[320] Complexity + Big O

Tyler Caraza-Harter

Complexity

Performance vs. Complexity

Things that affect performance (total time to run):

- ????

Performance vs. Complexity

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- speed of the computer (CPU, etc)

- speed of Python (quality+efficiency of interpretation)

- **algorithm:** strategy for solving the problem

- **input size:** how much data do we have?

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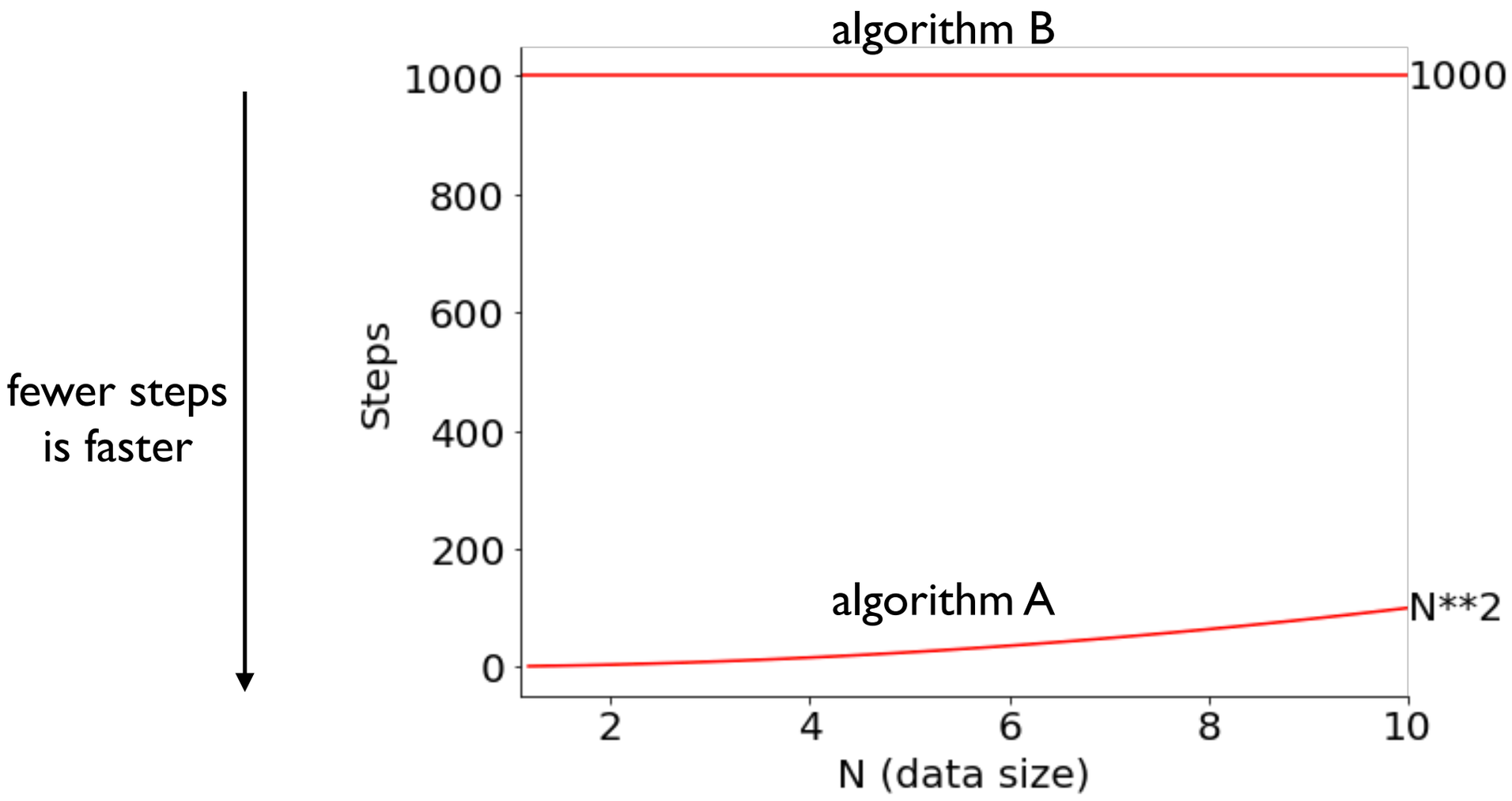
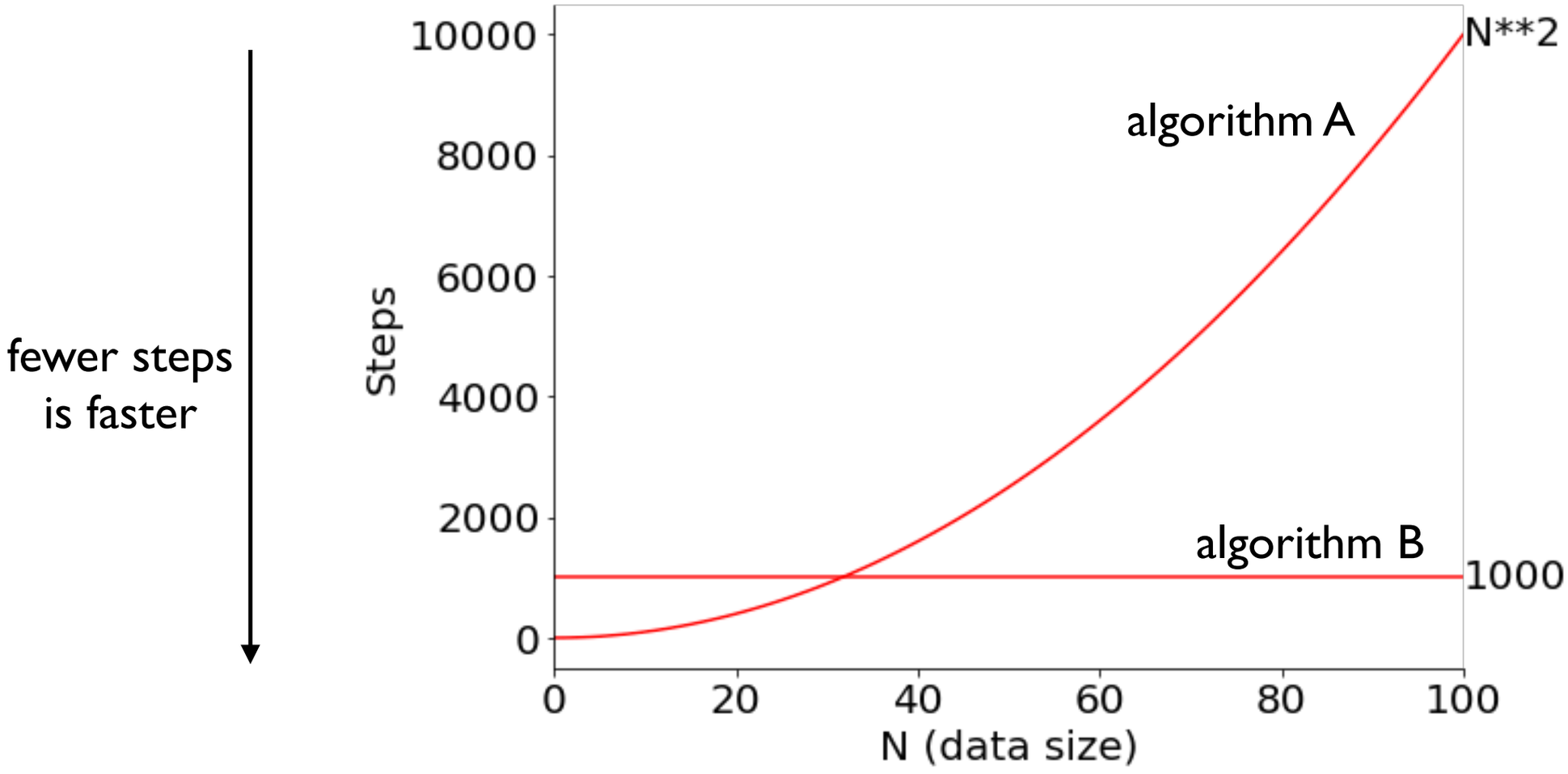
complexity analysis: how many steps must the algorithm perform, as a function of input size?

Which algorithm is better?



Do you prefer A or B?

Which algorithm is better?



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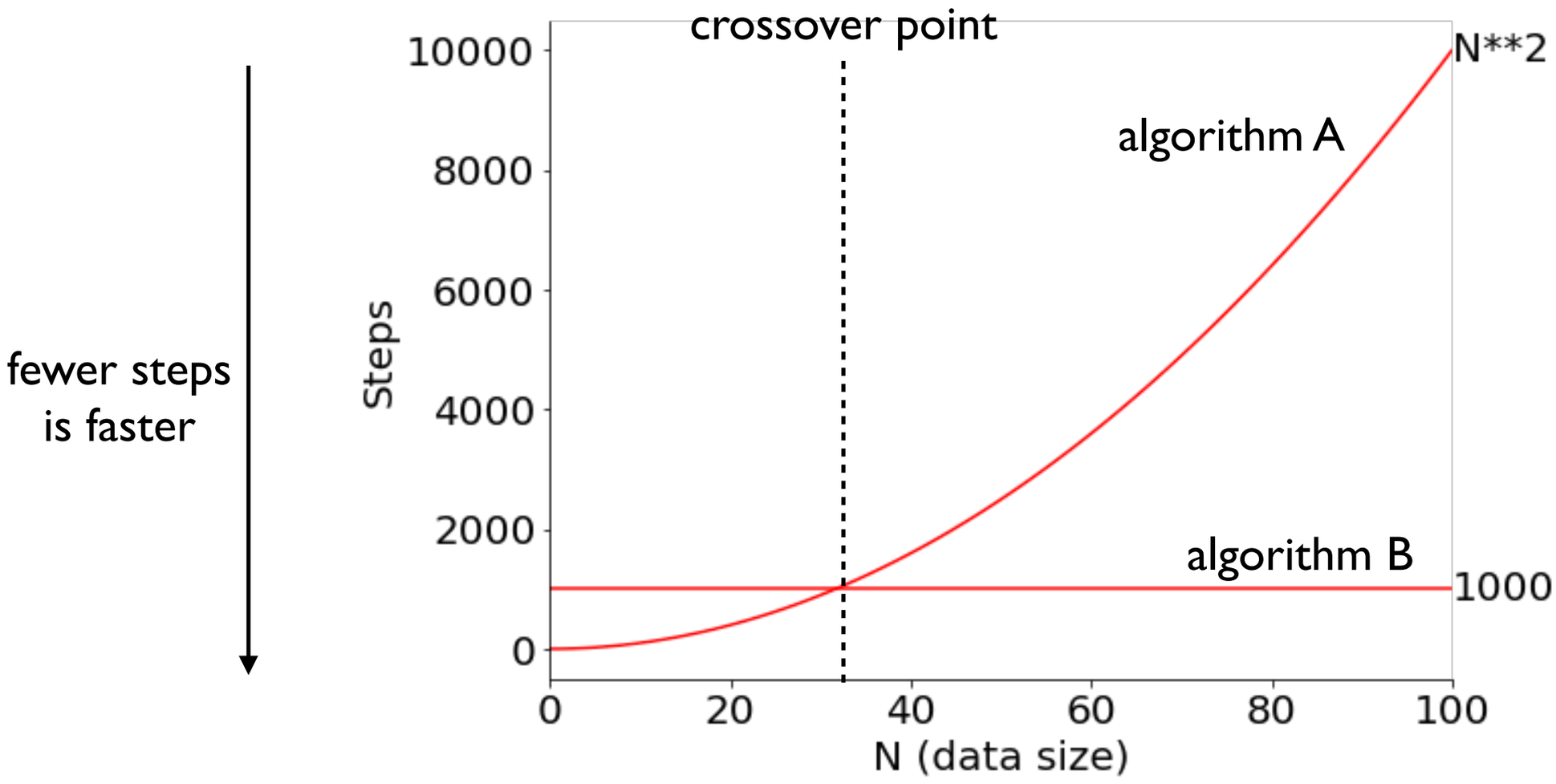
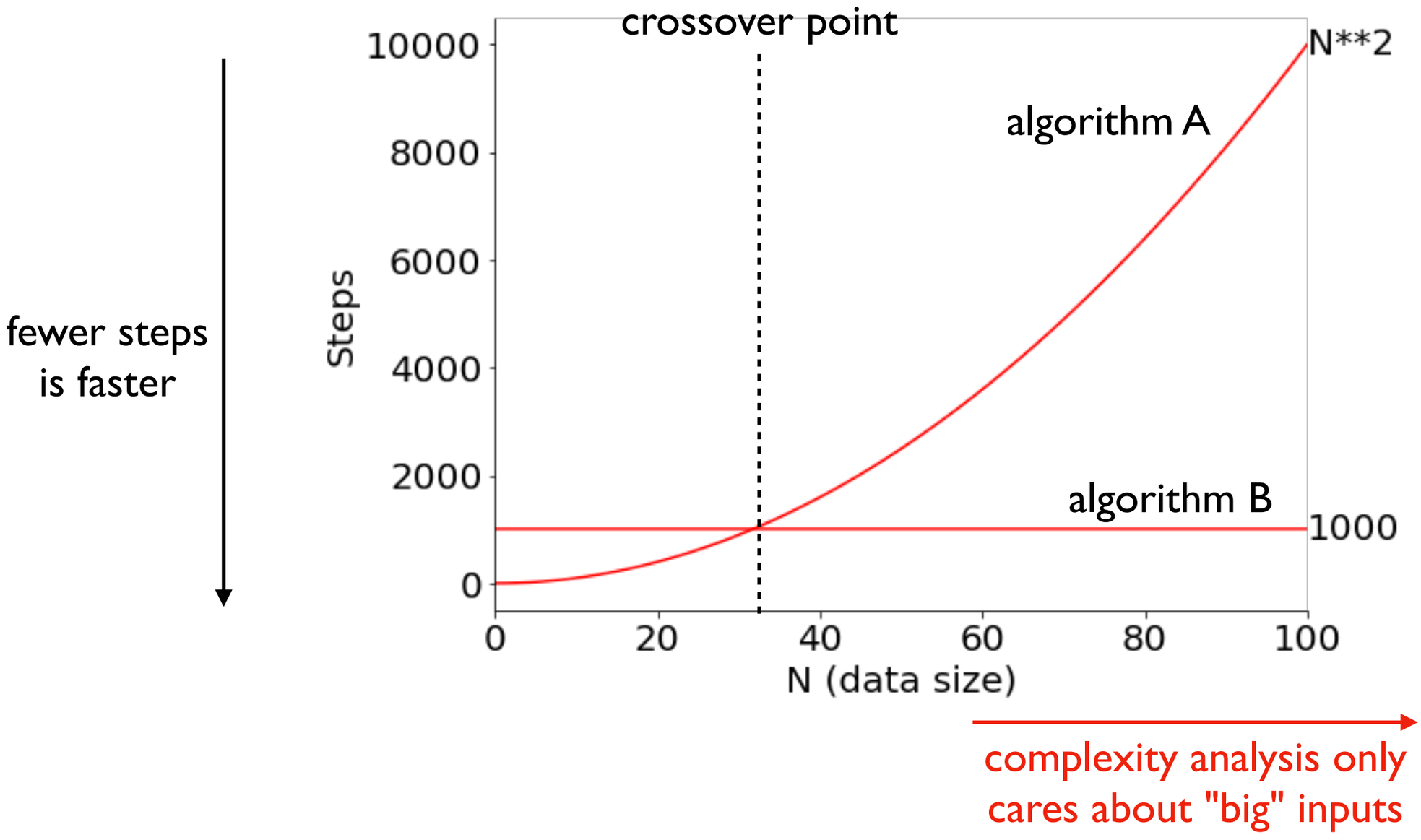


Which algorithm is better?

you might still reasonably
care about this portion!



What is the asymptotic behavior of the function?

Performance vs. Complexity

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Performance vs. Complexity

Things that affect performance (total time to run):

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- **algorithm:** strategy for solving the problem

- **input size:** how much data do we have?

what is this?

complexity analysis: how many **steps** must the algorithm perform, as a function of input size?

What is a "step"?

What is a step?

A **step** is any unit of work with bounded execution time
(it doesn't keep getting slower with growing input size)

input size is length of this list

```
input_nums = [2, 3, ...]
```

```
STEP  odd_count = 0
STEP  odd_sum = 0
STEP  for num in input_nums:
STEP      if num % 2 == 1:
STEP          odd_count += 1
STEP          odd_sum += num
STEP  odd_avg = odd_sum
STEP  odd_avg /= odd_count
```

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```
odd_count = 0
odd_sum = 0
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STEP
```
for num in input_nums:
```

STEP
```
    if num % 2 == 1:
```

STEP
```
        odd_count += 1
        odd_sum += num
```

STEP
```
odd_avg = odd_sum
odd_avg /= odd_count
```

also a valid breakdown into steps

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    if num % 2 == 1:
```

STEP
```
        odd_count += 1
        odd_sum += num
```

STEP
```
odd_avg = odd_sum / odd_count
```

One line can do a lot, so no reason to
have lines and steps be equivalent

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for num in input_nums:
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    if num % 2 == 1:
```

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```
        odd_count += 1
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```

STEP
```
odd_avg = odd_sum / odd_count
```

Sometimes a single line is not a single step:
`found = X in L`

What is a step?

A **step** is any unit of work with bounded execution time
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for num in input_nums:
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    if num % 2 == 1:
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```

STEP
```
odd_avg = odd_sum / odd_count
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What is a step?

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for num in input_nums:
    if num % 2 == 1:
        odd_count += 1
        odd_sum += num
```

STEP

```
odd_avg = odd_sum / odd_count
```

???

is this a valid way to identify steps?

What is a step?

A **step** is any unit of work with bounded execution time
(it doesn't keep getting slower with growing input size)

```
input_nums = [2, 3, ...]
```

STEP

```
odd_count = 0
odd_sum = 0
```

not a "step", because exec time depends on input size

STEP

```
for num in input_nums:
    if num % 2 == 1:
        odd_count += 1
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```

STEP

```
odd_avg = odd_sum / odd_count
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What is a step?

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```
for num in input_nums:
    if num % 2 == 1:
        odd_count += 1
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```

STEP

```
odd_avg = odd_sum / odd_count
```

Note! A loop that iterates a bounded number of times (not proportional to input size) COULD be a single step.

Counting Executed Steps

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A **step** is any unit of work with bounded execution time
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for num in input_nums:
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    if num % 2 == 1:
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```

STEP
```
odd_avg = odd_sum / odd_count
```

How many total steps will **execute** if
`len(input_nums) == 10`?

Counting Executed Steps

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input_nums = [2, 3, ...]

odd_count = 0
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odd_avg = odd_sum / odd_count
```

1 STEP

+ 11 STEP

+ 10 STEP

+ 1 STEP

= 23 steps

For **N** elements, there will be **2*N+3** steps

Counting Executed Steps

A **step** is any unit of work with bounded execution time
(it doesn't keep getting slower with growing input size)

```
input_nums = [2, 3, ...]
```

1	STEP	`odd_count = 0`
1	STEP	`odd_sum = 0`
11	STEP	`for num in input_nums:`
10	STEP	`    if num % 2 == 1:`
0 to 10	STEP	`        odd_count += 1`
0 to 10	STEP	`        odd_sum += num`
1	STEP	`odd_avg = odd_sum`
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Counting Executed Steps

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For **N** elements, there will be between
2*N+5 and **4*N+5** steps

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For **N** elements, there will be between
~~2*N+5~~ and **4*N+5** steps

usually we care about
the worst case

Counting Executed Steps

A **step** is any unit of work with bounded execution time
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2*N+3

answer 1

OR

4*N+5

answer 2

Counting Executed Steps

A **step** is any unit of work with bounded execution time
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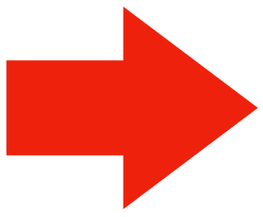
2*N+3 **OR** 4*N+5

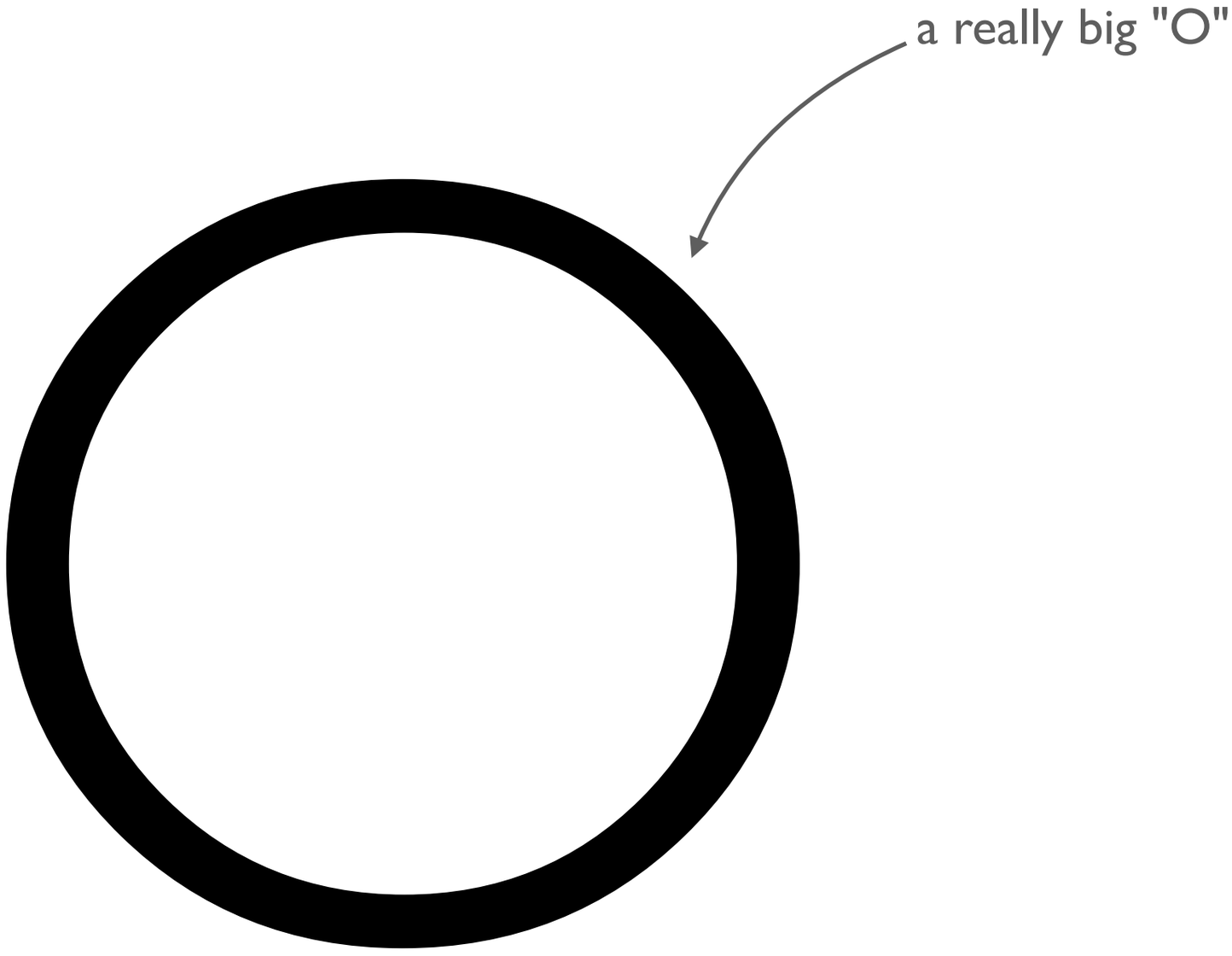
answer 1 answer 2

Answer 2 is never bigger than 2 times answer 1.
Answer 1 is never bigger than (1 times) answer 2.

Important: we might not identify steps the same, but our execution counts can at most differ by a constant factor!



can we broadly
(but rigorously)
categorize based on this?


a really big "O"

How fast?

Documentation

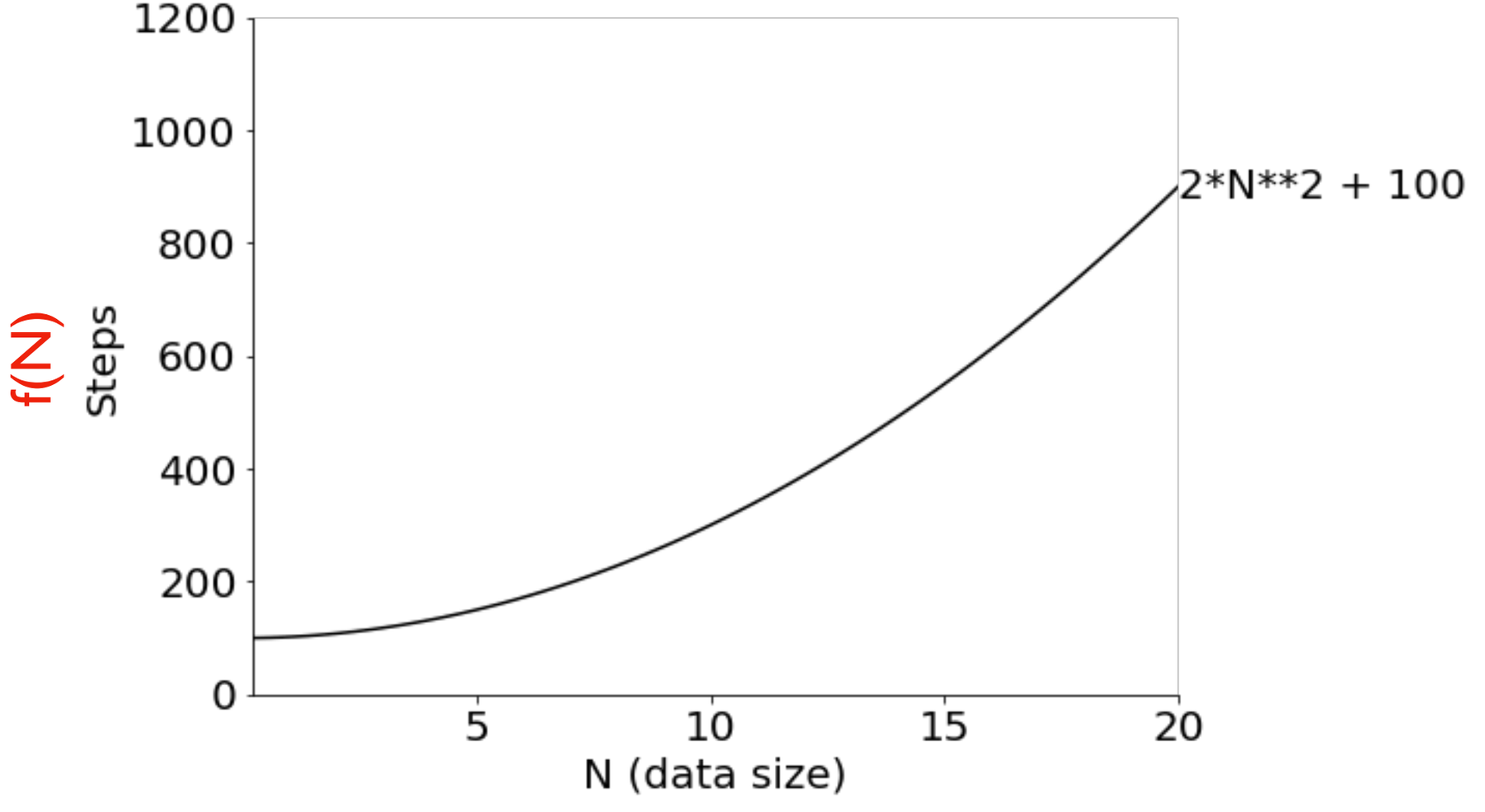
- https://scikit-learn.org/stable/modules/linear_model.html#ordinary-least-squares-complexity

- https://scikit-learn.org/stable/modules/tree.html#complexity

Big O Notation ("O" is for "order of growth")

Goal: categorize functions (and algorithms) by how fast they **grow**

- **do not care** about scale
- **do not care** about small inputs
- **care** about shape of the curve
- **strategy:** find some multiple of a general function is an upper bound



$f(N) == 2N^2 + 100$
is an $O(N^2)$ function

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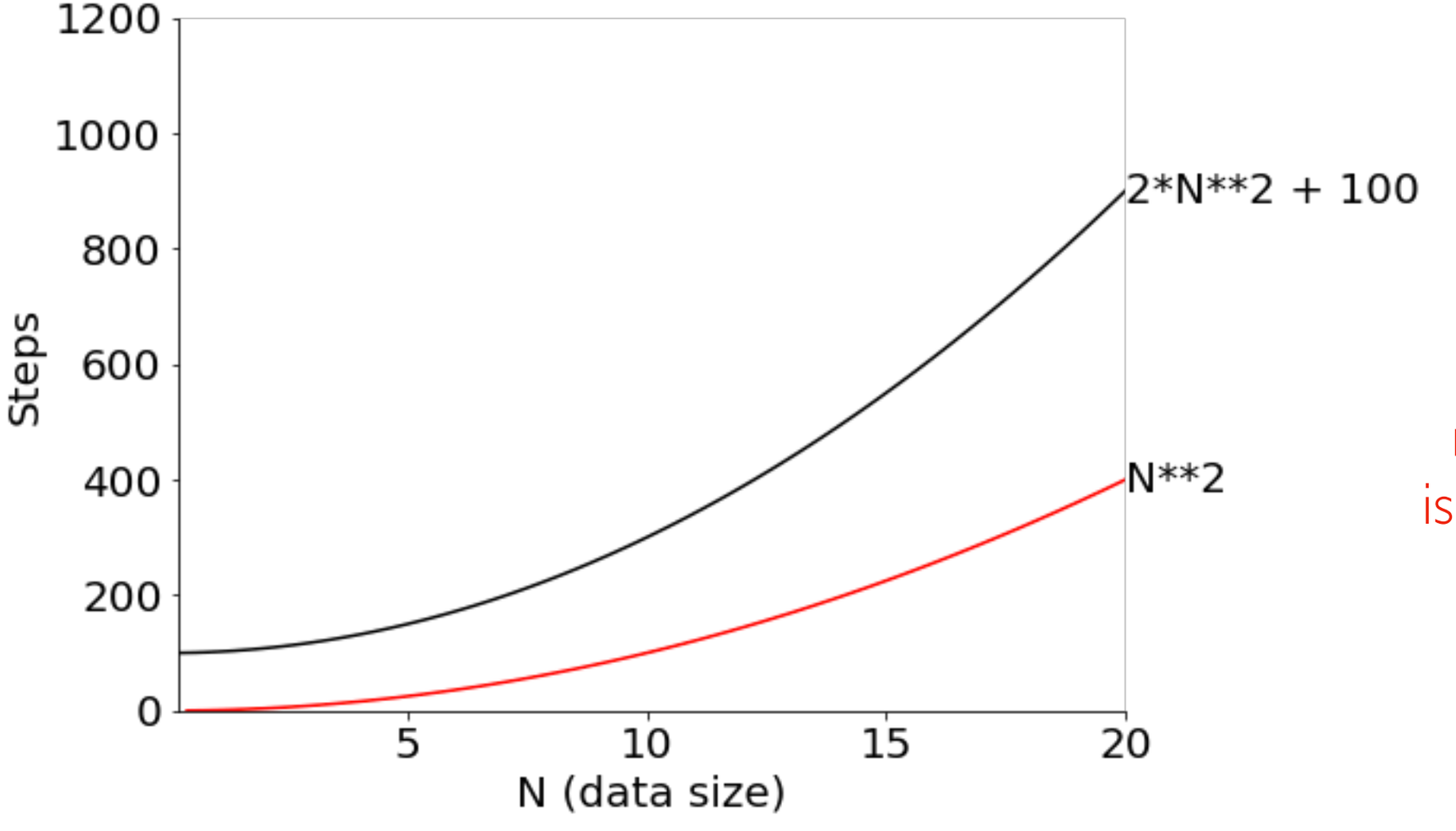

$f(N) == 2N^2 + 100$
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not because N^2
is an upper bound

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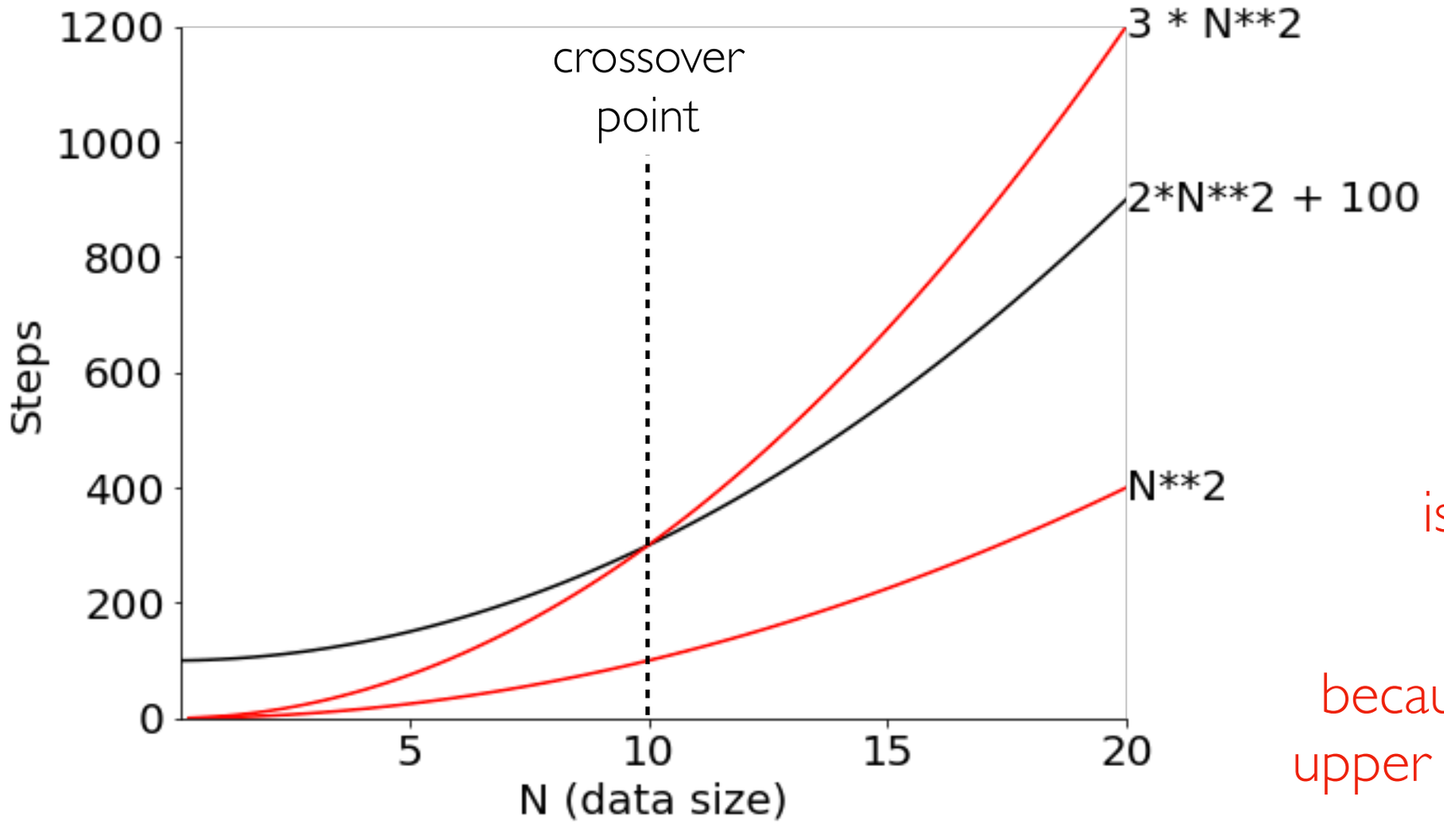

f(N) == 2N² + 100
is an **O(N²)** function

not because **N²**
is an upper bound

because some multiple is an upper bound after some point

Defining Big O

care about shape of the curve

do not care about small inputs

do not care about scale

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

Defining Big O

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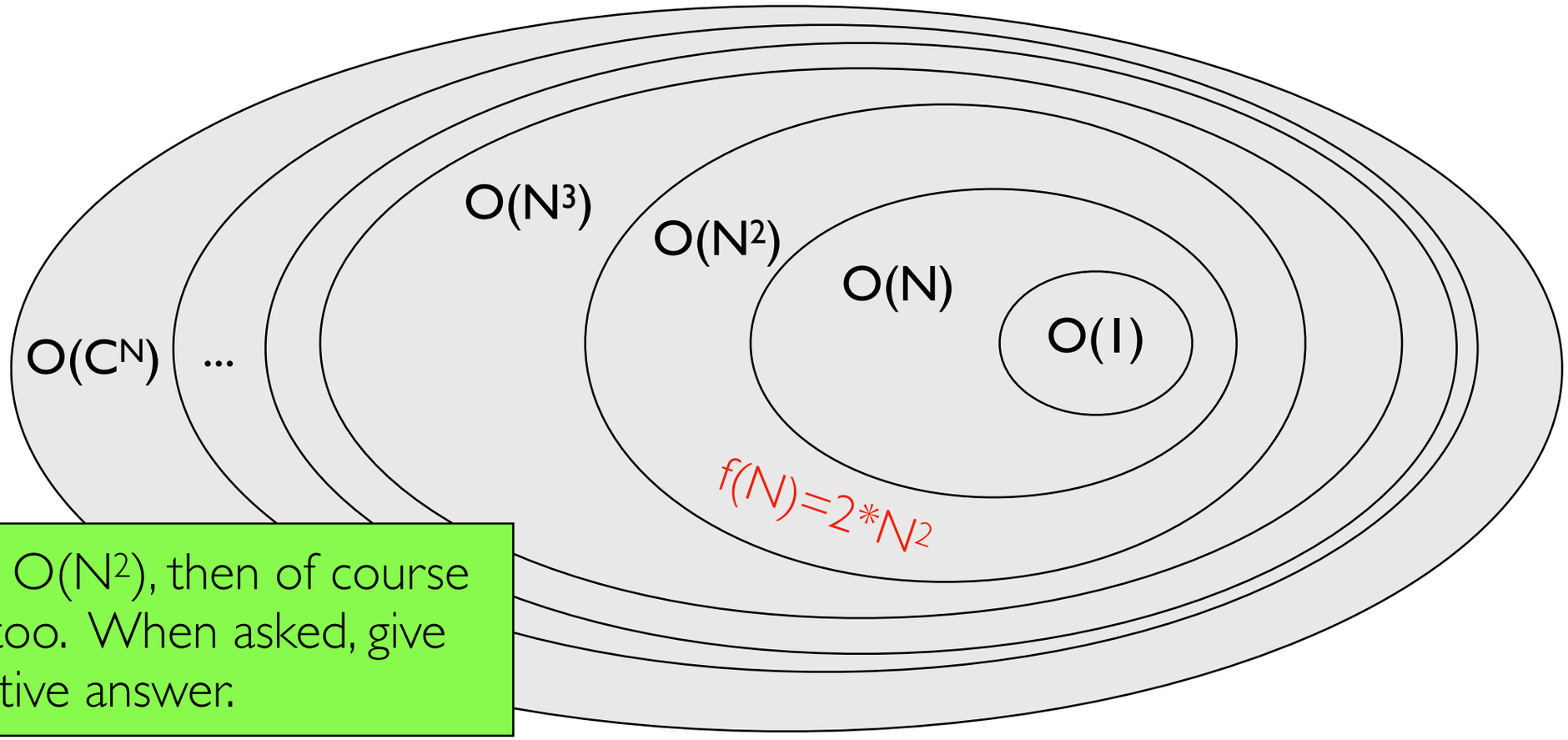
Sets



Note: if f(N) is in $O(N^2)$, then of course f(N) is in $O(N^3)$ too. When asked, give the most informative answer.

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

---

which ones
are true?

$f(N) = 2N \in O(N)$

$f(N) = 100N \in O(N^2)$

$f(N) = N^2 \in O(1000000N)$

Defining Big O

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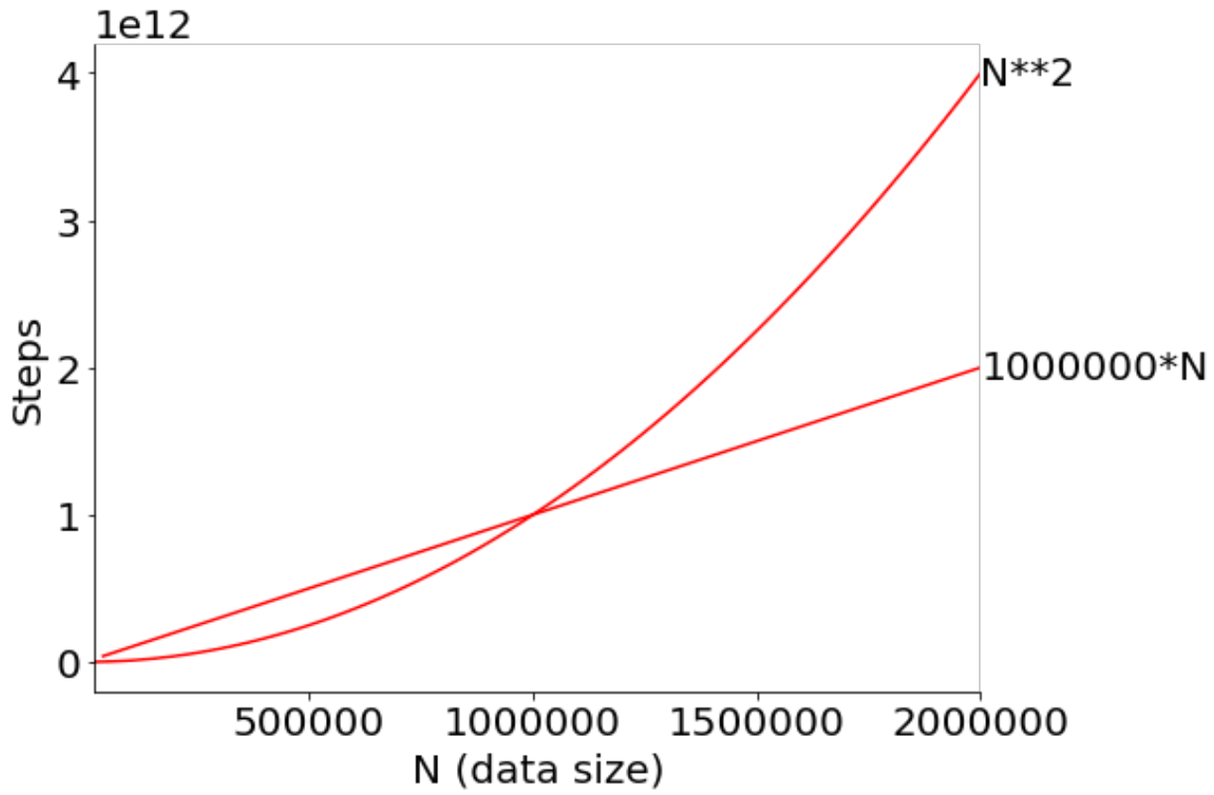

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Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

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---

which ones are true?

$N^2 \in O(N^2+N+1)$

$N^2+N+1 \in O(N^2)$

$N^5 \in O(N^4 + N^3 + N^2 + N)$

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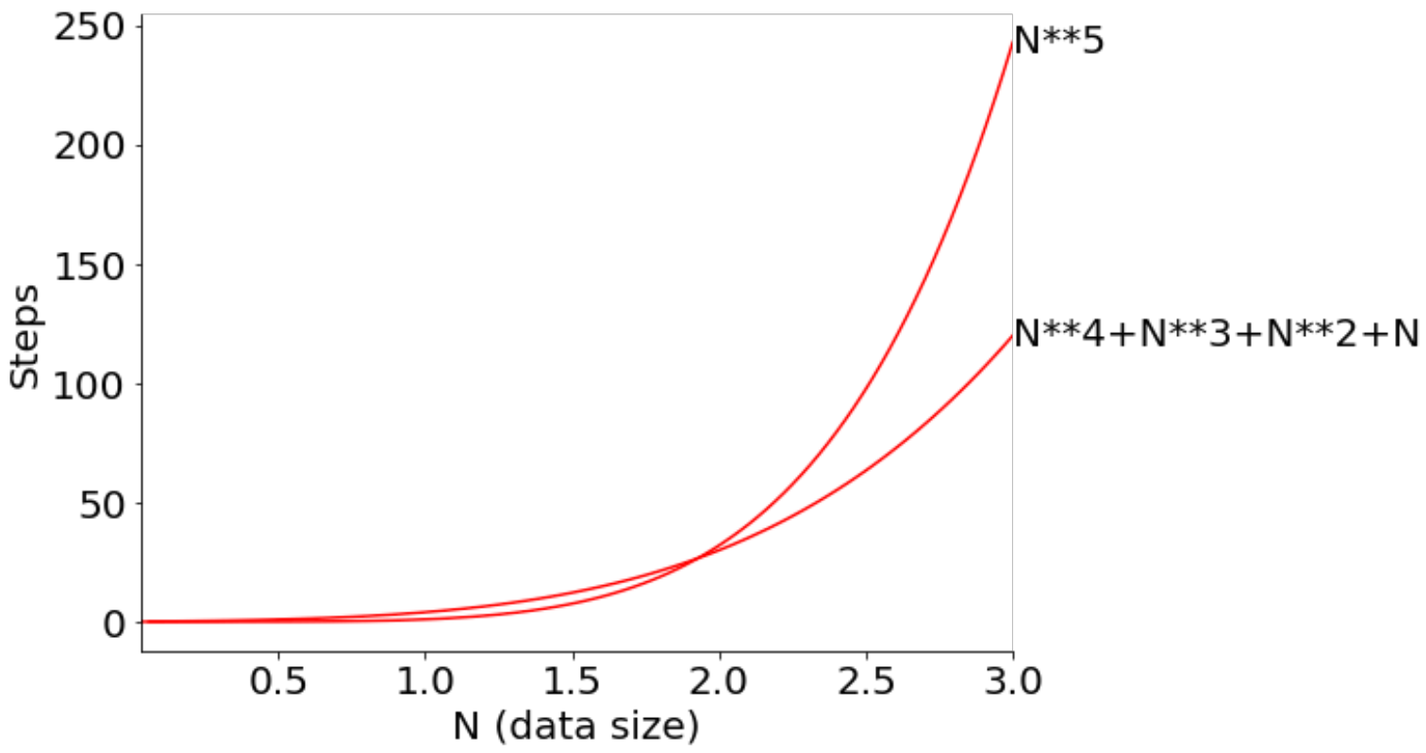

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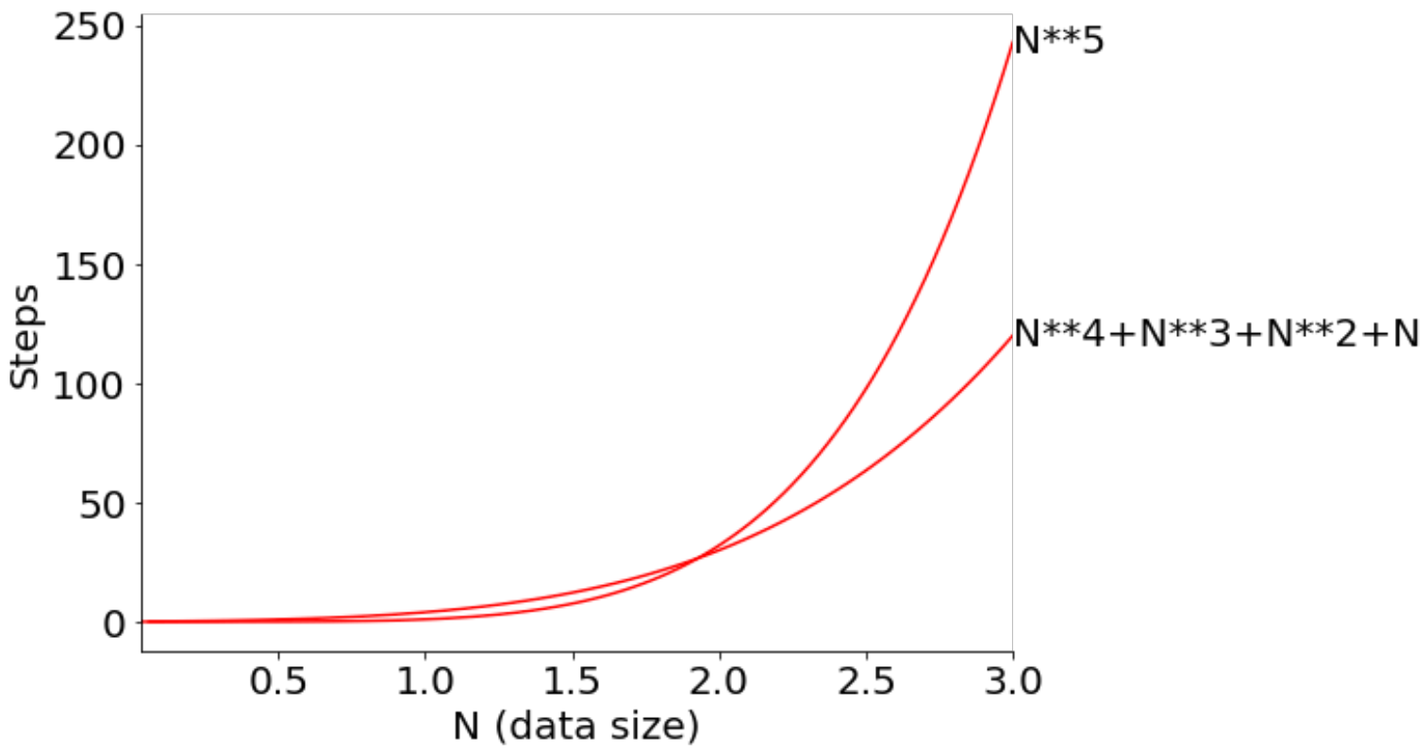

simplify when possible

$N^2 \in O(N^2+N+1)$

$N^2+N+1 \in O(N^2)$

~~$N^5 \in O(N^4 + N^3 + N^2 + N)$~~

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

We'll let **f(N)** be the number of steps that some **Algorithm A** needs to perform for input size **N**.

When we say **Algorithm A** $\in O(g(N))$,
we mean that $f(N) \in O(g(N))$

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

STEP
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odd_count = 0
odd_sum = 0
```
STEP
```
for num in input_nums:
```
STEP
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    if num % 2 == 1:
        odd_count += 1
        odd_sum += num
```
STEP
```
odd_avg = odd_sum / odd_count
```

For N elements, there will be 2*N+3 steps

$2*N+3 \leq 3 * N$

[for big N values]

therefore

this code is O(N)

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

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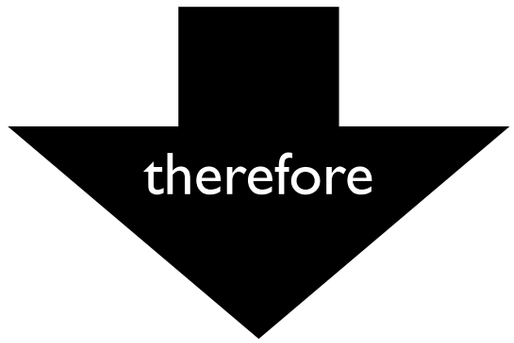
STEP  odd_avg = odd_sum
STEP  odd_avg /= odd_count
```

For N elements, there will be between 2*N+5 and 4*N+5 steps

$4*N+5 \leq 5 * N$

[for big N values]



this code is O(N)

Examples

Coding/Plotting Example

```
def is_prime(N):
    prime = True
    for factor in range(2, N):
        steps += 1
        if N % factor == 0:
            prime = False
    return prime
```

what is the complexity of each function

```
def find_primes(cap):
    primes = []
    for i in range(cap+1):
        if is_prime(i):
            primes.append(i)
    return primes
```

Coding/Plotting Example

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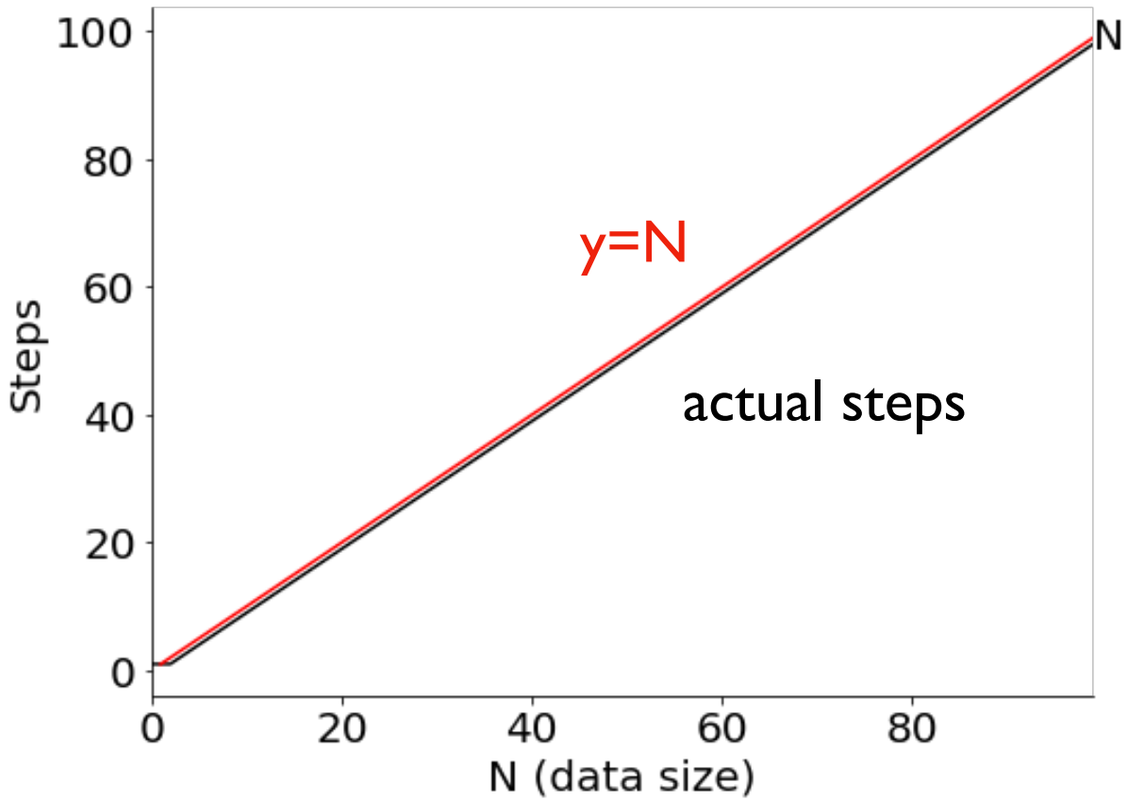

Coding/Plotting Example

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def is_prime(N):
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        if N % factor == 0:
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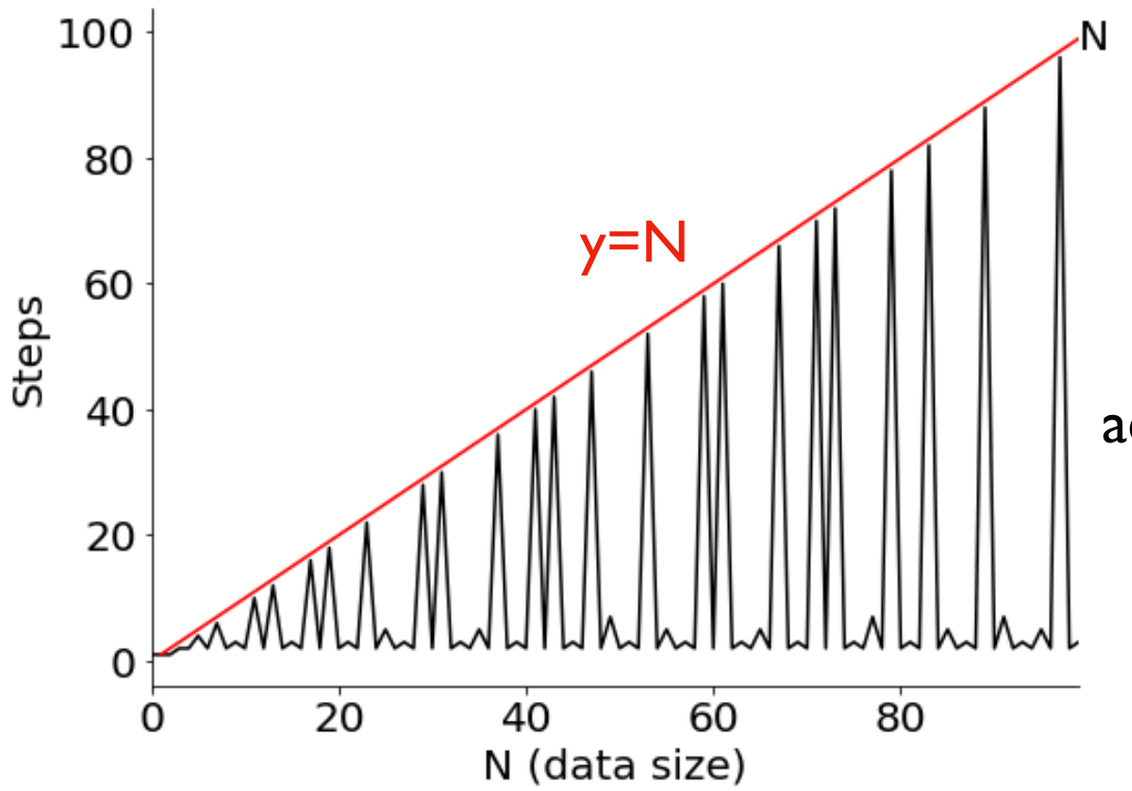
    return prime
```

what if we add a break here?

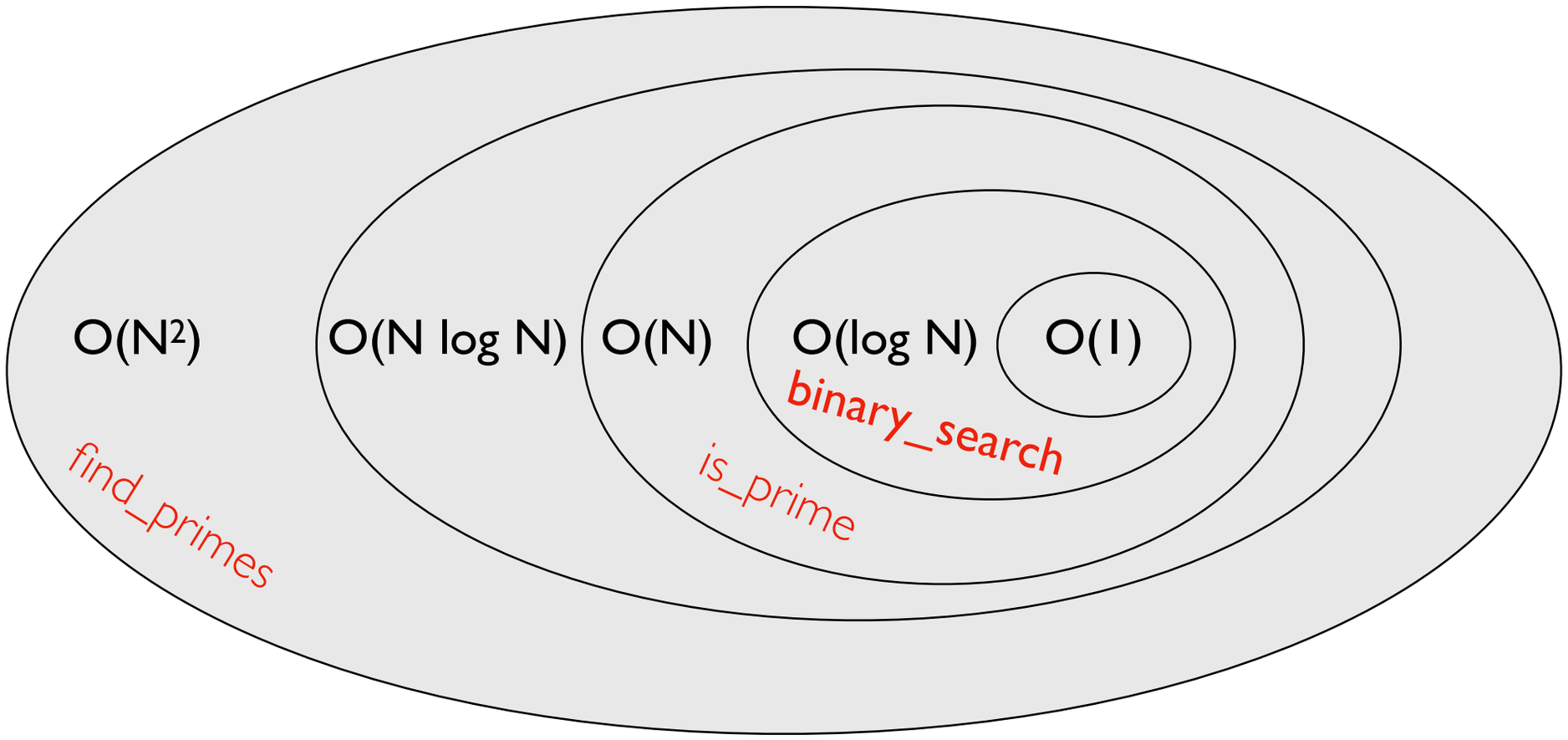


actual steps

for simplicity, we'll usually do a worst-case analysis, under which this would still be O(N)

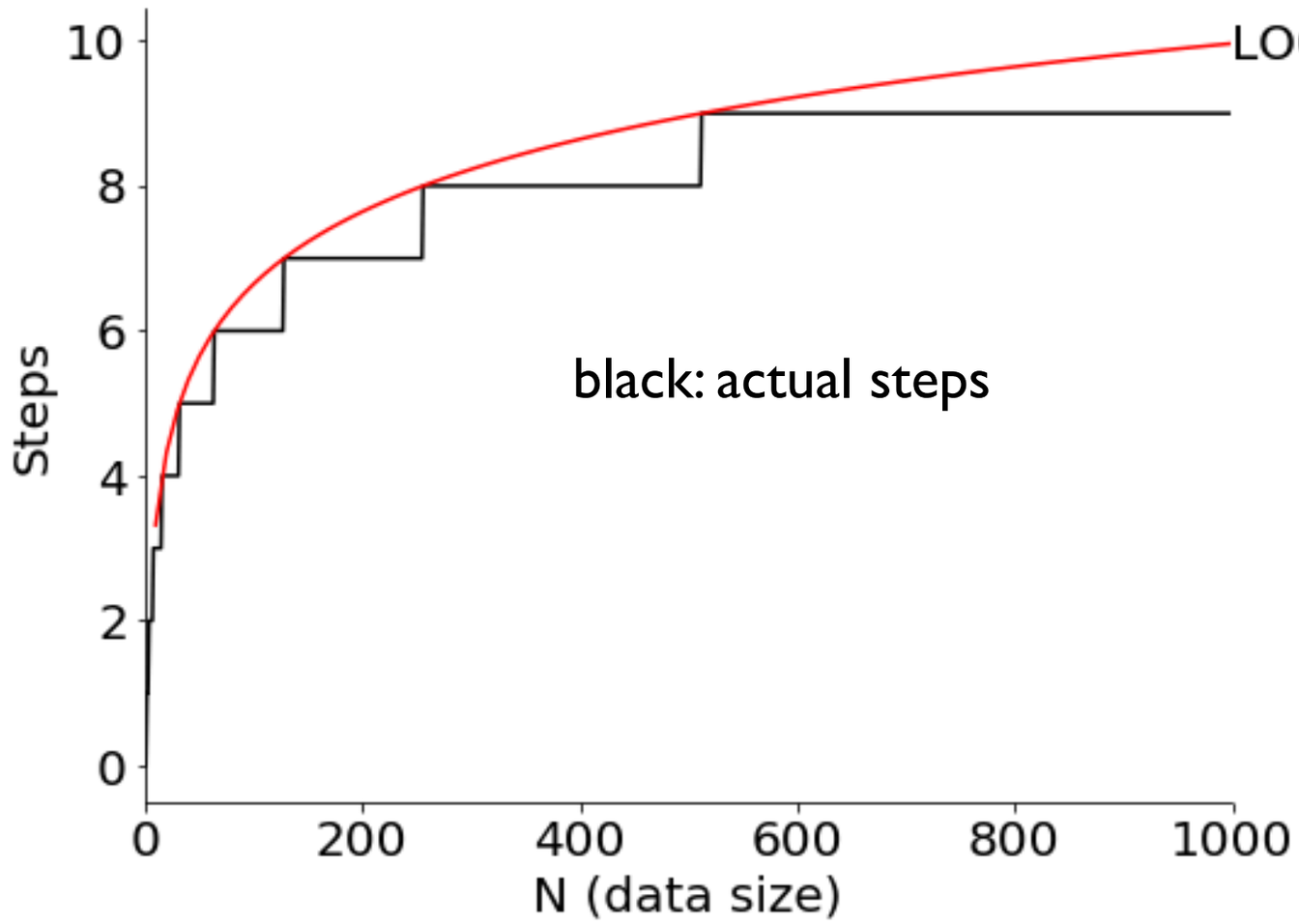
implications for `X in L`?

Binary Search: Coding Example

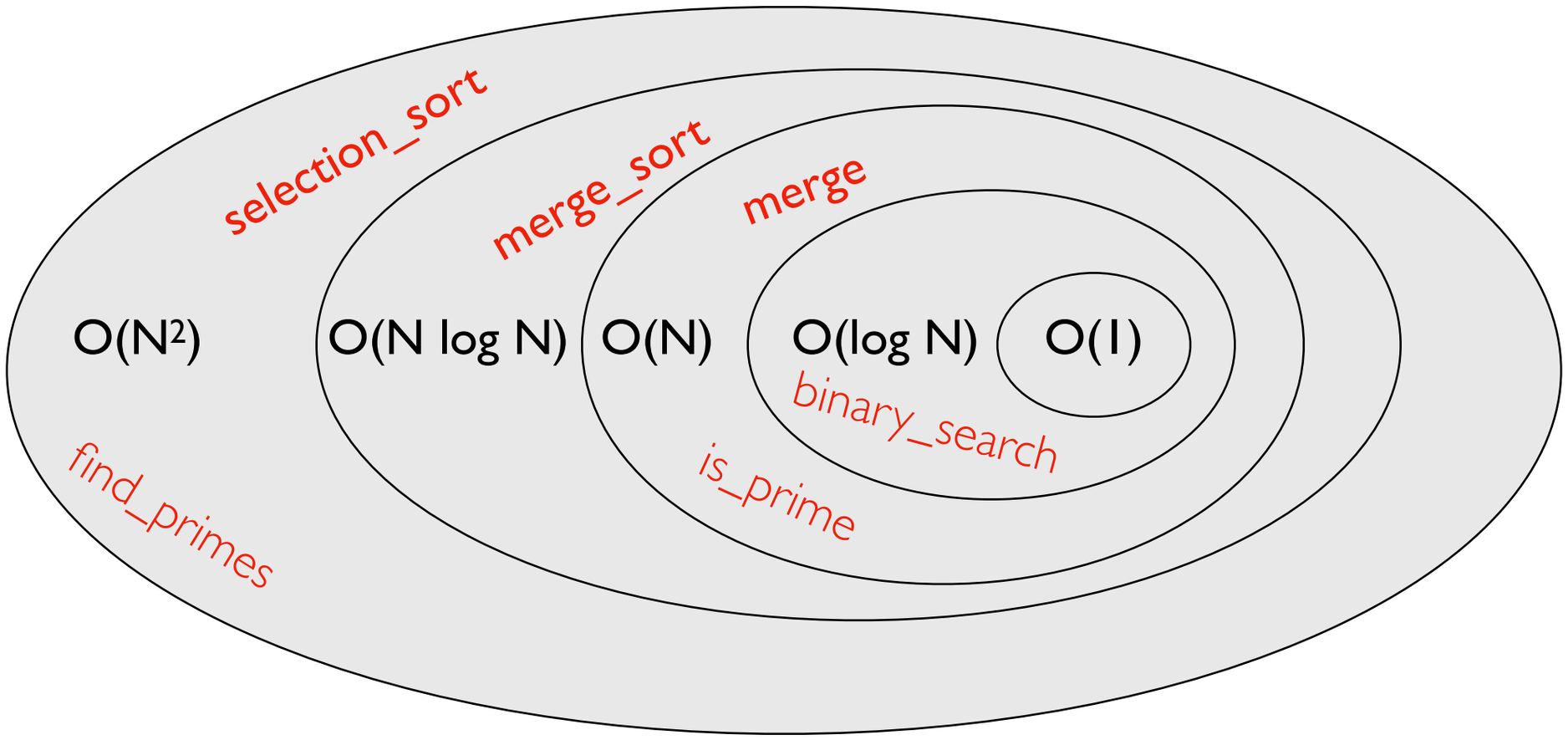


Binary Search

Binary Search: Coding Example



Sorting: Coding Examples



Analysis of Algorithms: Key Ideas

complexity: relationship between input size and steps executed

step: an operation of bounded cost (doesn't scale with input size)

asymptotic analysis: we only care about very large N values for complexity (for example, assume a big list)

worst-case: we'll usually assume the worst arrangement of data because it's harder to do an average case analysis (for example, assume search target at the end of a list)

big O: if $\mathbf{f(N) \leq C * g(N)}$ for large N values and some fixed constant **C**, then $\mathbf{f(N) \in O(g(N))}$