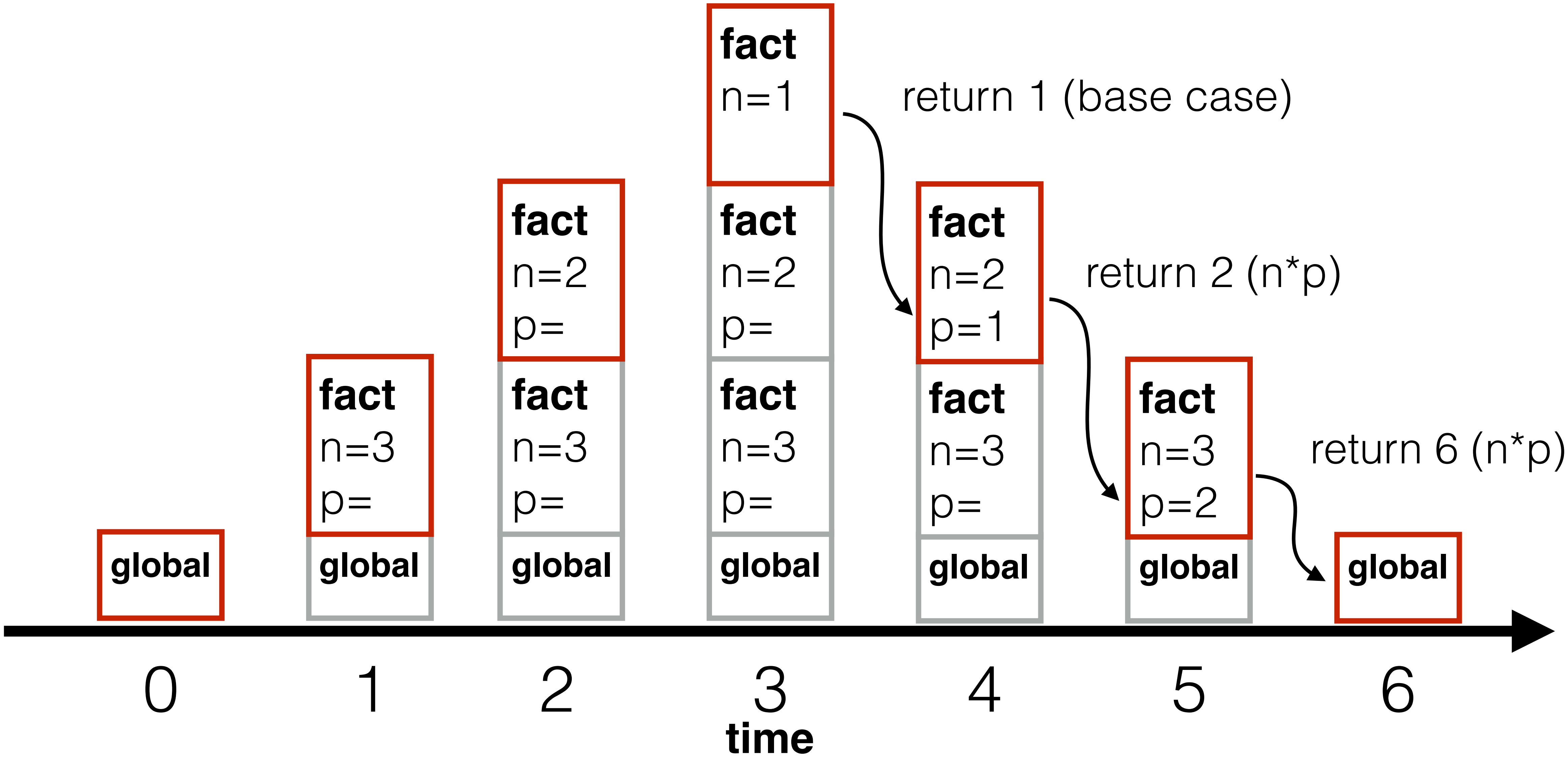
Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```



“Infinite” Recursion Bugs

What happens if:

-
-

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

“Infinite” Recursion Bugs

What happens if:

- we forgot the “n == 1” check?
-

```
def fact(n):
    ~~if n == 1:~~
        ~~return 1~~
    p = fact(n-1)
    return n * p
```

“Infinite” Recursion Bugs

What happens if:

- we forgot the “n == 1” check?
- factorial is called with a negative number?

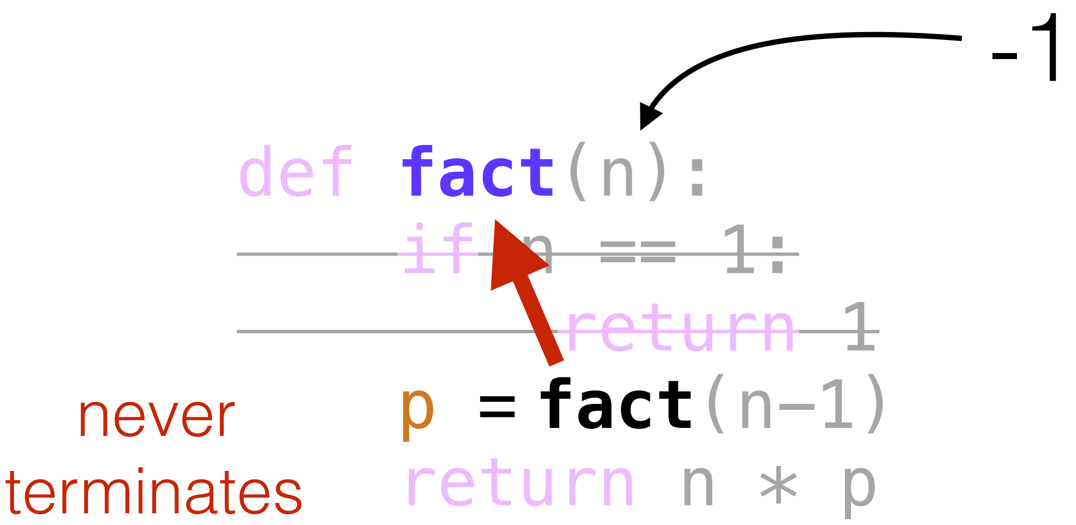
-1

```
def fact(n):
~~    if n == 1:~~
~~        return 1~~
    p = fact(n-1)
    return n * p
```

“Infinite” Recursion Bugs

What happens if:

- we forgot the “n == 1” check?
- factorial is called with a negative number?



-1

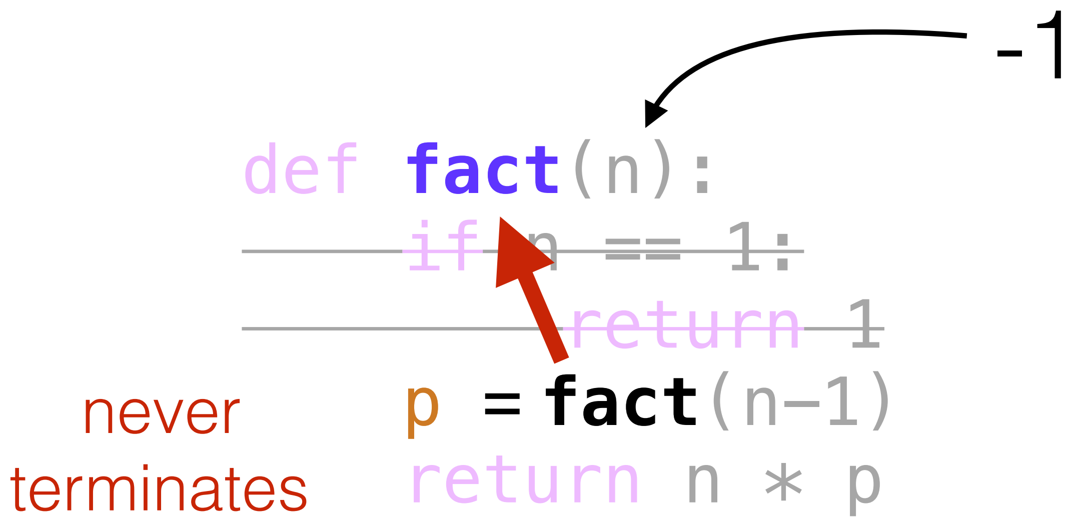
```
def fact(n):
    ~~if n == 1:~~
        ~~return 1~~
    p = fact(n-1)
    return n * p
```

never terminates

“Infinite” Recursion Bugs

What happens if:

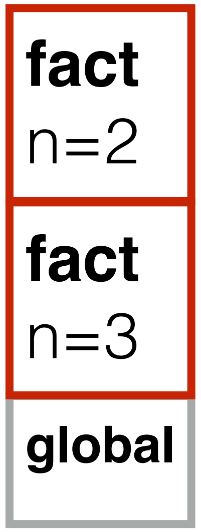
- we forgot the “n == 1” check?
- factorial is called with a negative number?



-1

```
def fact(n):
    ~~if n == 1:~~
        ~~return 1~~
    p = fact(n-1)
    return n * p
```

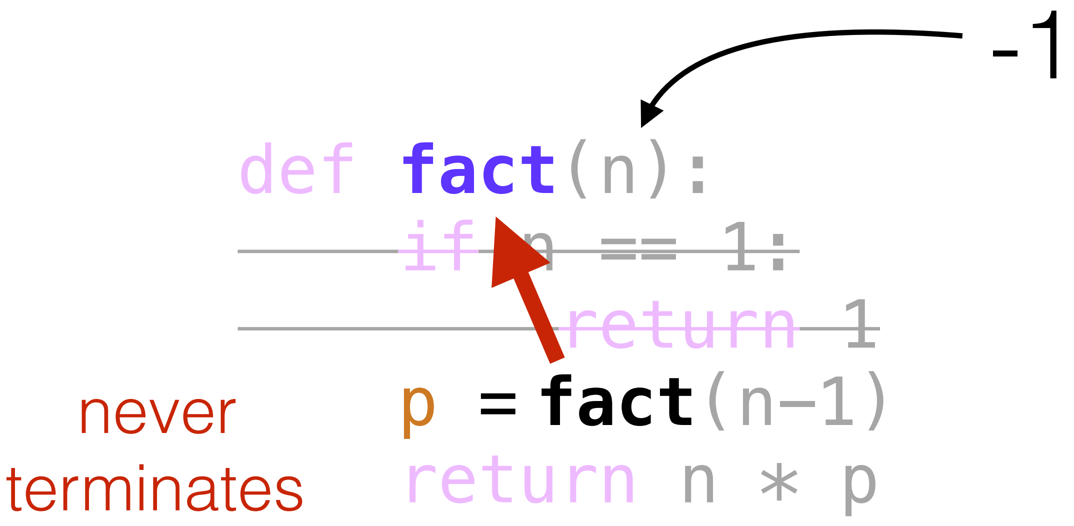
never terminates



“Infinite” Recursion Bugs

What happens if:

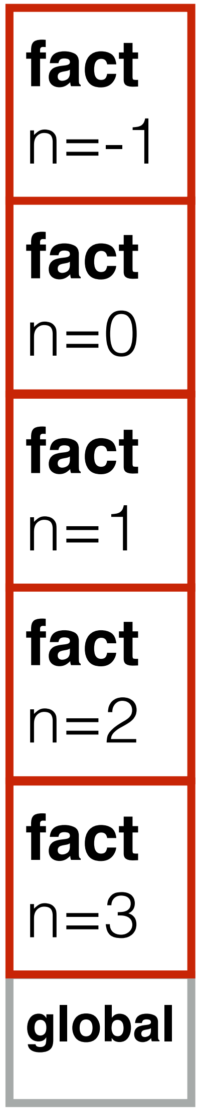
- we forgot the “n == 1” check?
- factorial is called with a negative number?



-1

```
def fact(n):
    ~~if n == 1:~~
        ~~return 1~~
    p = fact(n-1)
    return n * p
```

never terminates



fact
n=-1

fact
n=0

fact
n=1

fact
n=2

fact
n=3

global

“Infinite” Recursion Bugs

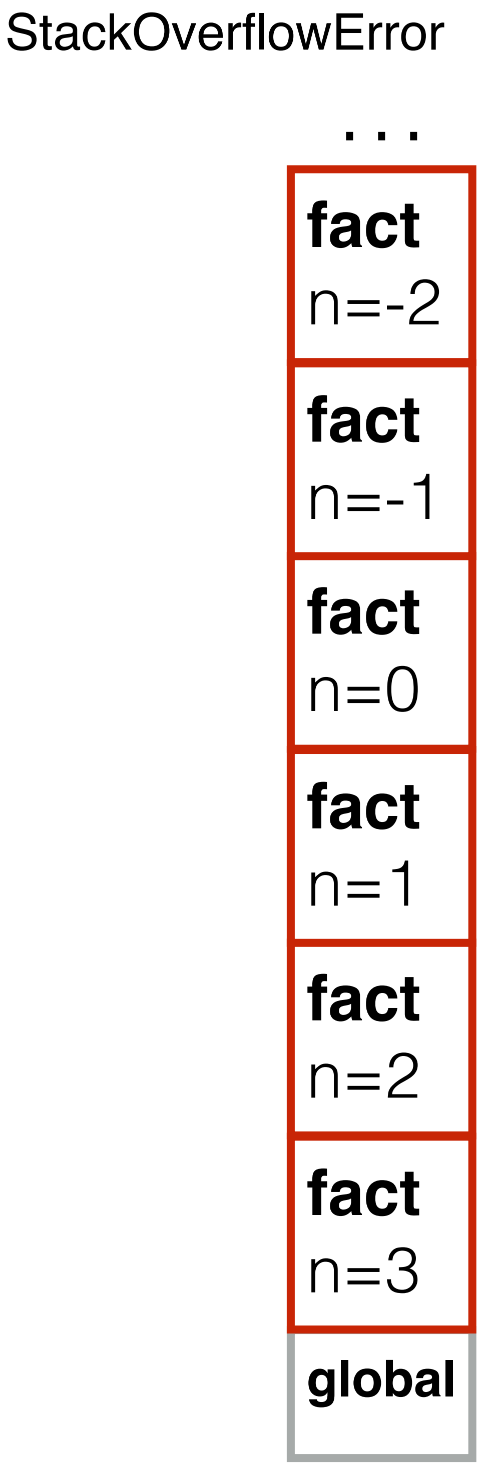
What happens if:

- we forgot the “n == 1” check?
- factorial is called with a negative number?

-1

```
def fact(n):
    ~~if n == 1:~~
        ~~return 1~~
    p = fact(n-1)
    return n * p
```

never terminates



StackOverflowError

...

fact
n=-2

fact
n=-1

fact
n=0

fact
n=1

fact
n=2

fact
n=3

global

Coding Demos

Demo 1: Pretty Print

Goal: format nested lists of bullet points

Input:

- The recursive lists

Output:

- Appropriately-tabbed items

Example:

```
>>> pretty_print([“A”, [“1”, “2”, “3”,],
                  “B”, [“4”, [“i”, “ii”]]])
*A
  *1
  *2
  *3
*B
  *4
    *i
    *ii
```

Demo 2: Recursive List Search

Goal: does a given number exist in a recursive structure?

Input:

- A number
- A list of numbers and lists (which contain other numbers and lists)

Output:

- True if there's a list containing the number, else False

Example:

```
>>> contains(3, [1,2,[4,[[3],[8,9]],5,6]])
True
>>> contains(12, [1,2,[4,[[3],[8,9]],5,6]])
False
```

Conclusion: Review Learning Objectives

Learning Objectives: Recursive Information

What is a recursive definition/structure?

- Definition contains term
- Structure refers to others of same type
- Example: a dictionary contains dictionaries (which may contain...)

/ recursive case

🍃 base case

Learning Objectives: Recursive Code

What is recursive code?

- Function that sometimes itself (maybe indirectly)

Why write recursive code?

- Real-world data/structures are recursive; intuitive for code to reflect data

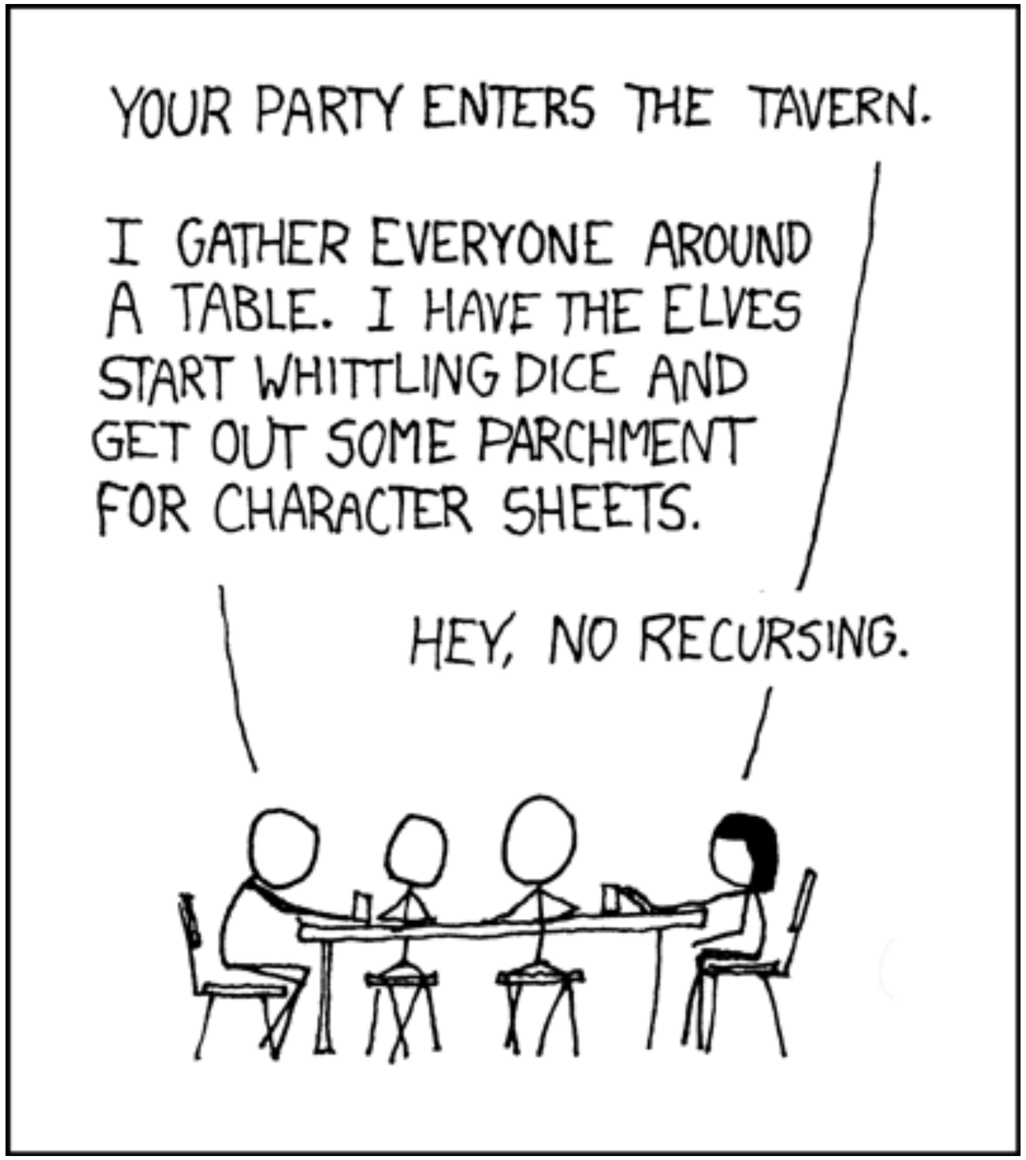
Where do computers keep local variables for recursive calls?

- In a section of memory called a “frame”
- Only one function is **active** at a time, so keep frames in a stack

What happens to programs with infinite recursion?

- Calls keep pushing more frames
- Exhaust memory, throw **StackOverflowError**

Questions?


YOUR PARTY ENTERS THE TAVERN.
I GATHER EVERYONE AROUND A TABLE. I HAVE THE ELVES START WHITTLING DICE AND GET OUT SOME PARCHMENT FOR CHARACTER SHEETS.
HEY, NO RECURSING.

https://xkcd.com/244/