Worksheet: Complexity Analysis

1

```
def search(L, target):
    for x in L:
        if x == target: #line A
            return True
    return False
```

assume this is asked unless otherwise stated

Let **f(N)** be the number of times line A executes, with `N=len(L)`. What is **f(N)** in each case?

Worst Case (target is at end of list): f(N) = _____

Best Case (target is at beginning of list): f(N) = _____

Average Case (target in middle of list): f(N) = _____

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size).

We classify algorithm complexity by classifying the **order of growth** of a function f(N), where f gives the number of steps the algorithm must perform for a given input size.

Big O definition: if **f(N) ≤ C * g(N)** for large **N** values and some fixed constant **C**, then **f(N) ∈ O(g(N))**

2

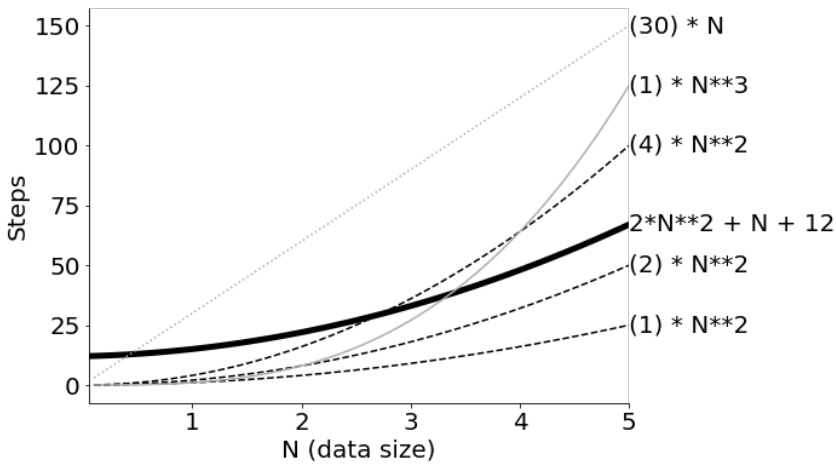
Let **f(N) = 2N² + N + 12**

If we want to show **f(N) ∈ O(N³)**, what is a good lower bound on N? Let's have C=1.

To show **f(N) ∈ O(N²)**, do we pick 1, 2, or 4 for the C? After picking C, what should we choose for N's lower bound?

What is more informative to show?
f(N) ∈ O(N³) or **f(N) ∈ O(N²)**?

Somebody claims **f(N) ∈ O(N)**, offering C=30 and N>0. Suggest an N value to counter their claim.



3

```
nums = [...]

first100sum = 0

for x in nums[:100]:
    first100sum += x
print(first100sum)
```

If we increase the size of nums from 20 items to 100 items, the code will probably take _____ times longer to run.

If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

The complexity of the code is O(_____), with N=len(nums).

4

Each of the following list operations are either **O(1)** or **O(N)**, where N is len(L). Circle those you think are **O(N)**.

```
L.insert(0, x)    L.pop(0)     x = L[0]      x = max(L)     x = len(L)

L.append(x)       L.pop(-1)    L2.extend(L)  x = sum(L)     found = X in L
```

5

```
L = [...]
for x in L:
    avg = sum(L) / len(L)
    if x > 2*avg:
        print("outlier", x)
```

What is the big O complexity?

Is there a way to optimize the code?

6

```
A = [...]
B = [...]

for x in A:
    for y in B:
        print(x*y)
```

how would you define the variable(s) to describe the size of the input data?

The complexity of code is

O(_____)

7

```
def merge(A, B):
    combined = []
    while len(A) > 0 or len(B) > 0:
        if len(B) == 0:
            combined.append(A.pop())
        elif len(A) == 0:
            combined.append(B.pop())
        elif A[-1] > B[-1]:
            combined.append(A.pop())
        else:
            combined.append(B.pop())
    combined.reverse() # assume O(N)
    return combined
```

without arguments, pop removes (and returns) the last value.

The complexity of code is

O(_____)

8

```
def selection_sort(L):
    for i in range(len(L)):
        idx_min = i
        for j in range(i, len(L)):
            if L[j] < L[idx_min]:
                idx_min = j
        # swap values at i and idx_min
        L[idx_min], L[i] = L[i], L[idx_min]

nums = [2, 4, 3, 1]
selection_sort(nums)
print(nums)
```

if this runs **f(N)** times, where **N=len(L)**,

then **f(N)** = _____

The complexity of selection sort is

O(_____)

9

```
# assume L is already sorted, N=len(L)
def binary_search(L, target):
    left_idx = 0 # inclusive
    right_idx = len(L) # exclusive
    while right_idx - left_idx > 1:
        mid_idx = (right_idx + left_idx) // 2
        mid = L[mid_idx]
        if target >= mid:
            left_idx = mid_idx
        else:
            right_idx = mid_idx

    return right_idx > left_idx and L[left_idx] == target
```

how many times does this step run when **N = 1? N = 2? N = 4? N = 8?**

If **f(N)** is the number of times this step runs, then f(N) = _____

The complexity of binary search is

O(_____)