[320] Complexity + Big O

Tyler Caraza-Harter

Outline

Performance and Complexity

What is a step?

Counting Executed Steps

Big O: for functions/curves

Big O: for algorithms

Performance vs. Complexity

Things that affect performance (total time to run):

- ????

Performance vs. Complexity

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- speed of the computer (CPU, etc)
- speed of Python (quality+efficiency of interpretation)
- **algorithm**: strategy for solving the problem
- **input size:** how much data do we have?

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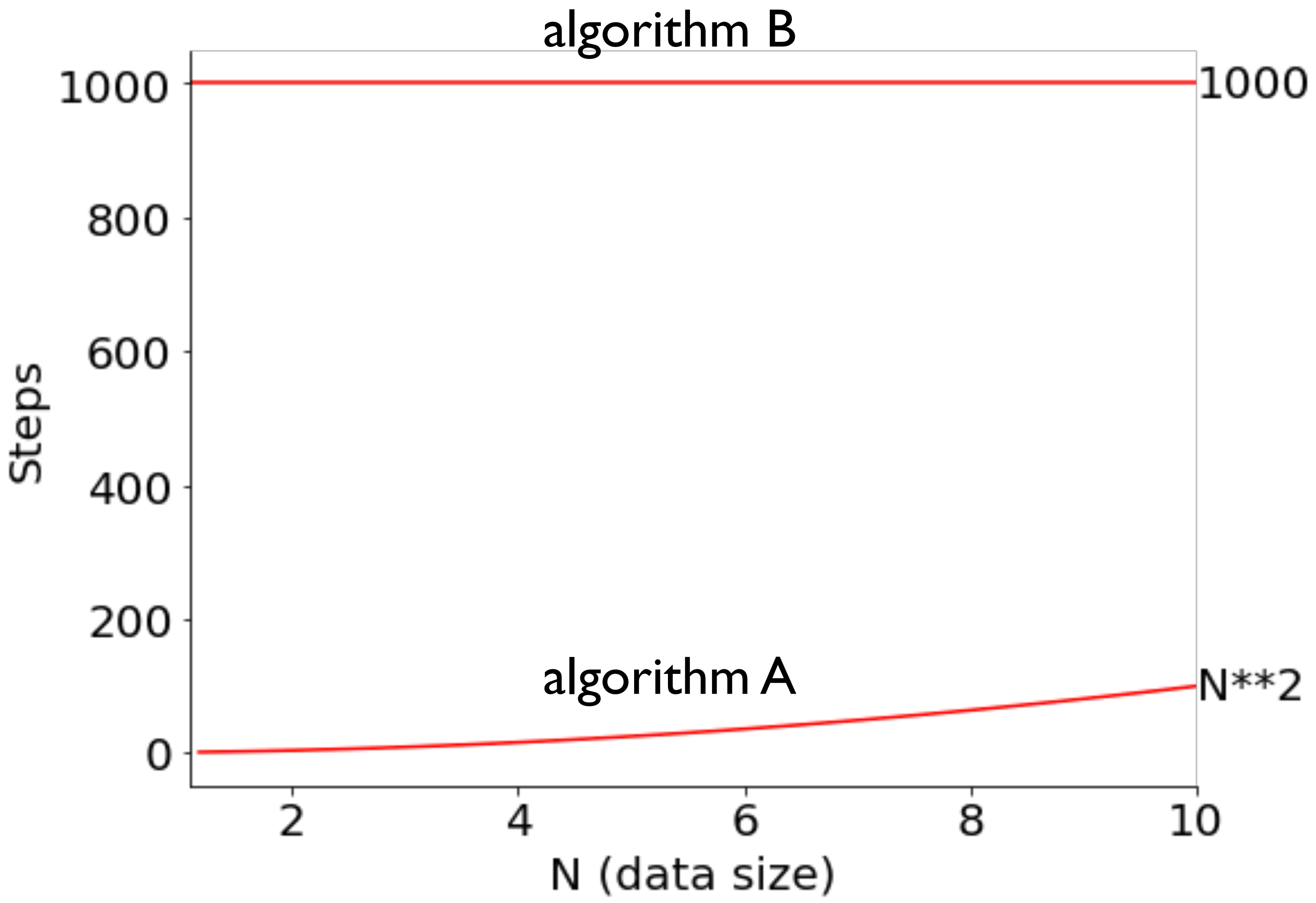
complexity analysis: how many steps must the algorithm perform, as a function of input size?

Which algorithm is better?

fewer steps is faster



Do you prefer A or B?

Which algorithm is better?

fewer steps
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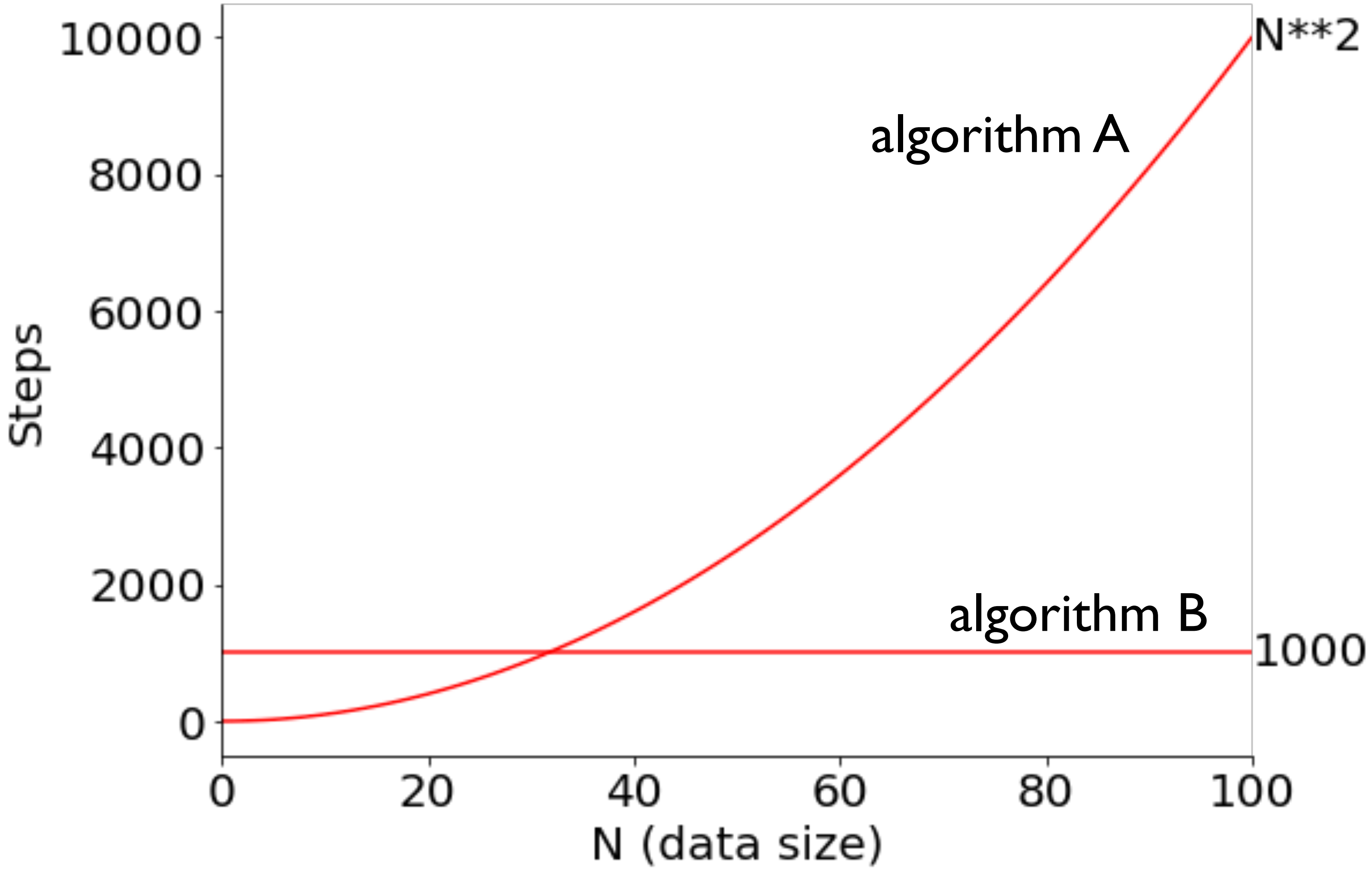

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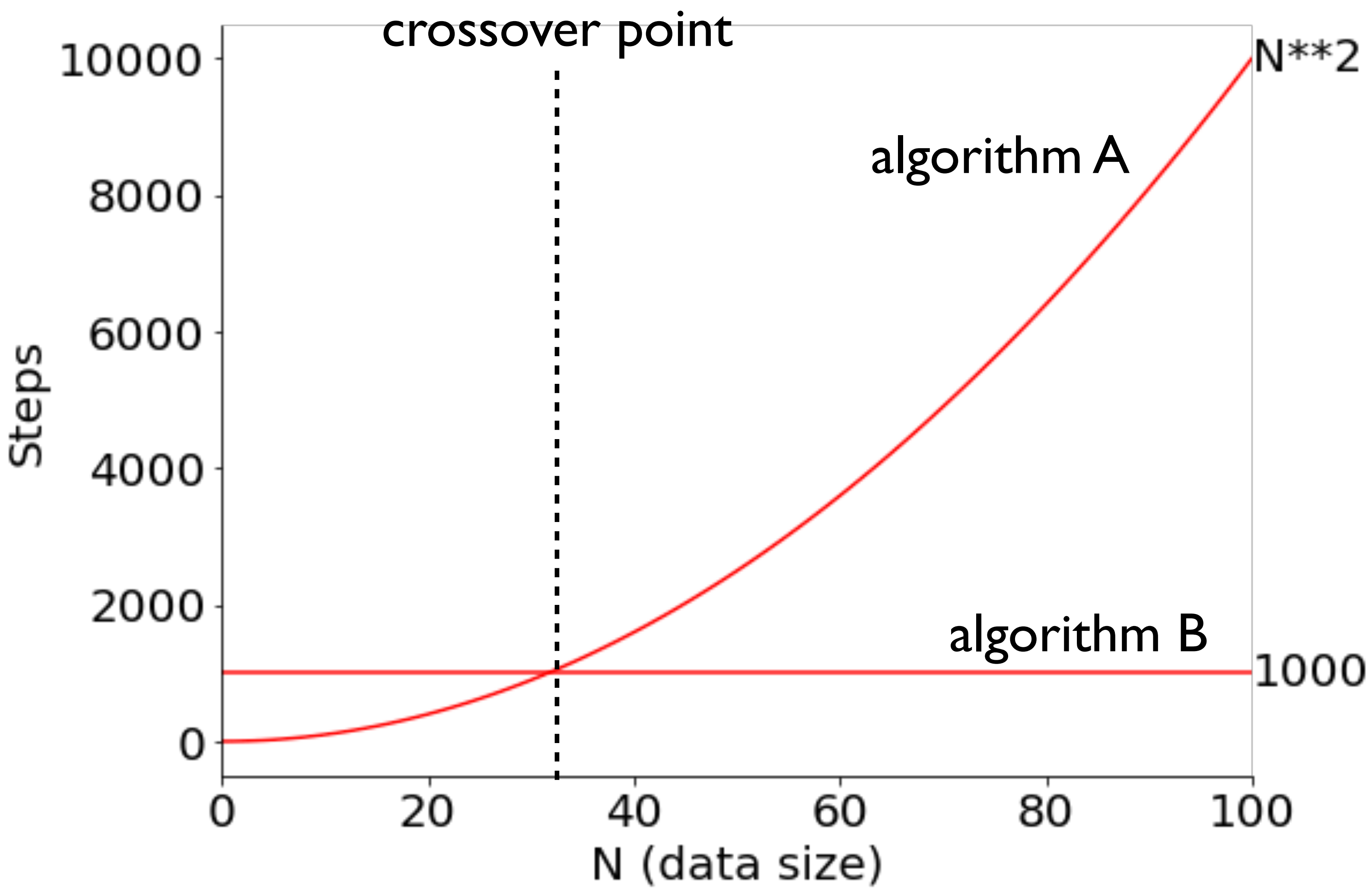
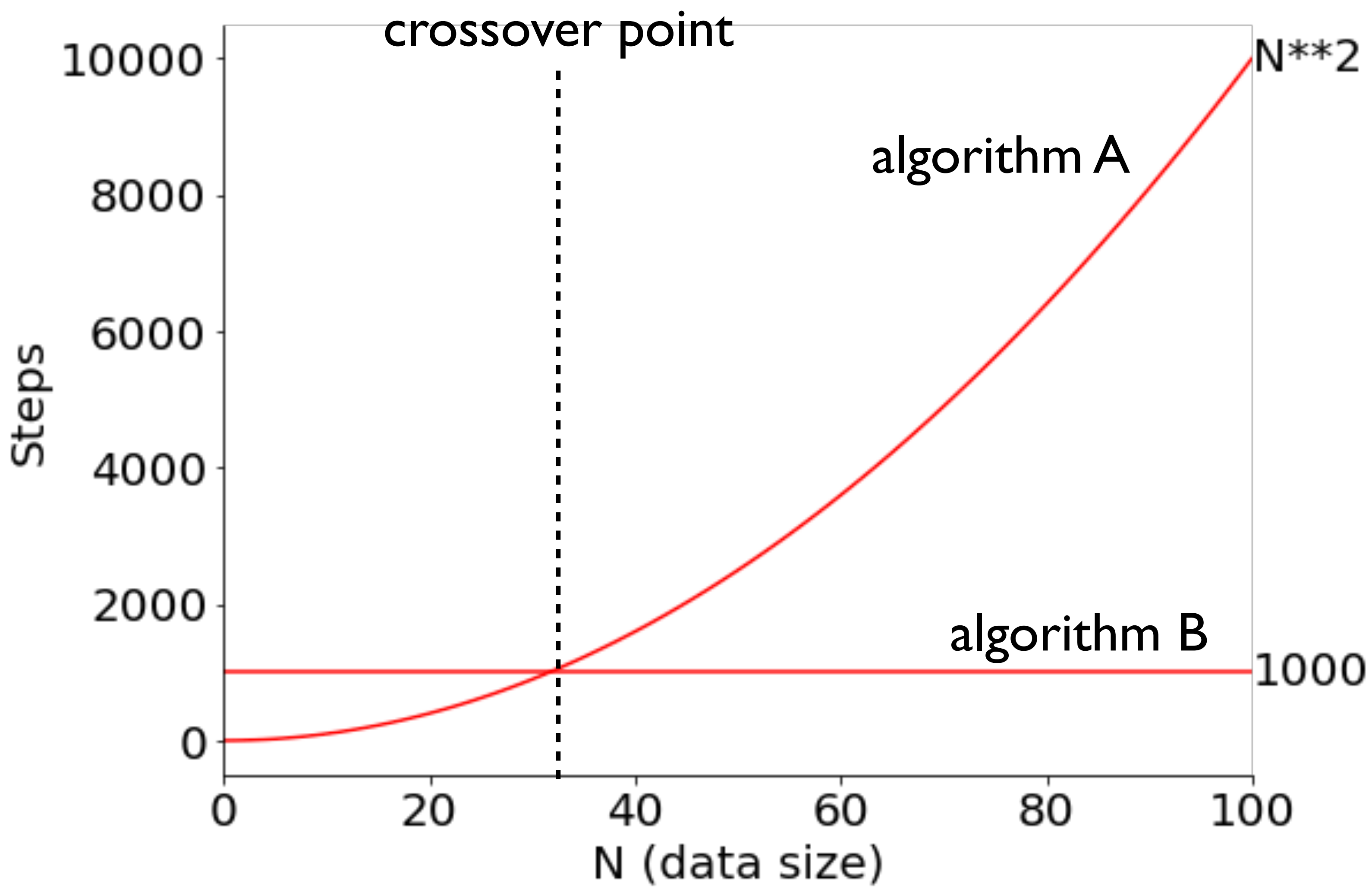


Which algorithm is better?

you might still reasonably care about this portion!



fewer steps is faster

complexity analysis only cares about "big" inputs

What is the asymptotic behavior of the function?

Performance vs. Complexity

Things that affect performance (total time to run):

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- speed of Python (quality+efficiency of interpretation)
- **algorithm**: strategy for solving the problem
- **input size:** how much data do we have?

what is this?

complexity analysis: how many **steps** must the algorithm perform, as a function of input size?

Outline

Performance and Complexity

What is a step?

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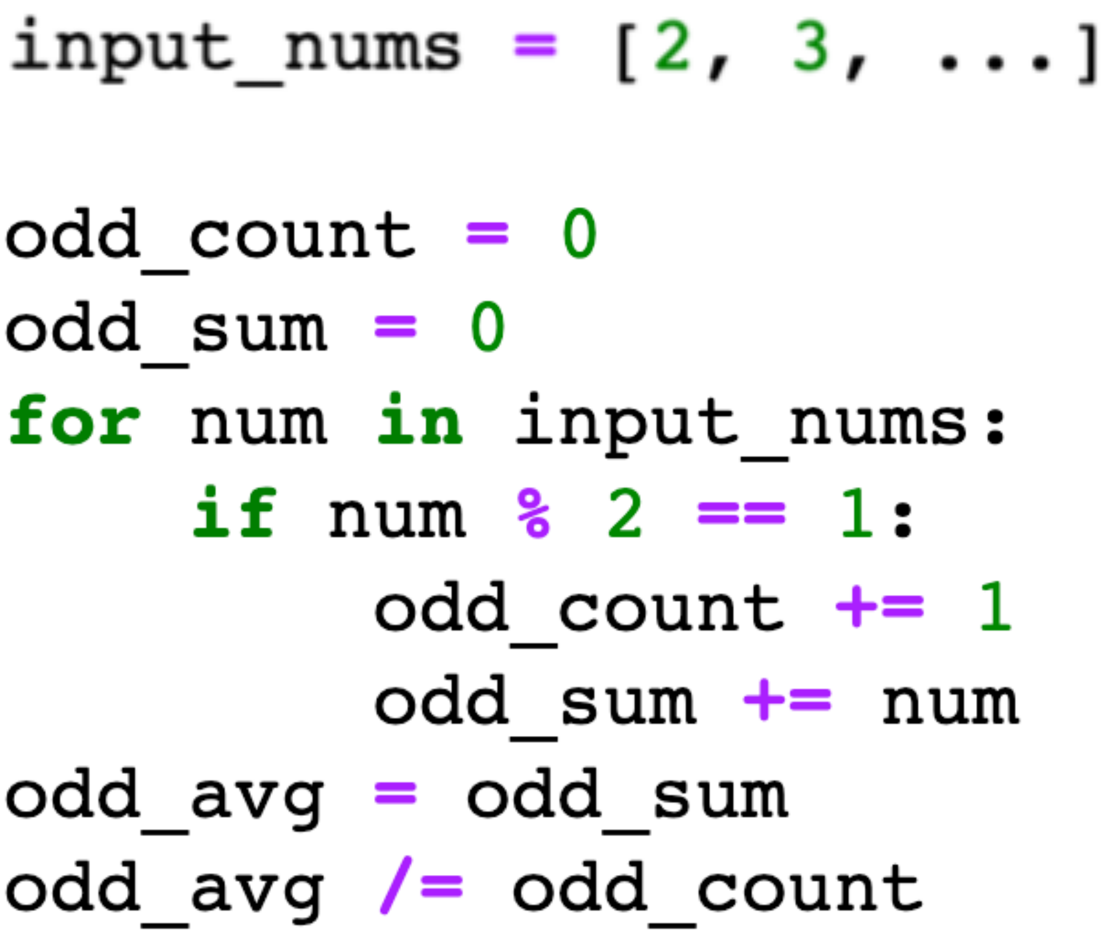
Big O: for algorithms

What is a step?

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size)

input size is length of this list



```
     input_nums = [2, 3, ...]

STEP odd_count = 0
STEP odd_sum = 0
STEP for num in input_nums:
STEP     if num % 2 == 1:
STEP         odd_count += 1
STEP         odd_sum += num
STEP odd_avg = odd_sum
STEP odd_avg /= odd_count
```

What is a step? 🐾

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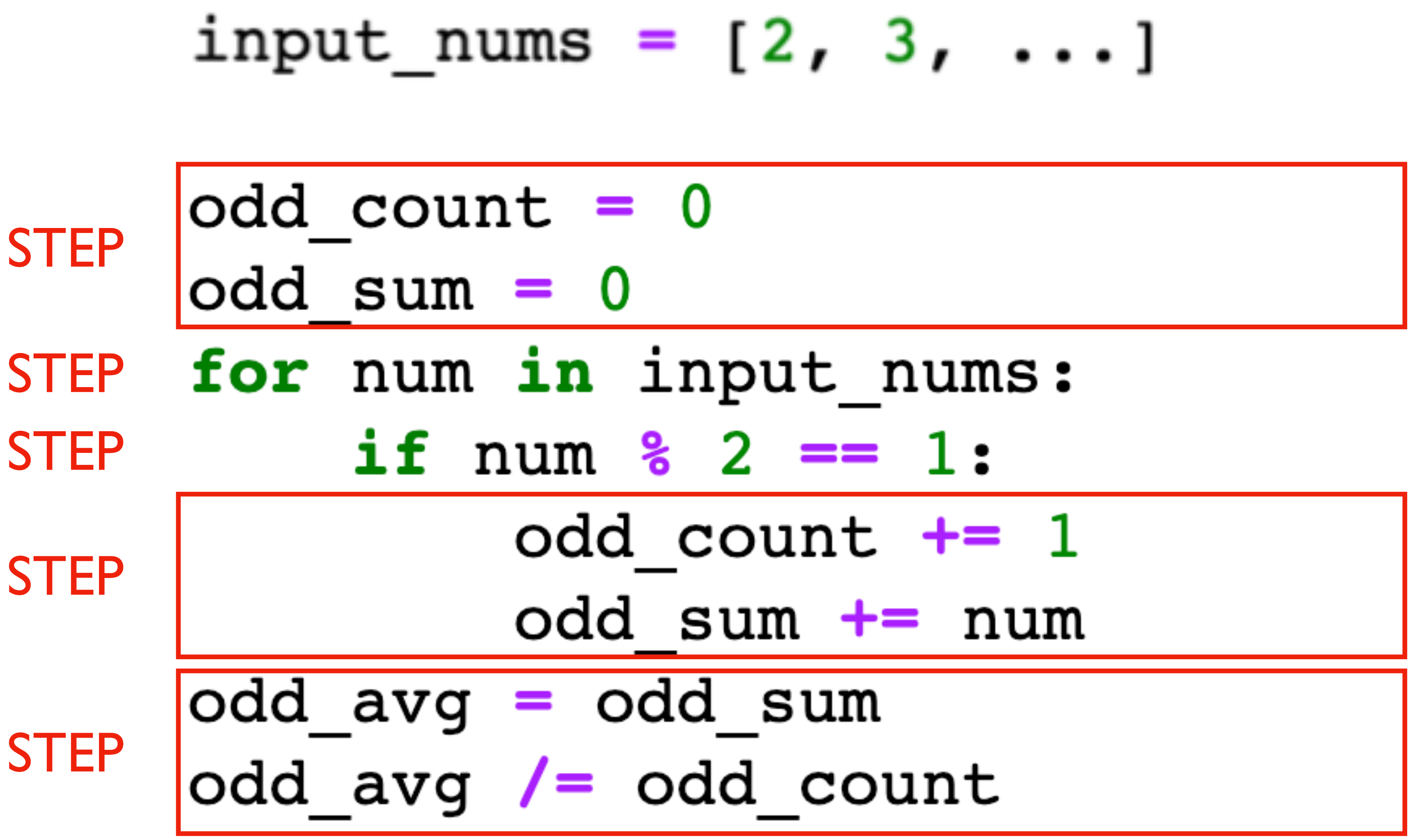

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STEP            odd_count += 1
                odd_sum += num
STEP    odd_avg = odd_sum
        odd_avg /= odd_count
```

✓

also a valid
breakdown
into steps

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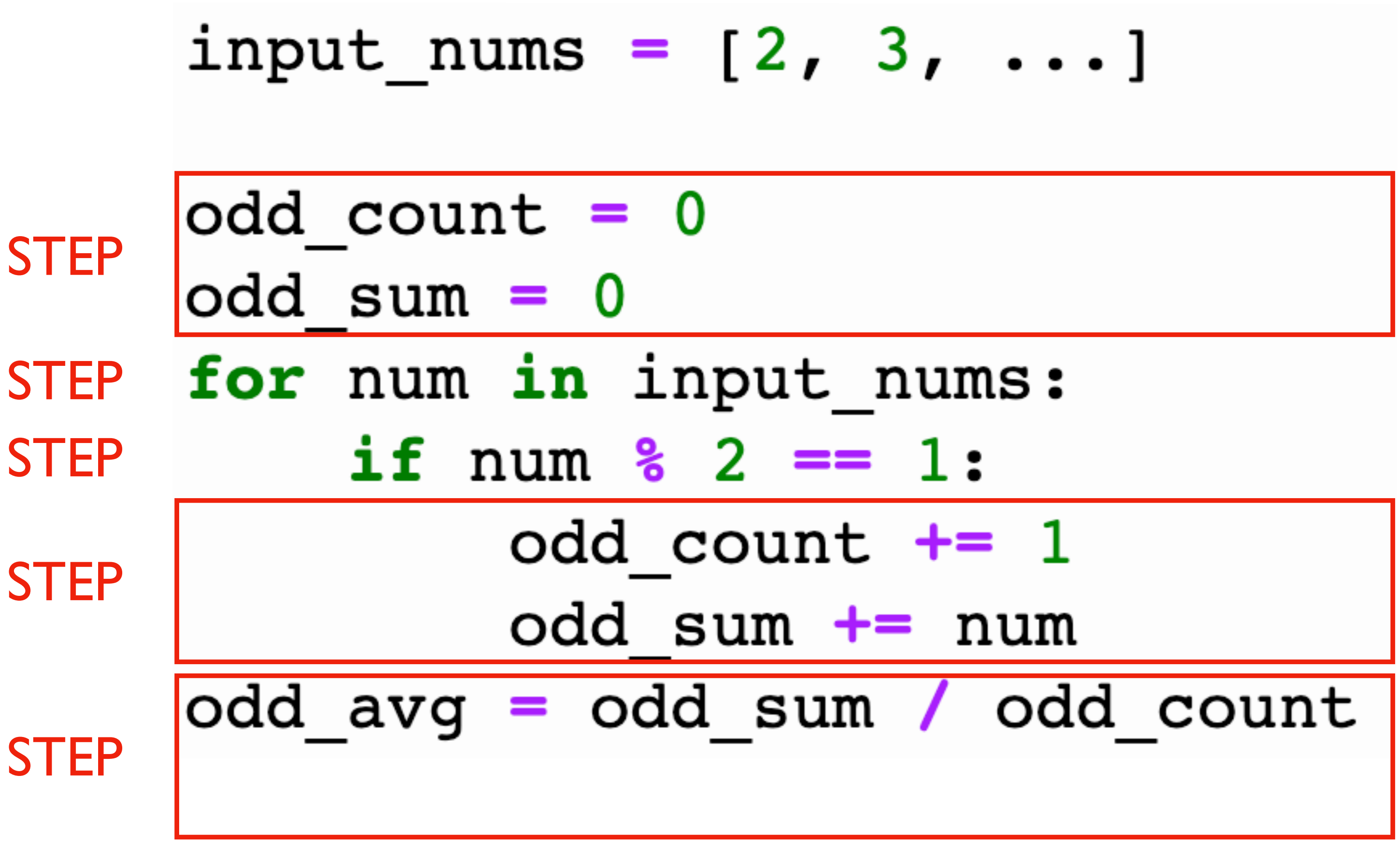

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```

One line can do a lot, so no reason to have lines and steps be equivalent

What is a step?

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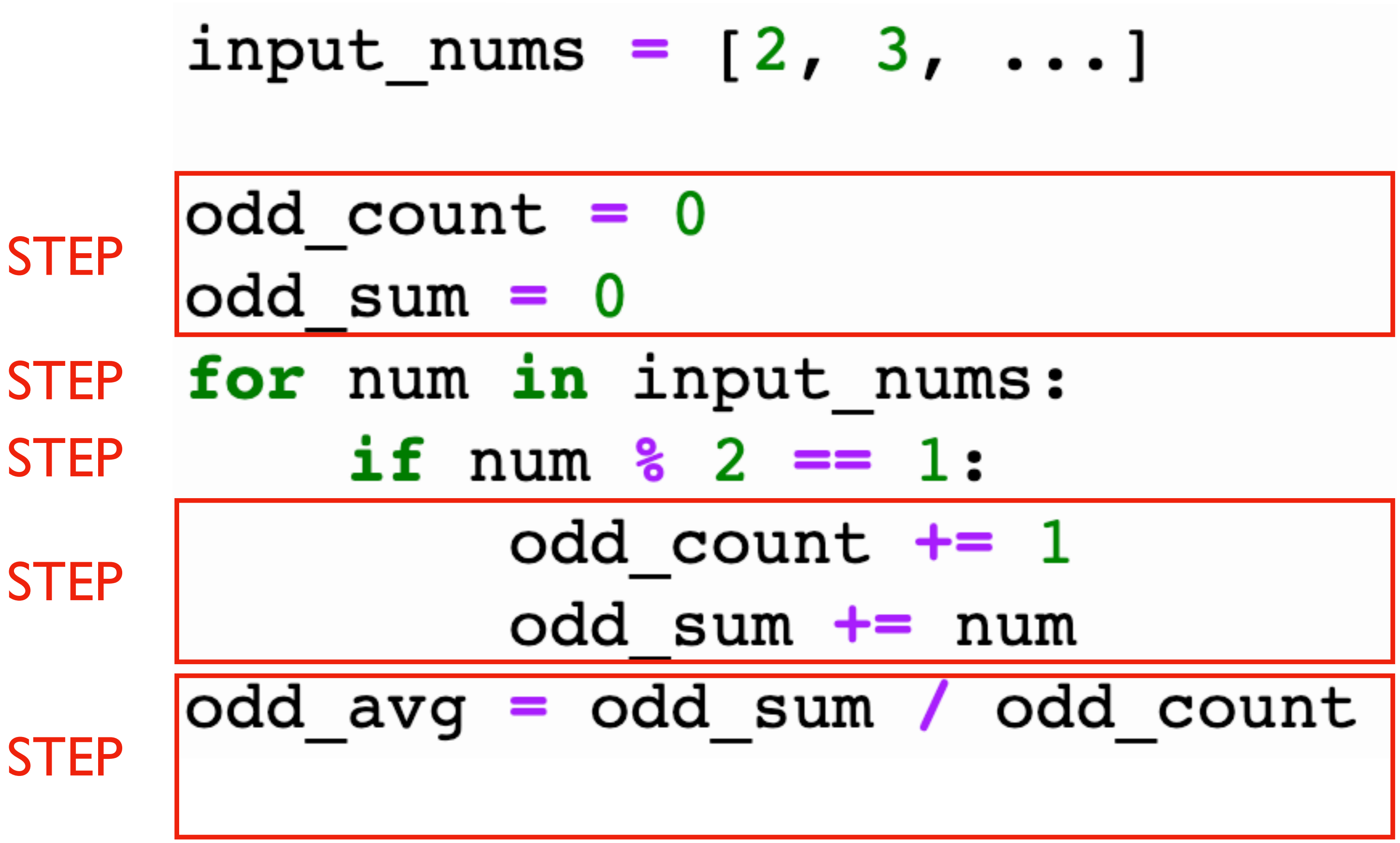

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```

Sometimes a single line is not a single step:
`found = X in L`

What is a step?

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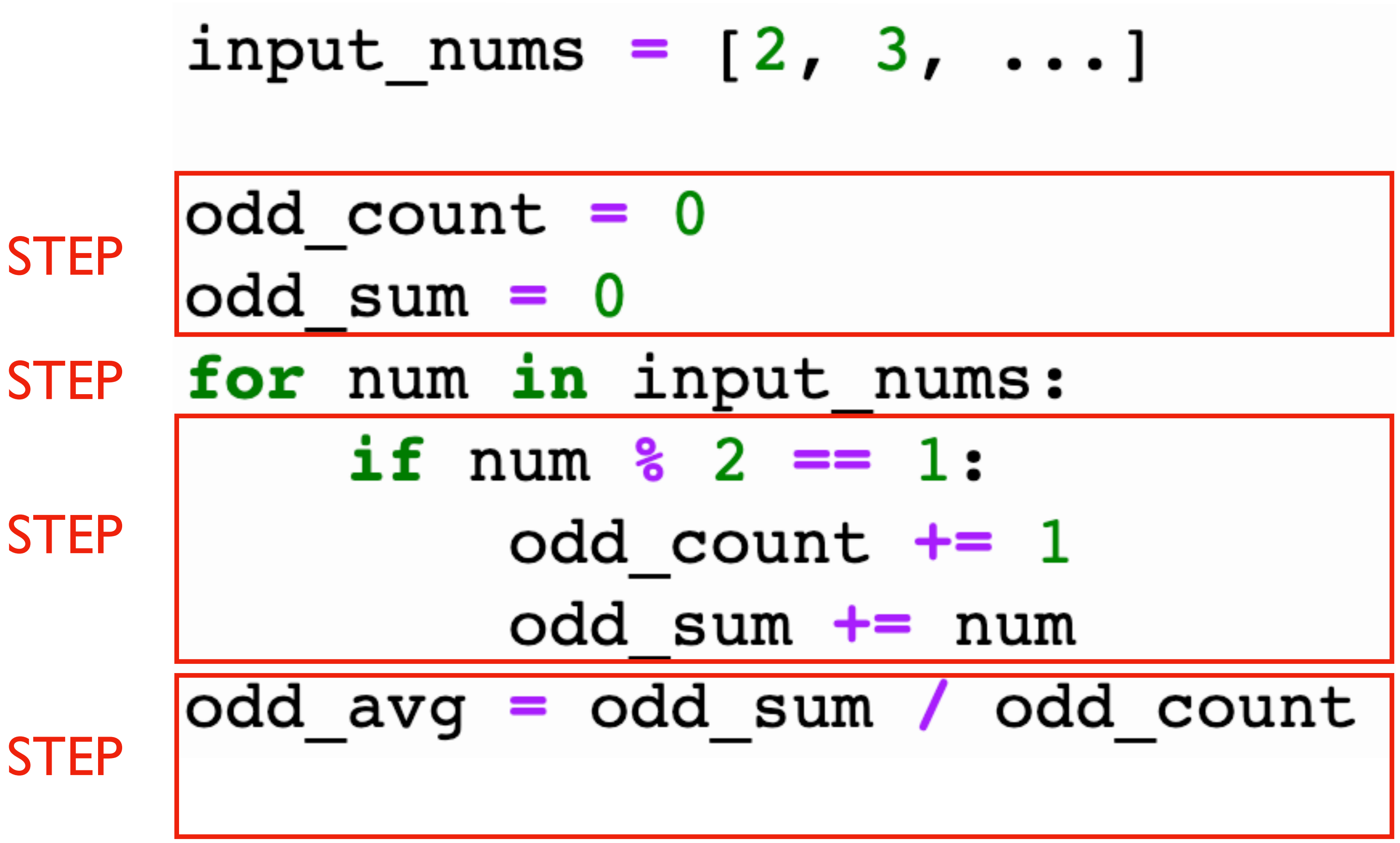

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```

STEP

STEP

STEP

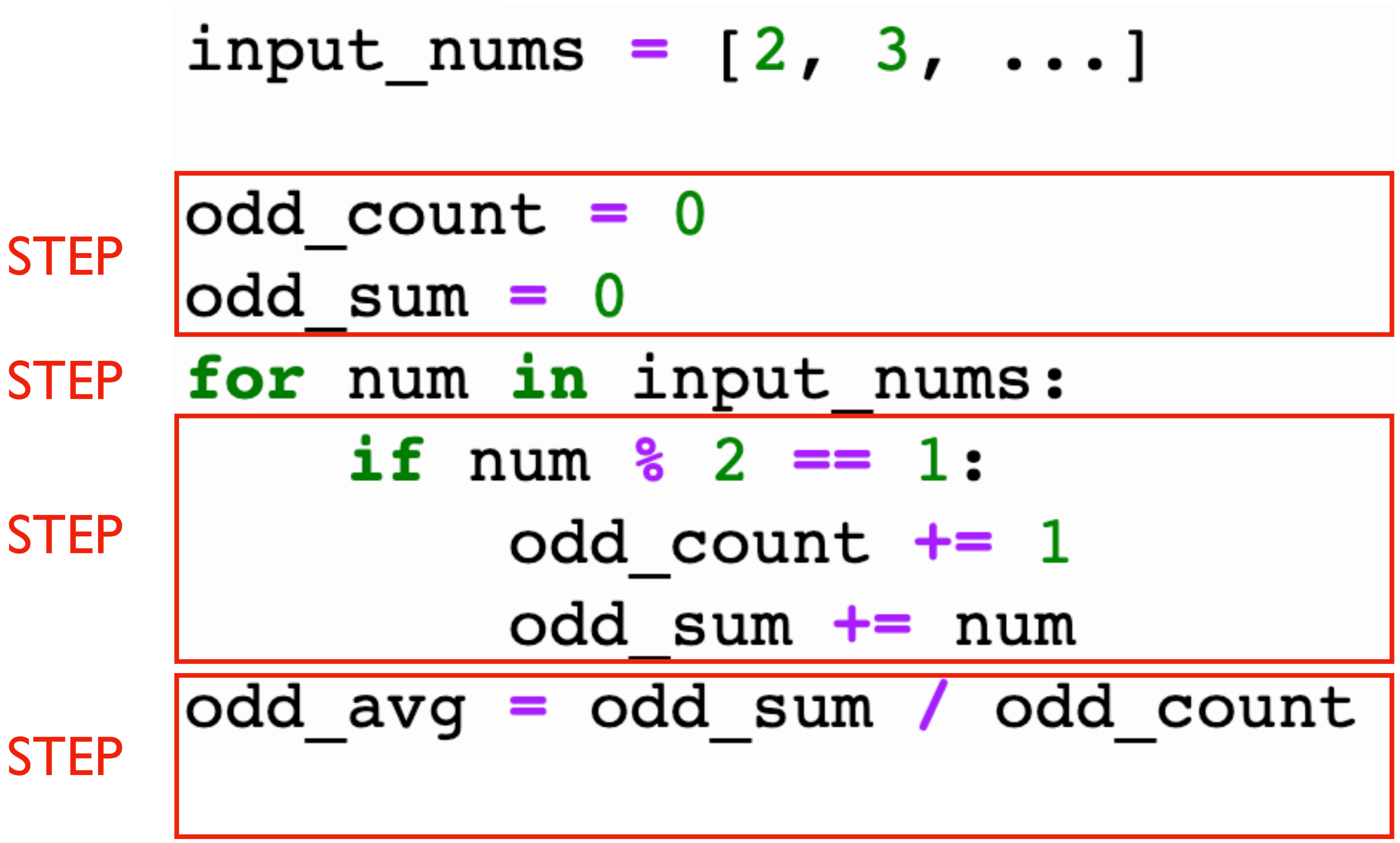
STEP

???

What is a step?

"bounded" doesn't mean "fixed"

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size)



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✓

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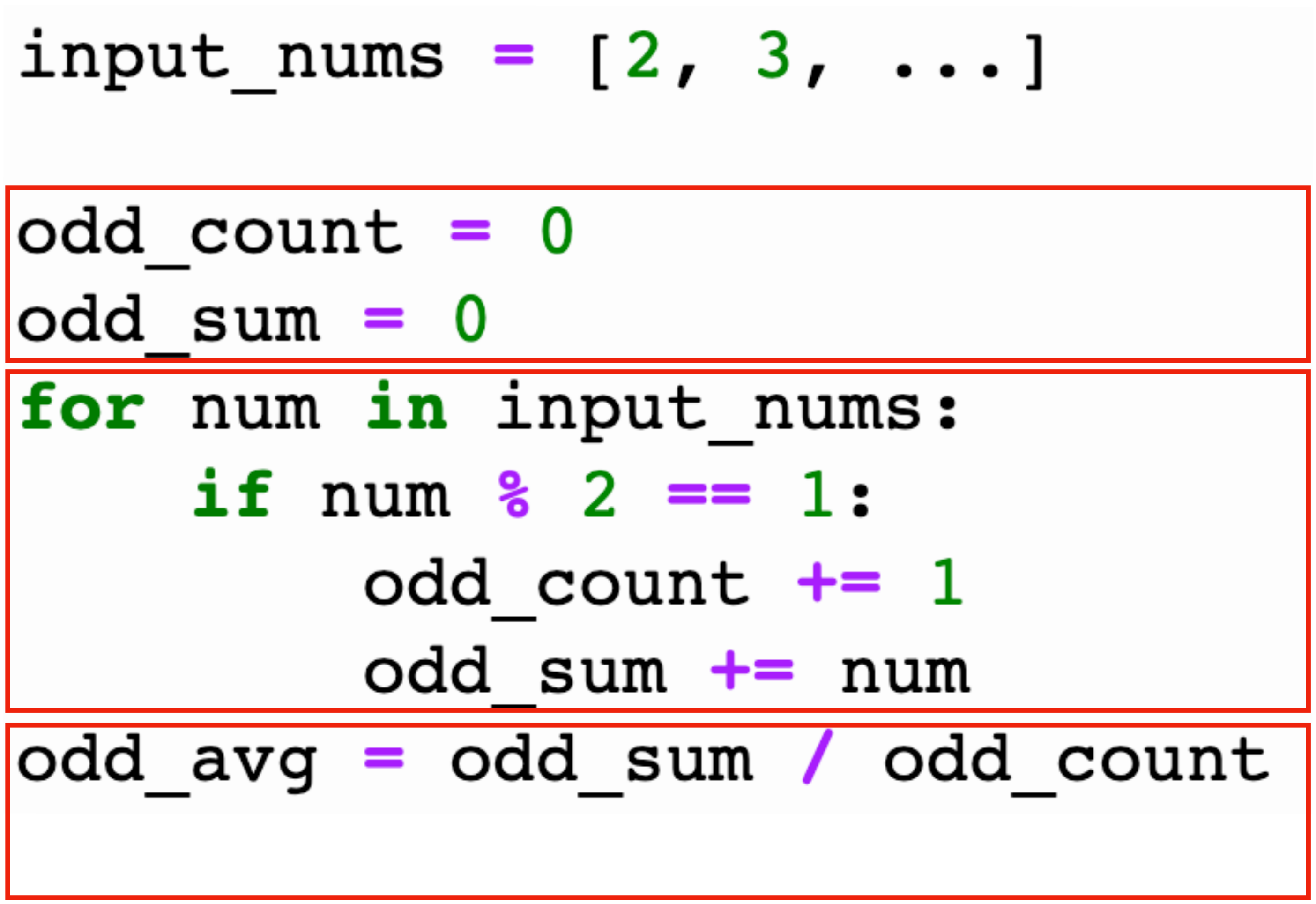

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```

STEP

STEP (whole loop execution, not one pass through)

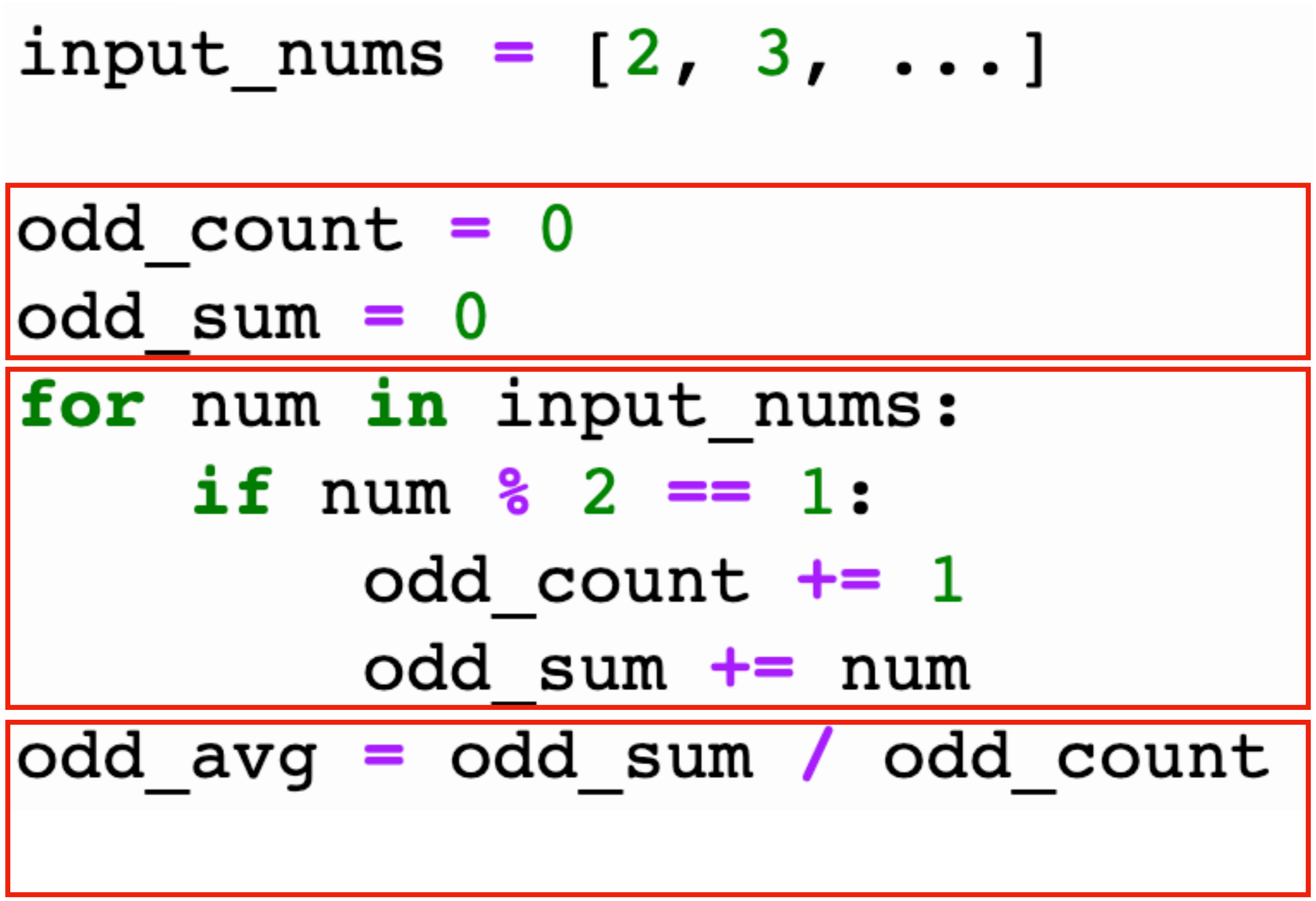
STEP

???

is this a valid way to identify steps?

What is a step?

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size)



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not a "step", because exec time depends on input size

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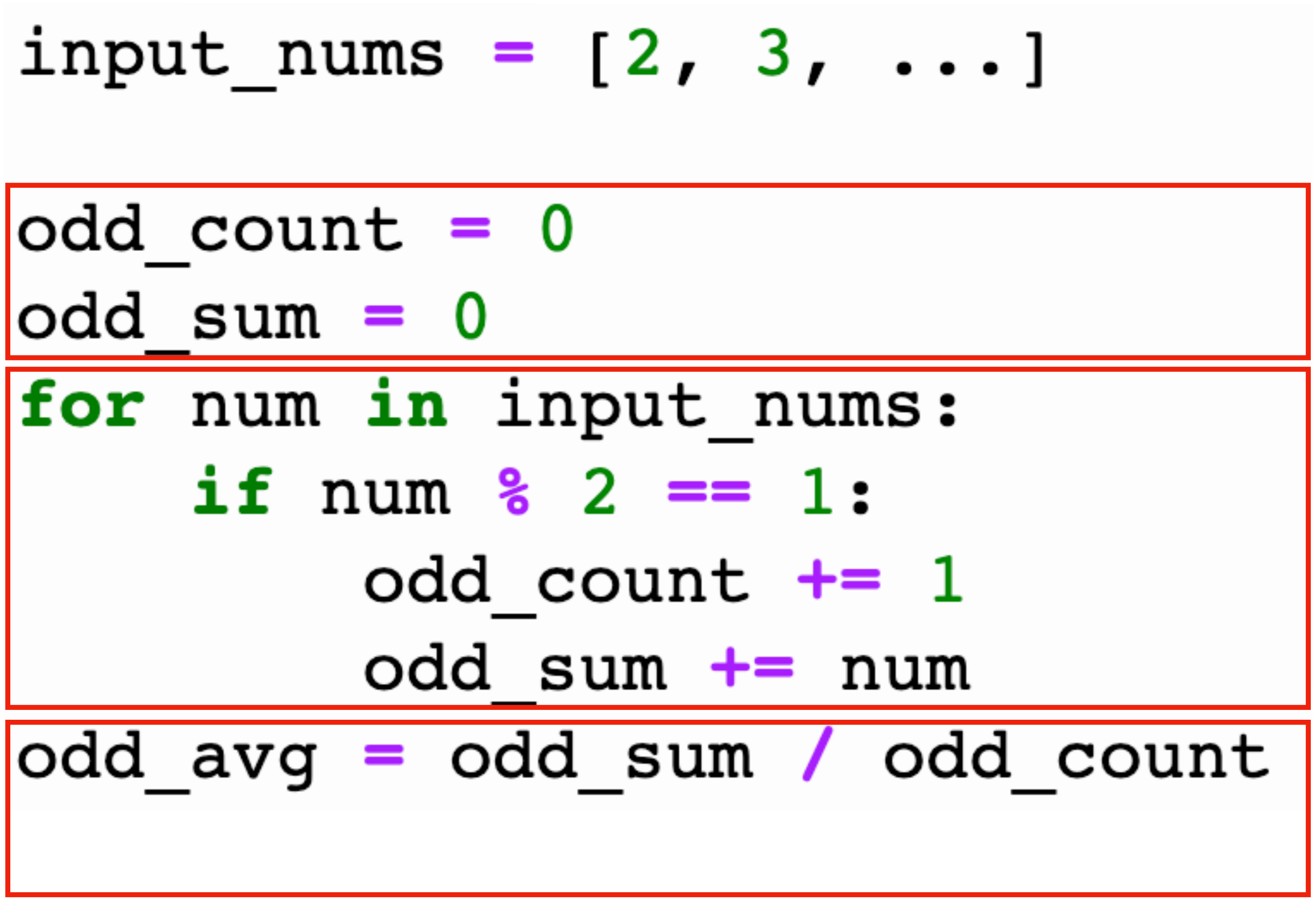

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STEP

not a "step", because exec time depends on input size

STEP

(whole loop execution, not one pass through)

STEP

Note! A loop that iterates a bounded number of times (not proportional to input size) COULD be a single step.

Outline

Performance and Complexity

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Big O: for functions/curves

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Counting Executed Steps

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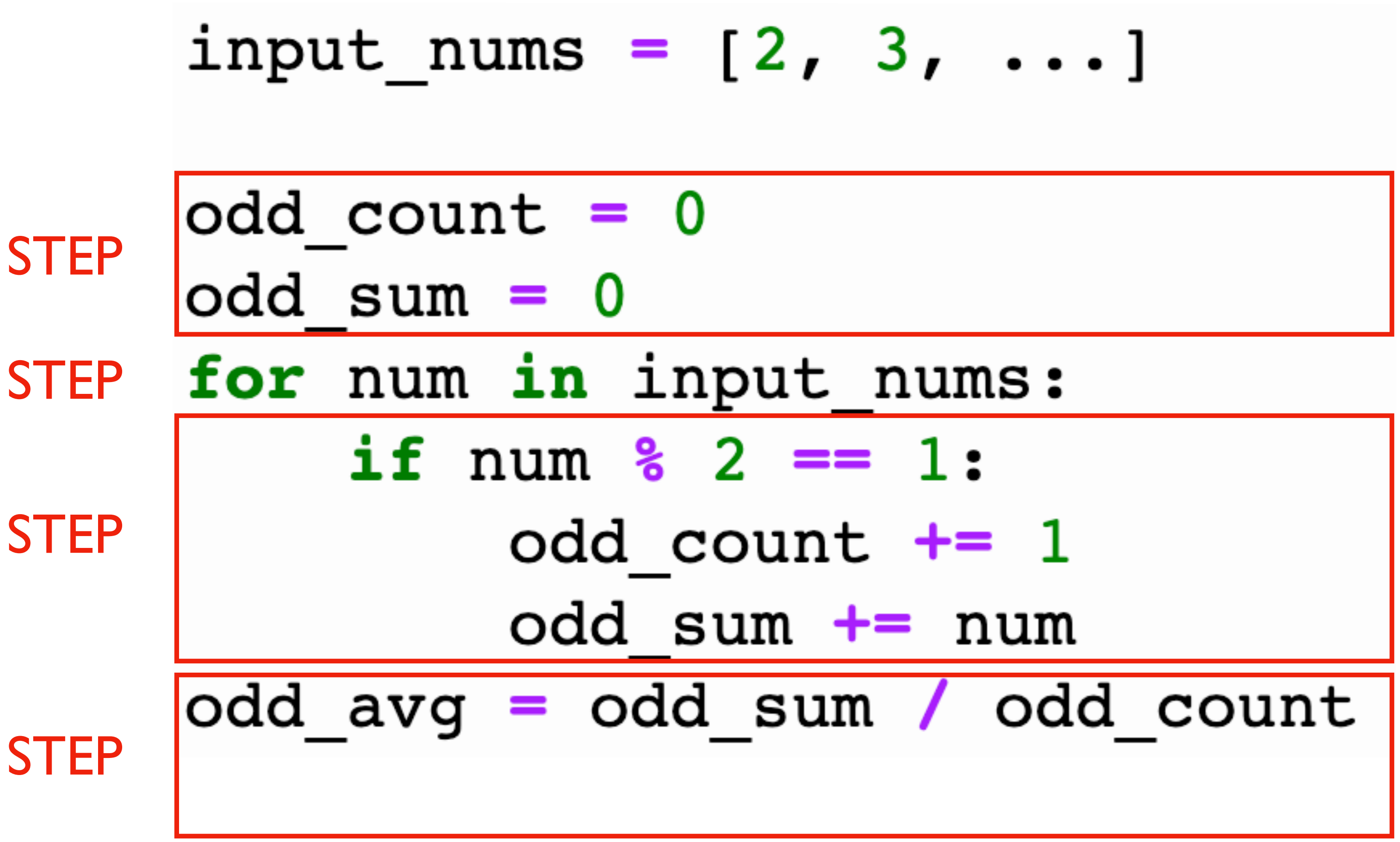

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How many total steps will **execute** if `len(input_nums) == 10`?

Counting Executed Steps

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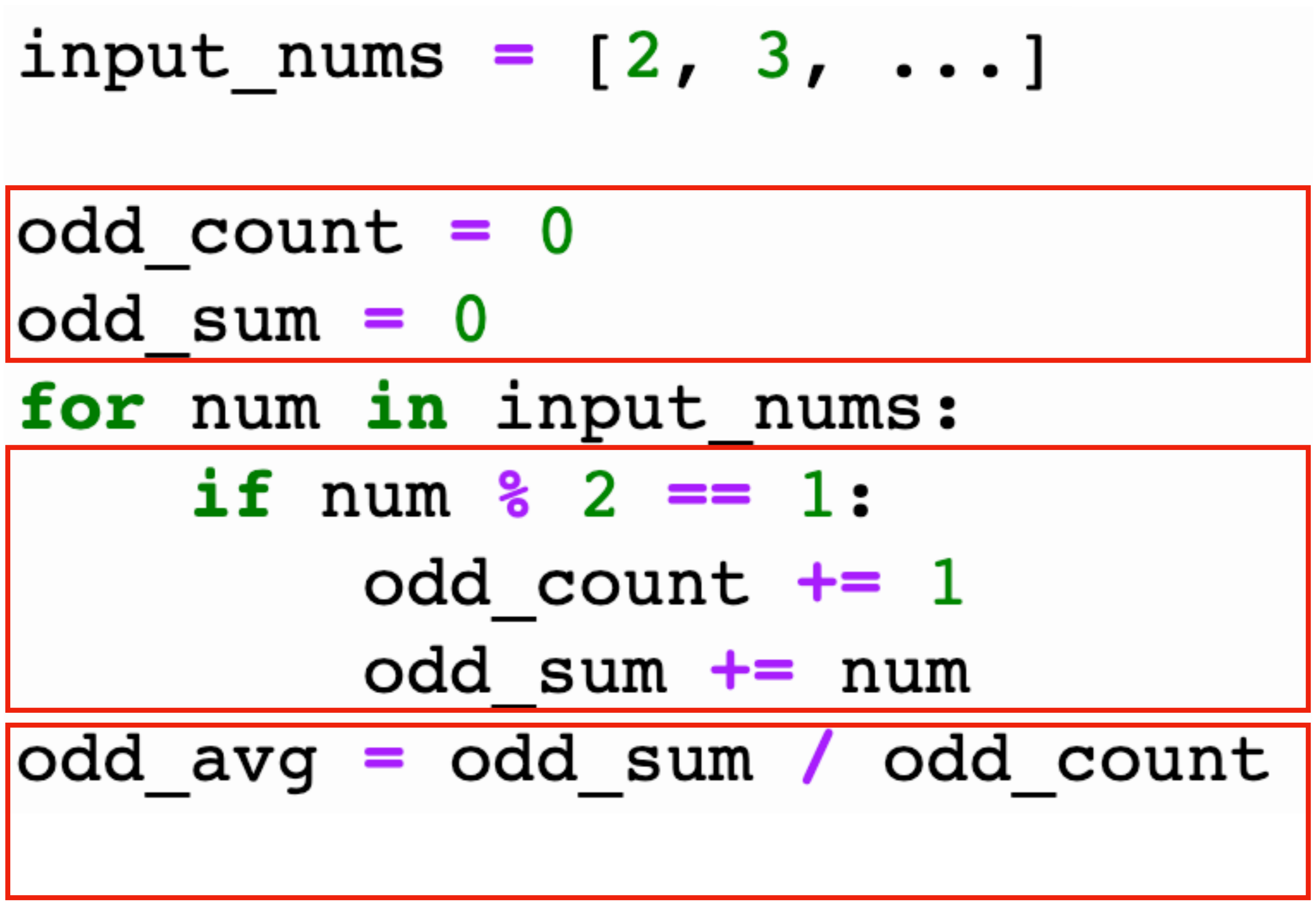

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```

1 STEP

+ 11 STEP

+ 10 STEP

+ 1 STEP

= 23 steps

For **N** elements, there will be **2*N+3** steps

Counting Executed Steps

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size)

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     11   STEP   for num in input_nums:
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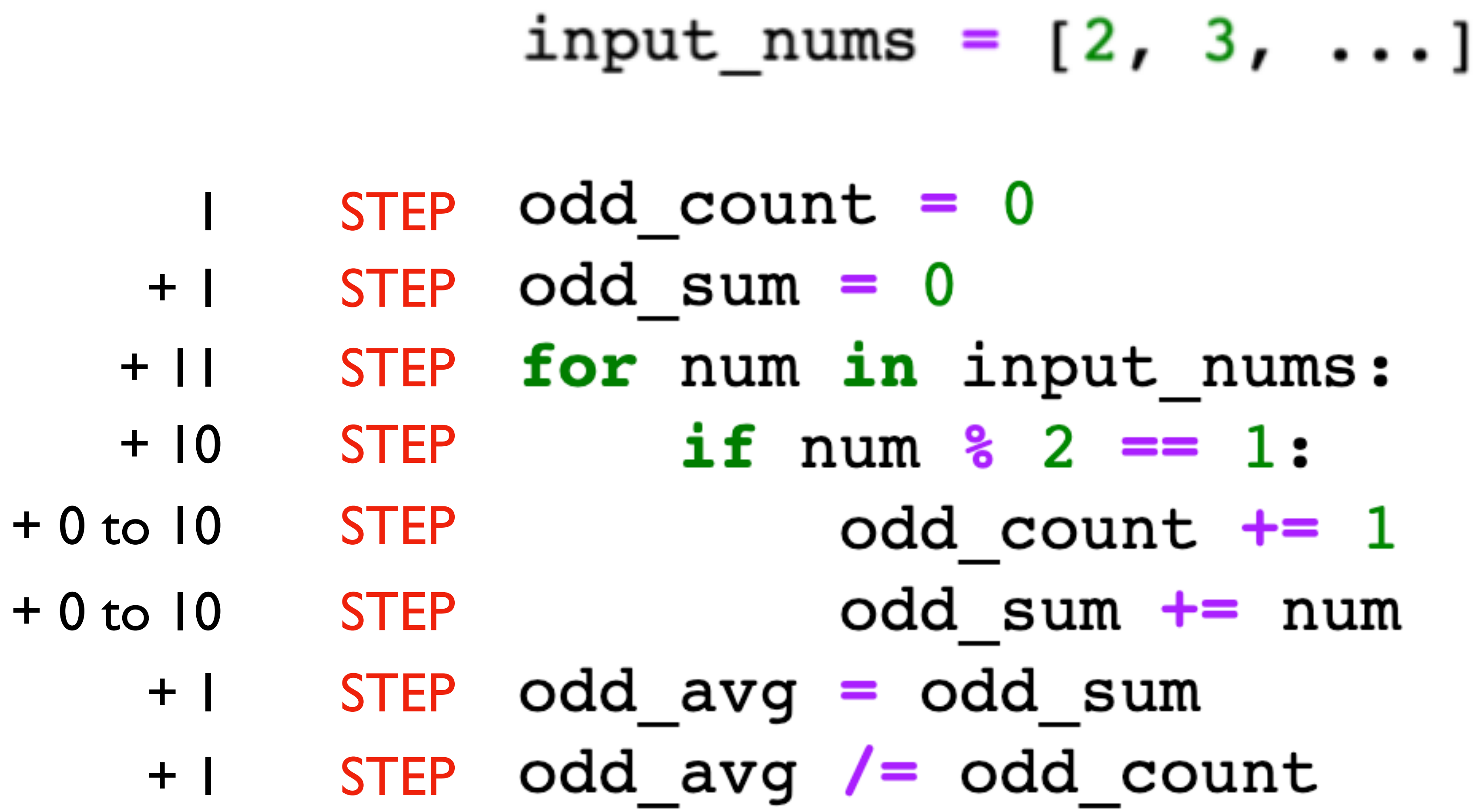

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For **N** elements, there will be between
2*N+5 and **4*N+5** steps

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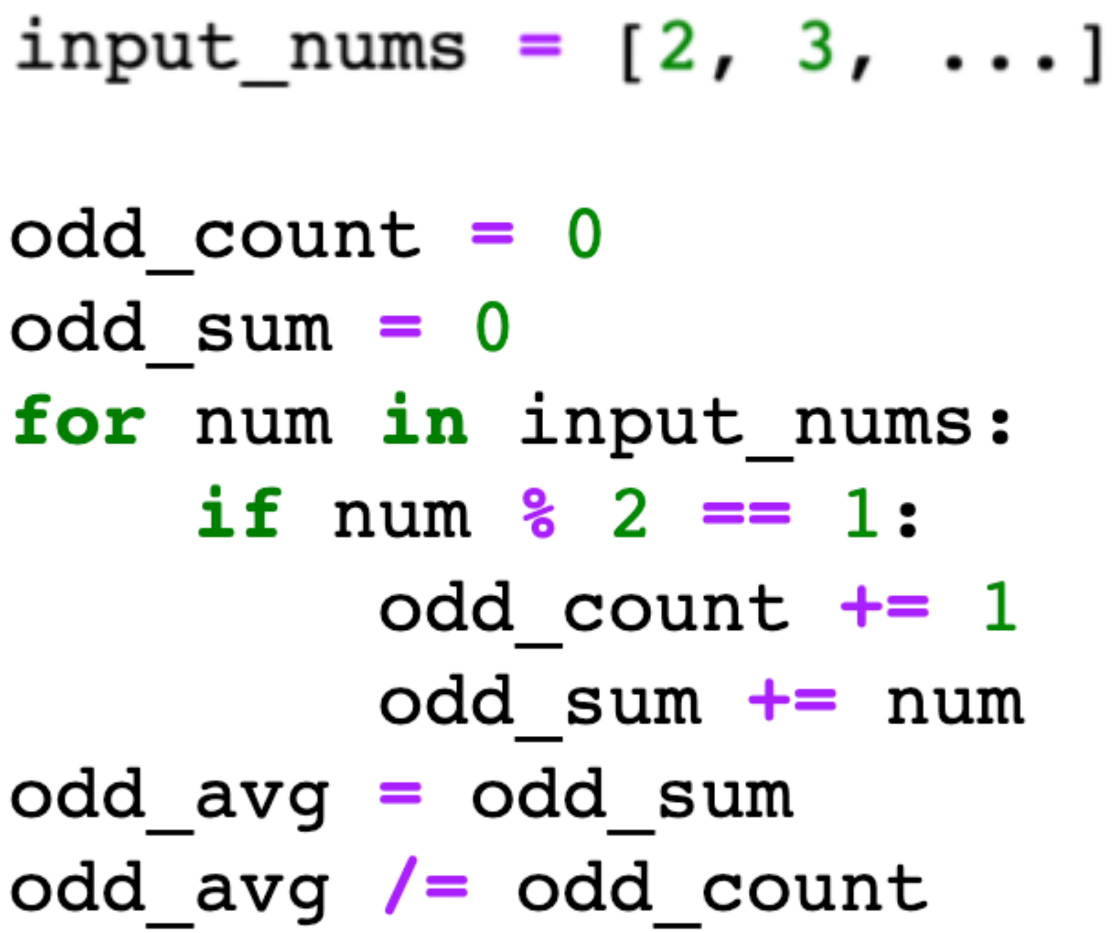

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For **N** elements, there will be between ~~2*N+5~~ and **4*N+5** steps

usually we care about the worst case

Counting Executed Steps

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size)

2*N+3 **OR** 4*N+5

answer 1 answer 2

Answer 2 is never bigger than 2 times answer 1.
Answer 1 is never bigger than answer 2.

Important: we might not identify steps the same, but our execution counts can at most differ by a constant factor! → can we broadly (but rigorously) categorize based on this?

Outline

Performance and Complexity

What is a step?

Counting Executed Steps

Big O: for functions/curves

Big O: for algorithms

How fast?

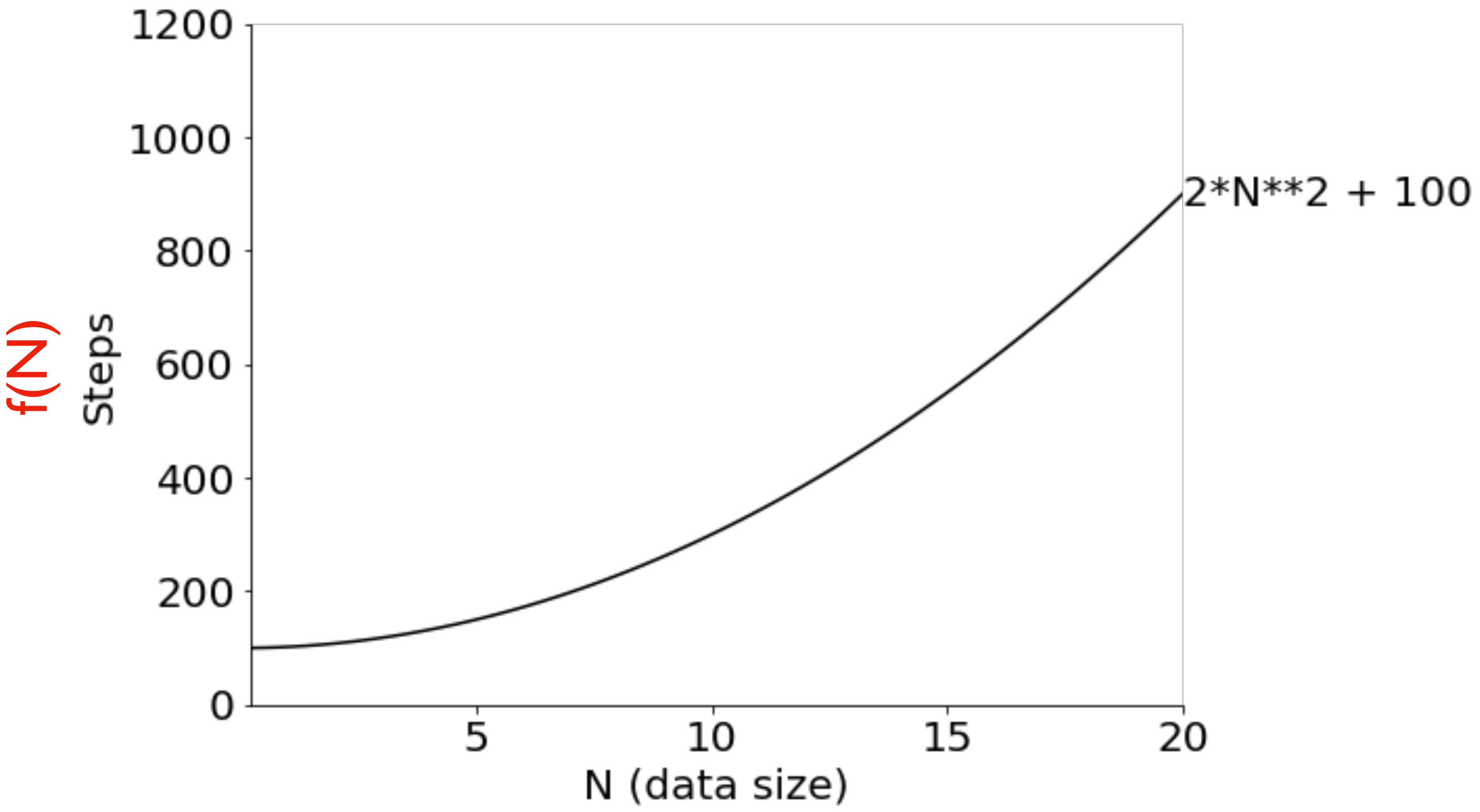
Documentation

- https://scikit-learn.org/stable/modules/linear_model.html#ordinary-least-squares-complexity
- https://scikit-learn.org/stable/modules/tree.html#complexity

Big O Notation ("O" is for "order of growth")

Goal: categorize functions (and algorithms) by how fast they **grow**

- **do not care** about scale
- **do not care** about small inputs
- **care** about shape of the curve
- **strategy:** find some multiple of a general function that is an upper bound



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is an **$O(N^2)$** function

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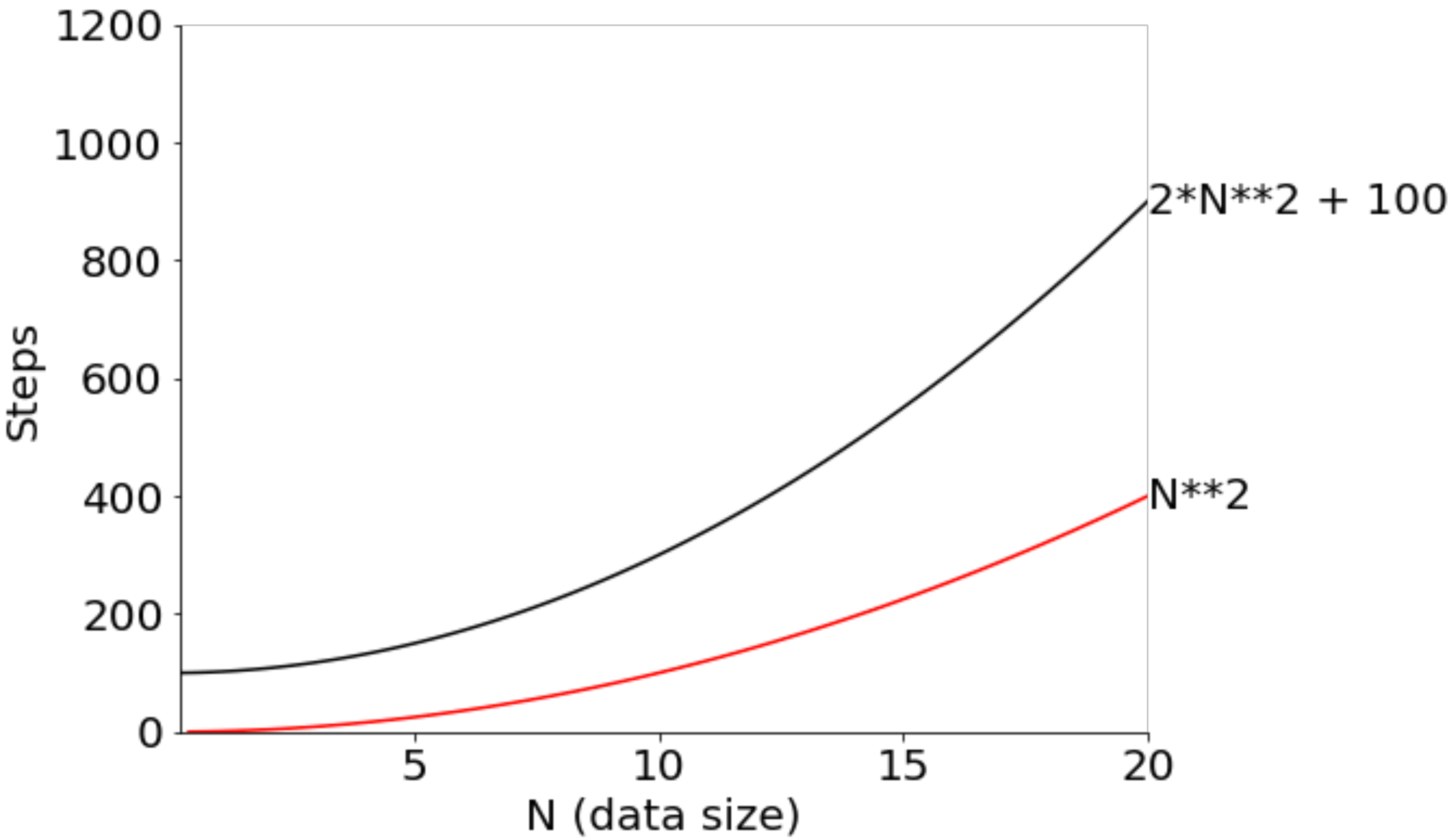

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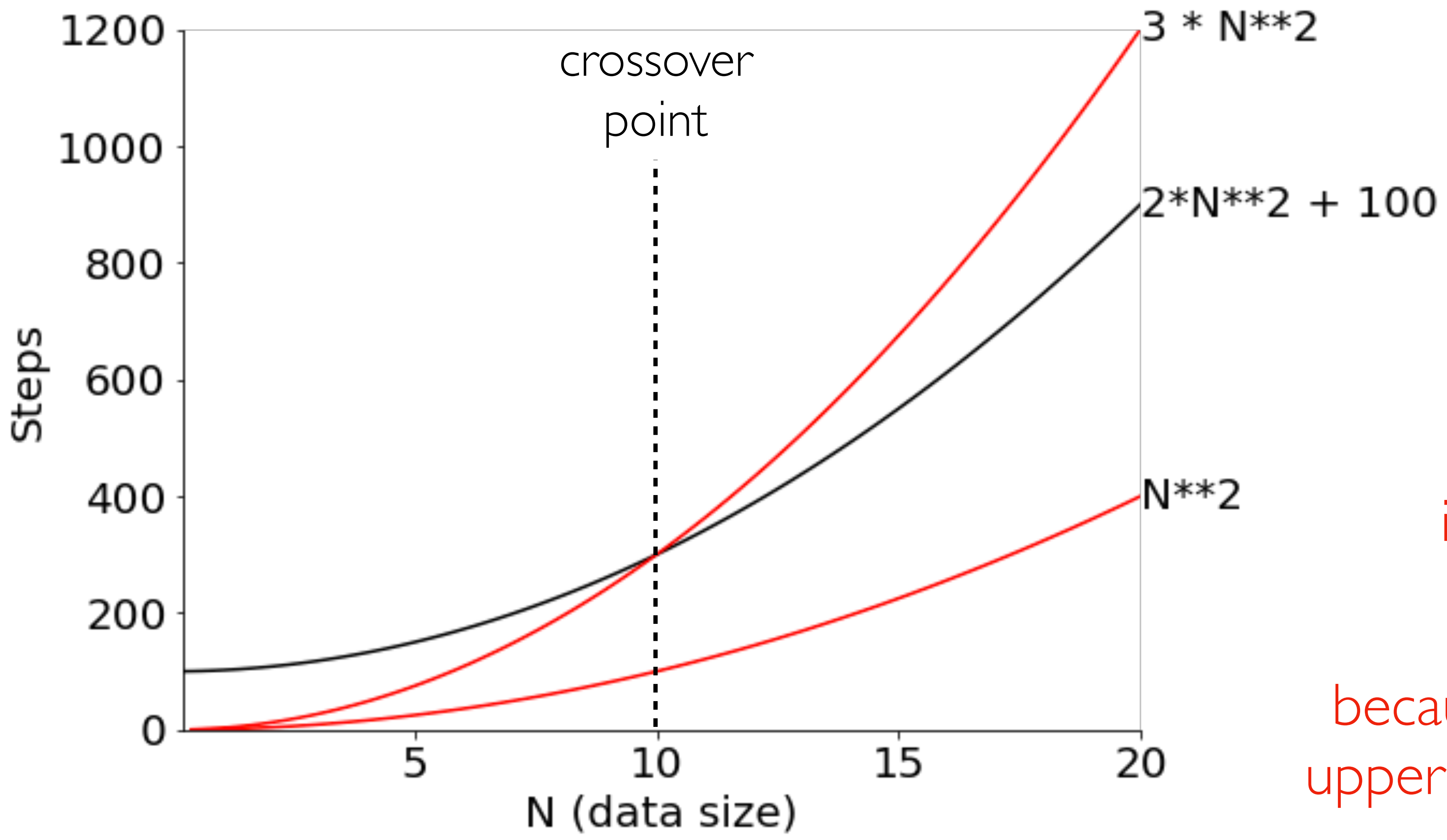

f(N) == $2N^2 + 100$
is an **O(N^2)** function

not because **N^2**
is an upper bound

because **some multiple** is an
upper bound after **some point**

Defining Big O

care about shape of the curve

do not care about small inputs

do not care about scale

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

Defining Big O

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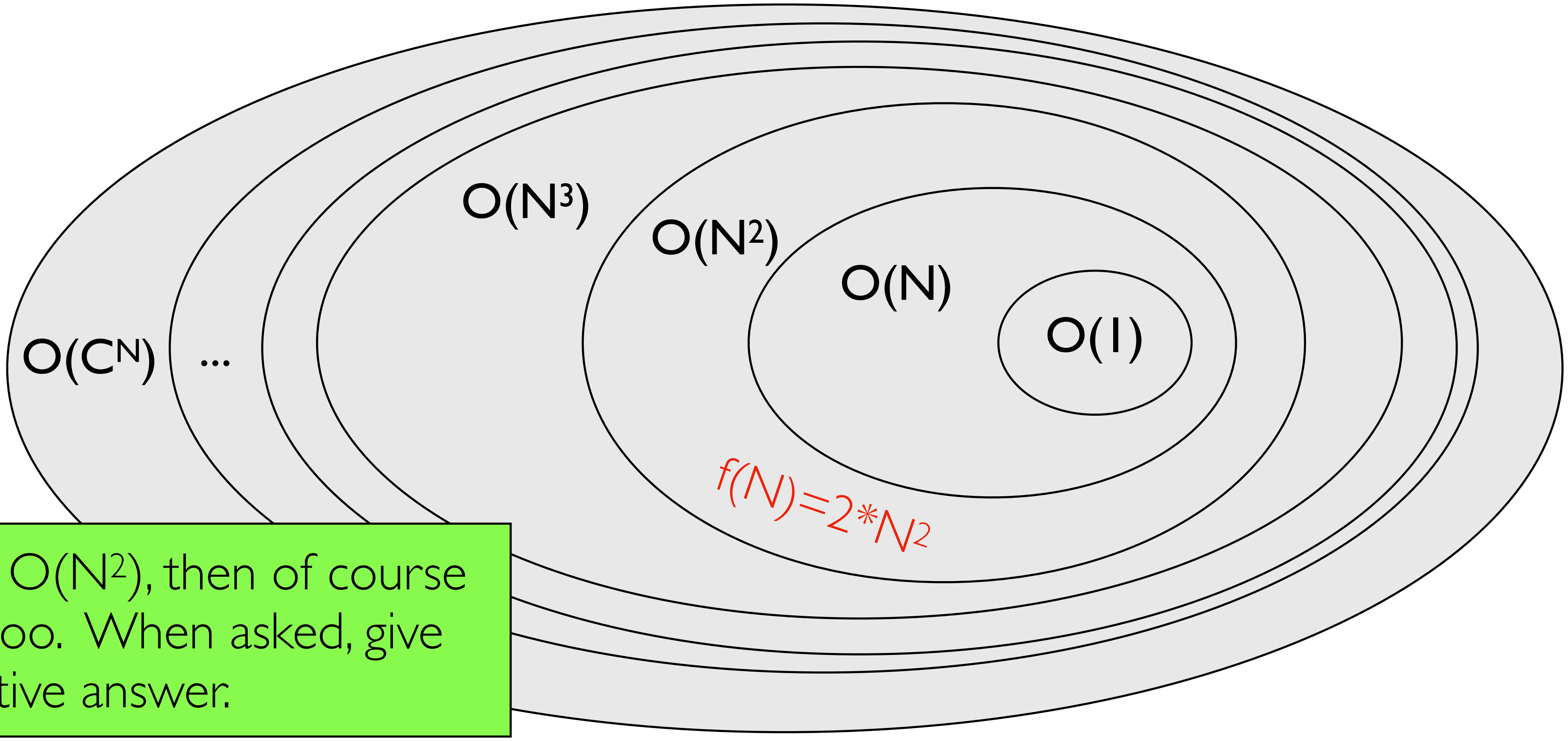
Sets



Note: if f(N) is in $O(N^2)$, then of course f(N) is in $O(N^3)$ too. When asked, give the most informative answer.

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

---

which ones are true?

$f(N) = 2N \in O(N)$

$f(N) = 100N \in O(N^2)$

$f(N) = N^2 \in O(1000000N)$

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

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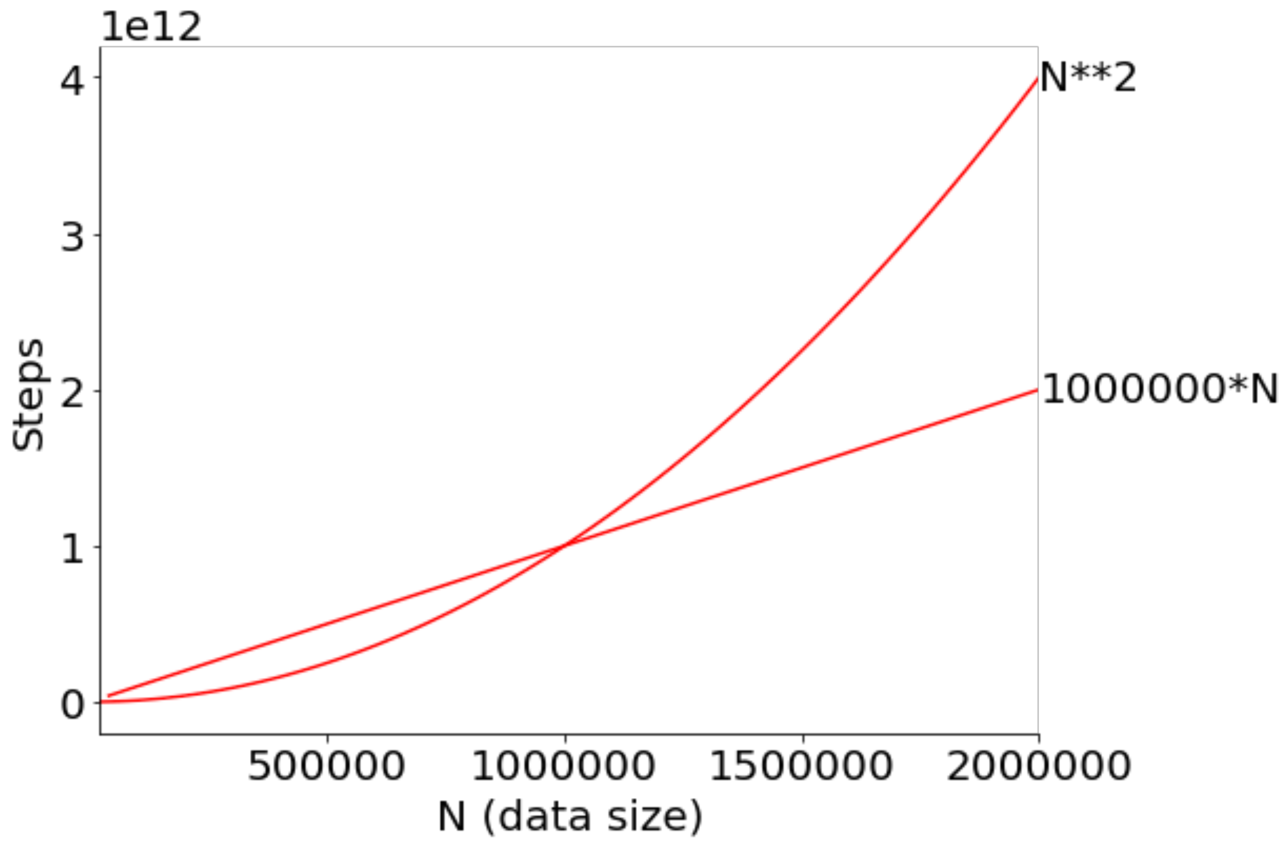
~~$f(N) = N^2 \in O(1000000N)$~~



Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

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---

shortcuts

$$O(3N^5 + N^4 + 8N^3 + 3N^2 + N + 5)$$

keep leading term
(if finite number)

$$O(3N^5)$$

drop coefficients

$$O(N^5)$$

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

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worksheet: problem 2

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Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed constant C*

Then $f(N) \in O(g(N))$

We'll let f(N) be the number of steps that some Algorithm A needs to perform for input size N.

When we say Algorithm A ∈ O(g(N)), we mean that f(N) ∈ O(g(N))

Defining Big O

If $f(N) \le C * g(N)$ *for large N values and some fixed constant C*

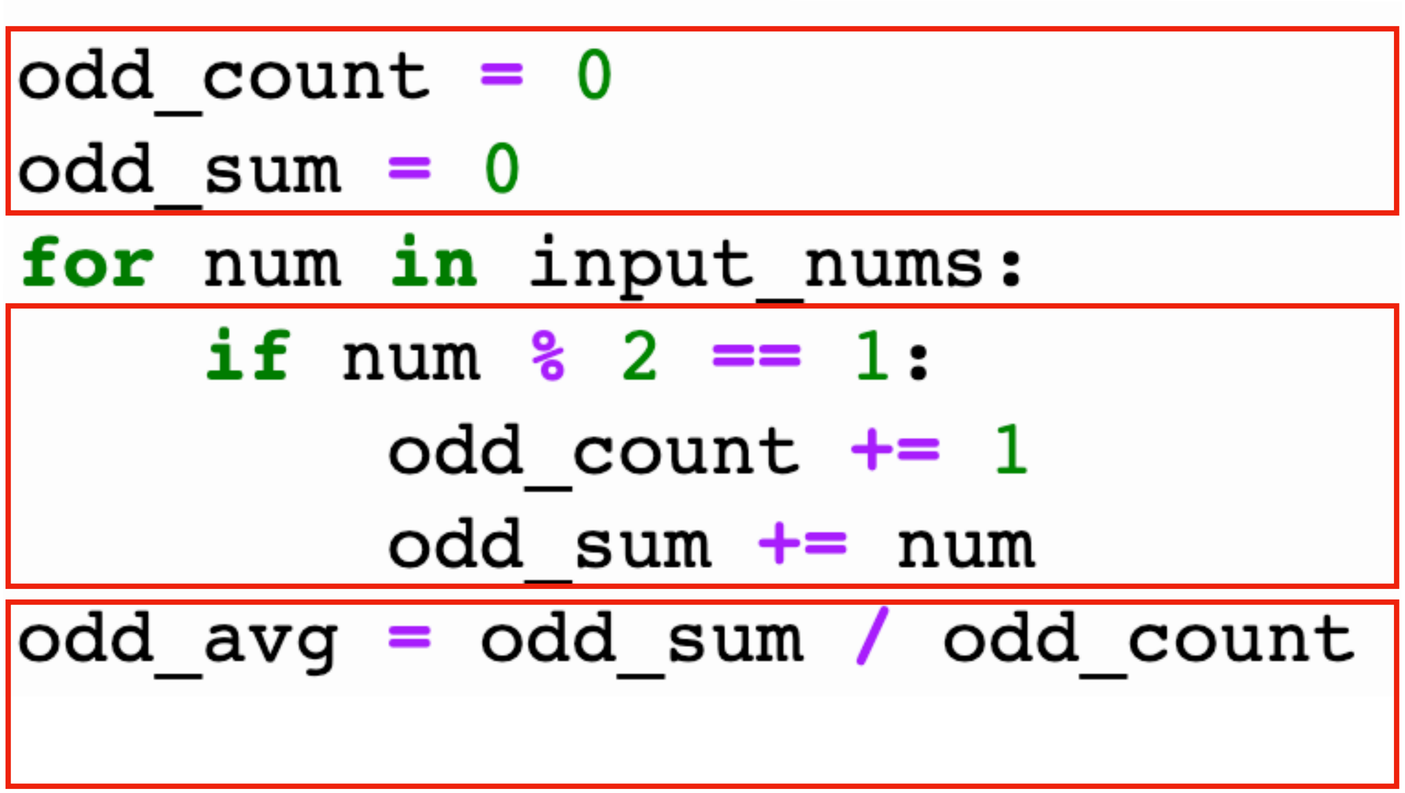
Then $f(N) \in O(g(N))$

STEP

STEP

STEP

STEP



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```

For N elements, there will be 2*N+3 steps

$2*N+3 \le 3 * N$

[for big N values]

therefore

this code is O(N)

Defining Big O

If $f(N) \leq C * g(N)$ *for large N values and some fixed <u>constant</u> C*

Then $f(N) \in O(g(N))$

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STEP  for num in input_nums:
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STEP  odd_avg = odd_sum
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```
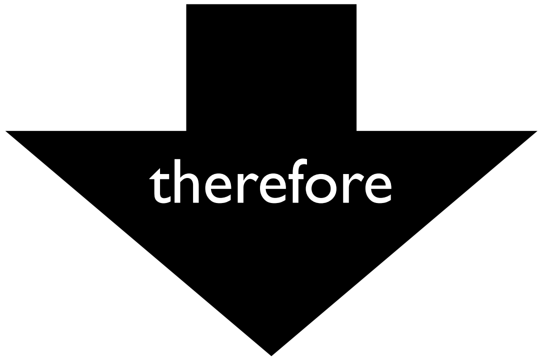

For N elements, there will be between 2*N+5 and 4*N+5 steps

$4*N+5 \leq 5 * N$

[for big N values]


therefore

this code is O(N)

Analysis of Algorithms: Key Ideas

complexity: relationship between input size and steps executed

step: an operation of bounded cost (doesn't scale with input size)

asymptotic analysis: we only care about very large N values for complexity (for example, assume a big list)

worst-case: we'll usually assume the worst arrangement of data because it's harder to do an average case analysis (for example, assume search target at the end of a list)

big O: if $\mathbf{f(N) \le C * g(N)}$ for large N values and some fixed constant $\mathbf{C}$,
then $\mathbf{f(N) \in O(g(N))}$